QUANTUM MECHANICS WITH NEUTRAL KAONS* **

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We briefly illustrate a few tests of quantum mechanics which can be performed with entangled neutral kaon pairs at a Φ-factory. This includes a quantitative formulation of Bohr's complementarity principle, the quantum eraser phenomenon and various forms of Bell inequalities.

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1. Introduction

Some of the *Gedanken*-experiments discussed in the early days of Quantum Mechanics (QM) by its founding fathers have been recently reanalyzed in their original form or in slightly modified versions. These reanalyses have allowed an experimental confirmation of the QM predictions and deep insights into the heart of QM. Good examples are recent researches on basic subjects such as Bohr's complementarity principle or other subtle QM issues which featured when the theory had already been completed, *e.g.*, the

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so called Einstein, Podolsky and Rosen paradox and the closely related subject of Bell inequalities. Most of this progress has been achieved thanks to the many advances in quantum optics and photonic experiments, however, the improved technologies also allowed to perform such tests with atoms and ions.

The purpose of the present contribution is to show that high energy physics systems, such as kaons, can also be considered to discuss basic questions of QM and that sometimes they are more instructive and simple than their photonic or atomic analogues.

We review a series of recent papers dealing with QM issues using neutral kaons as the relevant quantum states. We start in Sec. 2 with a short summary of the required neutral kaon formalism paying attention to the quantum measurements and to entangled kaon states. In Sec. 3 we discuss a quantitative version of Bohr's complementarity for single neutral kaon states. Entangled kaon pairs are then used to introduce a kaonic "quantum eraser" in Sec. 4 which may be tested at the Φ-factory. We then end with the subtle topic of Bell inequalities with neutral kaons in Sec. 5.

2. The neutral kaon system

2.1. Two bases: $\{K^0, \bar{K}^0\}$ *and* $\{K_{\rm S}, K_{\rm L}\}$

Neutral kaons are pseudoscalar mesons consisting of a quark–antiquark bound state, $K^0 \sim d\bar{s}$ and $\bar{K}^0 \sim s\bar{d}$. These two states define the "strangeness" or "strong-interaction" basis: $|K^0\rangle$ and $|\bar{K}^0\rangle$ with strangeness $S = +1$ and $S = -1$, respectively. This is the suitable basis to analyze S-conserving electromagnetic and strong interaction processes, such as the creation of $K^0\bar{K}^0$ systems from non-strange initial states (*e.g.*, $e^+e^- \to \phi(1020) \to K^0\bar{K}^0$ and $p\bar{p} \to K^0\bar{K}^0$), and the detection of neutral kaons via strong kaon–nucleon interactions. This "strangeness" basis is orthonormal, $\langle K^0|\bar{K}^0\rangle = 0$.

Weak interaction phenomena allow for strangeness non-conservation thus introducing new effects — such as K^0–$\bar{K}^0$ oscillations — as well as neutral kaon time evolution and decay. All these phenomena, together with kaon propagation in a medium with its associated regeneration effects, require the use of other relevant bases.

The "free-space" basis, $\{K_{\rm S}, K_{\rm L}\}$, consists of the so called K-short and K-long states which are the normalized eigenvectors of the effective weak Hamiltonian $H_{\rm free}$ governing neutral kaon time evolution in free-space:

$$i\frac{d}{d\tau}|K_{\rm S,L}(\tau)\rangle = H_{\rm free}\,|K_{\rm S,L}(\tau)\rangle\,, \quad H_{\rm free} = \begin{pmatrix} \lambda_+ & \lambda_-/r \\ r\lambda_- & \lambda_+ \end{pmatrix}, \tag{1}$$

where $r \equiv (1-\epsilon)/(1+\epsilon)$, ϵ is the CP violation parameter [1] and τ is the kaon proper time. The (complex) eigenvalues of the (non-hermitian) Hamiltonian

$H_{\rm free}$ are $\lambda_{\rm S,L} = \lambda_+ \pm \lambda_- = m_{\rm S,L} - i\Gamma_{\rm S,L}/2$, where $m_{\rm S,L}$ and $\Gamma_{\rm S,L}$ are the masses and decay widths. The eigenstates are

$$|K_{\rm S,L}\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle \pm (1-\epsilon)|\bar{K}^0\rangle\right] \to \frac{1}{\sqrt{2}}\left[|K^0\rangle \pm |\bar{K}^0\rangle\right] ,$$

with $|K_{\rm S,L}\rangle \equiv |K_{\rm S,L}(\tau = 0)\rangle$. In the final expressions, CP-violating effects have been ignored. This is reasonable due to the smallness of the CP violation parameter ϵ and defines the two orthogonal CP eigenstates $|K_1\rangle$ (CP $= +1$) and $|K_2\rangle$ (CP $= -1$). The $K_{\rm S,L}$ time evolution shows no oscillation between these two states and, according to Eq. (1), it is given by

$$|K_{\rm S,L}(\tau)\rangle = e^{-im_{\rm S,L}\tau} e^{-\frac{1}{2}\Gamma_{\rm S,L}\tau}|K_{\rm S,L}\rangle \equiv e^{-i\lambda_{\rm S,L}\tau}|K_{\rm S,L}\rangle \,. \tag{2}$$

The $|K_{\rm S,L}\rangle$ states define a quasi-orthonormal basis: $\langle K_{\rm S}|K_{\rm S}\rangle = \langle K_{\rm L}|K_{\rm L}\rangle = 1$ and $\langle K_{\rm S}|K_{\rm L}\rangle = \langle K_{\rm L}|K_{\rm S}\rangle = 2{\rm Re}\{\epsilon\}/(1+|\epsilon|^2) \simeq 0$. While the $\{K_{\rm S}, K_{\rm L}\}$ basis is useful to discuss free–space propagation, the CP basis describes weak kaon decays either into two or three final pions from the $K_1 \simeq K_{\rm S}$ or $K_2 \simeq K_{\rm L}$ states, respectively.

2.2. Two measurements: strangeness or lifetime

A generic neutral-kaon state is a "qubit" in a "quasispin space", *i.e.*, a quantum superposition of the two states of any of the previous bases, $\{K^0, \bar{K}^0\}$ or $\{K_{\rm S}, K_{\rm L}\}$, associated to the strangeness or lifetime quantum measurements, respectively. The former measurement requires the introduction of a nucleonic medium in the kaon trajectory, the latter is performed by allowing for kaon free-space propagation. Indeed, when a kaon–nucleon reaction occurs at a given place of the inserted medium, the distinct strong interactions of the $S = +1$ and $S = -1$ components on the bound nucleons inside the medium project the arbitrary state of an incoming kaon into one of the two members of the strangeness basis [2]. The quantum number S of the kaon state is determined by identifying the products of the strangeness conserving kaon–nucleon strong interaction. This measurement is then analogous to the projective von Neumann measurements with two-channel analyzers for polarized photons or Stern–Gerlach setups for spin-1/2 particles. Unfortunately, the efficiency for such strangeness measurements at moderate kaon energies as in $\phi \to K^0\bar{K}^0$ and $p\bar{p} \to K^0\bar{K}^0$ is certainly less than what one naively expects from the strong nature of these interactions [2, 3]. The reason, rather than being the difficulty in detecting the final state particles, stems from the low probability in initiating the strong reaction. Indeed, the efficiency to *induce* a kaon–nucleon interaction at a given time turns out to be close to 1 only for infinitely dense materials or for ultrarelativistic kaons.

To measure if a kaon is propagating in free-space as a $K_{\rm S}$ or $K_{\rm L}$ at a given time τ, one has to allow for further propagation in free-space and then detect at which time it subsequently decays. Kaons which show a decay vertex between times τ and $\tau + \Delta\tau$ have to be identified as $K_{\rm S}$'s, while those decaying later than $\tau + \Delta\tau$ have to be identified as $K_{\rm L}$'s. Since there are no $K_{\rm S}$–$K_{\rm L}$ oscillations, such subsequent decays do really identify the state at the desired previous time τ. The probabilities for wrong $K_{\rm S}$ and $K_{\rm L}$ identification are then given by $\exp(-\Gamma_{\rm S}\,\Delta\tau)$ and $1-\exp(-\Gamma_{\rm L}\,\Delta\tau)$, respectively. Choosing $\Delta\tau = 4.8\,\tau_{\rm S}$, both misidentification probabilities reduce to $\simeq 0.8\%$. Since the lifetime eigenstates are not strictly orthogonal to each other, their identification cannot be exact even in principle. However, ϵ is so small and the decay probabilities of the two components so different ($\Gamma_{\rm S} \simeq 579\,\Gamma_{\rm L}$) that the $K_{\rm S}$ *vs* $K_{\rm L}$ discrimination can effectively work [4, 5]. Note also that, contrary to strangeness measurements, $K_{\rm S}$ *vs* $K_{\rm L}$ identification is not affected by low efficiencies if detectors with very large solid angles are used.

2.3. Two measurement procedures: active or passive

The above measurement methods are appropriate to establish Bell inequalities and tests [6–10]. On the one hand, the two measurements correspond to complementary observables, with dichotomic outcomes in both cases. On the other hand, being performed by exerting the free will of the experimenter, they are *active* measurement procedures.

However, contrary to what happens in other two-level quantum systems, such as spin-1/2 particles or photons, *passive* measurements are also possible for neutral kaons [11] by randomly exploiting the QM dynamics of kaon decays. To this aim, one has to allow for complete free-space propagation and observe the various kaon decay modes. By neglecting CP violation effects, kaon decays into two and three pions permit the identification of $K_{\rm S}$'s and $K_{\rm L}$'s, respectively. Alternatively, the strangeness of a given kaon state is measured by observing its semileptonic decays. These decays obey the well tested $\Delta Q = \Delta S$ rule, which allows the modes $K^0 \to \pi^- l^+ \nu_{\rm l}$ and $\bar{K}^0 \to \pi^+ l^- \bar{\nu}_{\rm l}$, with $l = e, \mu$, but forbids decays into the respective charge-conjugate final states [1]. These procedures for the passive $K_{\rm S}$ *vs* $K_{\rm L}$ and K^0 *vs* $\bar{K}^0$ discriminations are unambiguous in the approximations given by CP conservation and the $\Delta Q = \Delta S$ rule, respectively. However, the experimenter has no control on the time when the measurement occurs, nor on the basis in which it is performed, in contrast with the previous active procedure. As a result, the so called Kasday construction [12] invalidates Bell inequality tests performed with such passive measurements [7].

2.4. Two-kaon systems: entanglement

The simplest and most often discussed bipartite state is the spin singlet state consisting of two photons or two spin-1/2 particles, as first proposed by Bohm [13]. Let us then first consider the analogous two-kaon entangled state [10, 14–16]. Both ϕ-resonance decays [17] and s-wave proton–antiproton annihilations [3] produce the $J^{\rm PC} = 1^{--}$ initial state:

$$
\begin{aligned}
|\phi(\tau=0)\rangle &= \frac{1}{\sqrt{2}}\left\{|K^0\rangle_{\rm l}|\bar{K}^0\rangle_{\rm r} - |\bar{K}^0\rangle_{\rm l}|K^0\rangle_{\rm r}\right\} \\
&= \frac{1}{\sqrt{2}}\frac{1+|\epsilon|^2}{|1-\epsilon^2|}\left\{|K_{\rm L}\rangle_{\rm l}|K_{\rm S}\rangle_{\rm r} - |K_{\rm S}\rangle_{\rm l}|K_{\rm L}\rangle_{\rm r}\right\}\ ,
\end{aligned} \tag{3}
$$

l and r denoting the "left" and "right" directions of motion of the two separating kaons. The weak, CP-violating effects enter only in the last equality.

After production, the left and right moving kaons evolve according to Eq. (2) up to left- and right-times $\tau_{\rm l}$ and $\tau_{\rm r}$, respectively. Once normalizing to surviving kaon pairs, this leads to the $\Delta\tau = \tau_{\rm l} - \tau_{\rm r}$ dependent state

$$
|\phi(\Delta\tau)\rangle = \frac{1}{\sqrt{1+e^{\Delta\Gamma\Delta t}}}\left\{|K_{\rm L}\rangle_{\rm l}|K_{\rm S}\rangle_{\rm r} - e^{i\Delta m\Delta\tau}e^{\frac{1}{2}\Delta\Gamma\Delta\tau}|K_{\rm S}\rangle_{\rm l}|K_{\rm L}\rangle_{\rm r}\right\}\ , \tag{4}
$$

in the lifetime basis, with $\Delta m \equiv m_{\rm L} - m_{\rm S}$ and $\Delta\Gamma \equiv \Gamma_{\rm L} - \Gamma_{\rm S}$. For $\Delta\tau = 0$ this state shows maximal entanglement in both lifetime and strangeness.

This state (4) is analogous to the polarization-entangled two-photon [idler (i) plus signal (s)] state used in different optical tests:

$$
|\Psi\rangle = \frac{1}{\sqrt{2}}\left\{|V\rangle_i|H\rangle_s - e^{i\Delta\phi}|H\rangle_i|V\rangle_s\right\}, \tag{5}
$$

where $\Delta\phi$ is an adjustable relative phase and V and H refer to vertical and horizontal polarizations. For entangled kaons, Δm plays the role of $\Delta\phi$ and induces strangeness oscillations in time which can be used to mimic the different orientations of polarization analyzers in photonic Bell-tests [14,15]. Note, however, that the two terms in the photonic state have the same weight but that this is not the case in the two-kaon states (4).

The entanglement of the kaonic state (4) has been tested experimentally at CPLEAR over macroscopic distances and using active strangeness measurements [3]. The non-separability of this state has also been observed at the Daphne Φ-factory using passive measurements [18]; with some modification of the set-up, this could be possible by active measurements too.

In Ref. [19] the authors analyzed the possibility of a spontaneous wave-function factorization, which was proposed by Schrödinger and Furry in 1935.

If a factorization in the $\{K_{\rm S}, K_{\rm L}\}$ basis is assumed, the QM interference term is simply multiplied by $(1-\zeta)$

$$P_\zeta\left[K^0(\tau_{\rm l}), K^0(\tau_{\rm r})\right] = \frac{1}{4}\left\{1 - 2(1-\zeta)\frac{\cos(\Delta m \Delta\tau) e^{-(\Gamma_{\rm S}+\Gamma_{\rm L})(\tau_{\rm l}+\tau_{\rm r})/2}}{e^{-\Gamma_{\rm S}\tau_{\rm l}-\Gamma_{\rm L}\tau_{\rm r}} + e^{-\Gamma_{\rm L}\tau_{\rm l}+\tau_{\rm r})}}\right\},$$

where ζ is the "decoherence parameter", $0 \leq \zeta \leq \zeta_{\rm Schrdinger-Furry} = 1$, characterizing the strength of the interaction of the entangled state with the environment. If a factorization in the $\{K^0, \bar{K}^0\}$ basis is assumed then it is a bit more complicated. Now one can compare these models with the experimental data from the CPLEAR experiment [3] and a recent experiment of the KLOE Collaboration at Daphne [18]

$$\begin{aligned} \zeta_{K_{\rm S},K_{\rm L}} &= 0.13 \pm 0.16 \;[3]; 0.018 \pm 0.040_{\rm stat} \pm 0.007_{\rm syst}\;[18]\,, \\ \zeta_{K^0,\bar{K}^0} &= 0.4 \pm 0.7\;[3]; (0.10 \pm 0.21_{\rm stat} \pm 0.04_{\rm syst}) \times 10^{-5}\;[18]\,. \end{aligned} \tag{6}$$

The results in the $\{K^0 \bar{K}^0\}$ basis of the KLOE experiment benefits from large cancellations between the interference term and the two terms that occur for the CP suppressed final state $\pi^+\pi^-$.

3. Quantitative complementarity

As it is well known since long time, the observation of an interference pattern and the acquisition of "which way" information are mutually excluded in interferometric devices. However, quantitative statements of this complementarity principle have become available only recently.

The quality of an interference pattern can be quantified in terms of the "fringe visibility" $\mathcal{V}_0 = (I_{\rm max} - I_{\rm min})/(I_{\rm max} + I_{\rm min})$, where $I_{\rm max,min}$ stand for the maximum and minimum measured intensities. In general, for a two-path interferometer the intensity is $I(\phi) \propto (1 + \mathcal{V}_0 \cos\phi)$, where ϕ is the phase difference between the two paths. The amount of which-path information is given by the "predictability" [20] $\mathcal{P} \equiv |w_{\rm I} - w_{\rm II}|$, where $w_{\rm I(II)}$ is the probability to take the interferometric path I (II). Then, complementarity can be expressed in the quantitative form

$$\mathcal{P}^2 + \mathcal{V}_0^2 \leq 1\,, \tag{7}$$

where the equal sign is valid for pure states.

The evolution of a single kaon state can be interpreted in terms of this duality relation [21,22]. By normalizing to kaons surviving up to time τ and neglecting CP violation effects, the time evolution of an initial K^0 state (an analogous discussion holds for initial $\bar{K}^0$'s) can be written as

$$|K^0(\tau)\rangle = \frac{1}{\sqrt{2}}\left[\frac{1 + e^{-\frac{1}{2}\Delta\Gamma\tau} e^{-i\Delta m\tau}}{\sqrt{1+e^{-\Delta\Gamma t}}}|K^0\rangle + \frac{1 - e^{-\frac{1}{2}\Delta\Gamma\tau} e^{-i\Delta m\tau}}{\sqrt{1+e^{-\Delta\Gamma t}}}|\bar{K}^0\rangle\right].$$

The strangeness oscillations in τ of $|K^0(\tau)\rangle$ are characterized by the phase $\phi(\tau) = \Delta m\,\tau$ and by a time dependent visibility and path predictability

$$\mathcal{V}_0(\tau) = \cosh^{-1}\left(\frac{\Delta\Gamma}{\tau/2}\right)\,, \quad \mathcal{P}(\tau) = \tanh\left(\frac{\Delta\Gamma}{\tau/2}\right)\,, \tag{8}$$

which satisfy Eq. (7), $\mathcal{P}^2(\tau)+\mathcal{V}_0^2(\tau) = 1$. This clearly shows the interferometric behaviour of neutral kaon evolution, where the $K_{\rm S}$ and $K_{\rm L}$ components play the role of the two interferometric paths [21, 22].

4. Quantum eraser

The quantum eraser is a subtle phenomenon originally proposed by Scully and Drühl [23] and recently reviewed by Aharonov and Zubairy [24]. It has been demonstrated in several atomic and photonic experiments but it can be performed, with some advantages, by using neutral kaons [25].

In this type of analyses one considers variations of the basic double-slit experiment. In a two-way experiment, interference patterns are observed if and only if it is impossible to know, *even in principle*, which way the particle took. Interference disappears if there is a way to know — through a *quantum marking* procedure — which way the *object* particle took; whether or not the outcome of the corresponding "which way" observation, performed on the *meter* particle, is actually read out, it does not matter: interference is in any way lost. For a two-particle entangled state, if the path of one member is marked, information on the path taken by its entangled partner is in principle available and no interference fringes can be observed. But, if that "which way" mark is erased by means of a suitable measurement — *quantum erasure* — interferences reappear in joint detection events.

For neutral kaons, the phenomenon of K^0–$\bar{K}^0$ oscillations plays the role of the standard interference fringes. Similarly, the $K_{\rm S}$ and $K_{\rm L}$ states, showing a distinct propagation in free-space, are the analogs of the two separated trajectories in interferometers.

Consider the two-kaon entangled state (4). The *object* kaon flying to the left hand side is always measured *actively* in the strangeness basis. This measurement is performed by placing the strangeness detector at different points of the left trajectory, thus scanning for oscillations along a certain $\tau_{\rm l}$ range. The kaon flying to the right hand side, the *meter*, is always measured *actively* at a fixed time $\tau_{\rm r}^0$, either in the strangeness or in the lifetime basis. In the latter case we obtain full "*which width*" information for the object kaon — analogously to the "*which way*" information in a double slit; consequently, no interference in the meter-object joint detections can be observed. This can be seen from Eq. (4) leading to the non-oscillating joint probabilities:

$$P\left[K^0(\tau_{\rm l}),K_{\rm S}(\tau_{\rm r}^0)\right]=P\left[\bar{K}^0(\tau_{\rm l}),K_{\rm S}(\tau_{\rm r}^0)\right]=\frac{1}{2\left(1+e^{\Delta\Gamma\Delta\tau}\right)}\,, \tag{9}$$

$$P\left[K^0(\tau_{\rm l}),K_{\rm L}(\tau_{\rm r}^0)\right]=P\left[\bar{K}^0(\tau_{\rm l}),K_{\rm L}(\tau_{\rm r}^0)\right]=\frac{1}{2\left(1+e^{-\Delta\Gamma\Delta\tau}\right)}\,. \tag{10}$$

However, the possibility to obtain "which width" information can be precluded by quantum erasure, *i.e.*, by measuring strangeness on the meter kaon thus making its $K_{\rm S}$ or $K_{\rm L}$ "mark" inoperative. The joint probabilities:

$$\begin{aligned} P\left[K^0(\tau_{\rm l}),K^0(\tau_{\rm r}^0)\right] &= P\left[\bar{K}^0(\tau_{\rm l}),\bar{K}^0(\tau_{\rm r}^0)\right] \\ &= \frac{1}{4}\left[1-\mathcal{V}(\Delta\tau)\cos(\Delta m\,\Delta\tau)\right]\,, \end{aligned} \tag{11}$$

$$\begin{aligned} P\left[K^0(\tau_{\rm l}),\bar{K}^0(\tau_{\rm r}^0)\right] &= P\left[\bar{K}^0(\tau_{\rm l}),K^0(\tau_{\rm r}^0)\right] \\ &= \frac{1}{4}\left[1+\mathcal{V}(\Delta\tau)\cos(\Delta m\,\Delta\tau)\right]\,, \end{aligned} \tag{12}$$

then show $\tau_{\rm l}$-dependent strangeness oscillations with visibility $\mathcal{V}(\Delta\tau)=\cosh^{-1}(\Delta\Gamma\Delta\tau/2)$.

The previous kaon quantum eraser has some advantages as compared to the standard photonic case. With kaon pairs, the two "paths" ($K_{\rm S}$ and $K_{\rm L}$ propagation) are already present and automatically "marked" ($\Gamma_{\rm S}\gg\Gamma_{\rm L}$) from the very beginning simplifying the state preparation. From a more theoretical point of view, one notes that the oscillating probabilities (11) and (12) are even functions of $\Delta\tau$. Which measurement, left or right, is first performed is then irrelevant and the quantum eraser can be operated in the so-called "delayed choice" mode [11]. In this mode, the decision to observe or not strangeness oscillations when scanning the object kaon can be taken once this kaon has already been detected by performing a future measurement of strangeness or lifetime on the meter kaon.

These latter comments also add some light to the very nature of the quantum eraser working principle: the way in which joint detection events are *classified* according to the available information. In the "delayed choice" mode, a series of strangeness measurements is performed at different $\tau_{\rm l}$ times on the object kaons and the corresponding outcomes are recorded. Later one can measure either lifetime or strangeness on the corresponding meter partner and, only then, full information allowing for a definite sorting of each pair is available. Choosing to perform strangeness measurements on the meter kaons amounts to completely erase the "which width" information on each pair in such a way that oscillations and complementary anti-oscillations appear in the corresponding subsets. The alternative choice of lifetime measurements on meter kaons, instead, does not offer the possibility to classify the events in oscillatory subsets as before.

5. Bell inequalities tests

In this section we briefly discuss some promising ideas to test Bell inequalities with kaons, *i.e.*, testing Local Realism (LR) *vs* QM. Further discussions on this subject can be found in Refs. [8, 9, 16, 26, 27] and references therein.

The derivation of the CHSH–Bell inequality for neutral kaons [8] follows the original proof by Clauser *et al.* in 1969, Ref. [28], which is an extension of Bell's proof under more realistic assumptions. One finds

$$\begin{aligned} S_{k_n,k_m,k_{n'},k_{m'}}(t_1,t_2,t_3,t_4) \;=\; & \left|E_{k_n,k_m}(t_1,t_2) - E_{k_n,k_{m'}}(t_1,t_3)\right| \\ & + \left|E_{k_{n'},k_m}(t_4,t_2) + E_{k_{n'},k_{m'}}(t_4,t_3)\right| \leq 2\,. \end{aligned} \quad (13)$$

For kaons, each experimenter has choices of two different kinds: one choice refers to the "quasispin" state and the other to the time the kaon propagates until the measurement. The situation is then somehow more interesting than in the photonic case but also more involved because of the kaon evolution and decay. As in the usual photon setup, Alice and Bob can choose among two settings, *i.e.*, Alice: $\{(k_n,t_1);(k_{n'},t_4)\}$ and Bob: $(\{(k_m,t_2);(k_{m'},t_3)\})$. The expectation value $E_{k_n,k_m}(t_1,t_2)$ denotes then that Alice chooses to measure the quasispin k_n at time t_1 on the kaon propagating to her side and Bob chooses to measure k_m at time t_2 on his kaon.

If we fix the quasispins to the strangeness eigenstates, we obtain a Bell inequality depending on four times, which is in close analogy to photonic cases. But, surprisingly, working with the maximally entangled state (3) no violation of the inequality (13) can be obtained [6, 8]. By contrast, for non-maximal entangled states violations of this Bell inequality (up to $S = 2.159$) can be found, see Ref. [9]. Discussions with the experimenters at Daphne on the feasibility of these states are ongoing.

Apart from the previous states of kaons, other non-maximally entangled states are of interest for Bell tests. In Ref. [5] we have proposed the state

$$|\Phi\rangle = \frac{1}{\sqrt{2+|R|^2}}\left[|K_{\rm S}\rangle|K_{\rm L}\rangle - |K_{\rm L}\rangle|K_{\rm S}\rangle + R|K_{\rm L}\rangle|K_{\rm L}\rangle\right] , \quad (14)$$

which can be produced at a Φ-factory with the use of a kaon regenerator. Here, $R \equiv -\eta \exp\{[-i\Delta m + \frac{1}{2}(\Gamma_{\rm S} - \Gamma_{\rm L})]T\}$, η being the regenerator parameter. The non–maximally entangled state Φ describes all kaon pairs with both left and right partners surviving up to a common proper time T, with $\tau_{\rm S} \ll T \ll \tau_{\rm L} \simeq 579\,\tau_{\rm S}$.

This state can be conveniently used for Bell-type tests. Following the approach of Ref. [5], for each kaon on each beam at time T we consider either a strangeness or a lifetime measurement. With the strategy of subsection

2.2 for lifetime measurements, requiring an extra interval time $\Delta T = 4.8\,\tau_{\rm S}$ after T, care has to be taken to choose T large enough to guarantee the space-like separation between left and right measurements. For kaon pairs from ϕ decays, this implies $T > 1.77\,\Delta T$.

The following Clauser–Horne (CH) inequalities have been derived under the assumption of perfectly efficient experimental apparata (fair sampling hypothesis) in Ref. [5]:

$$\frac{P(\bar{K^0},K_{\rm L}) - P(\bar{K^0},\bar{K^0}) + P(K_{\rm S},\bar{K^0}) + P(K_{\rm S},K_{\rm L})}{P(K_{\rm S},*) + P(*,K_{\rm L})} \le 1\,,$$
$$\frac{P(\bar{K^0},K_{\rm S}) - P(\bar{K^0},\bar{K^0}) + P(K_{\rm L},\bar{K^0}) + P(K_{\rm L},K_{\rm S})}{P(*,K_{\rm S}) - P(K_{\rm L},*)} \le 1\,, \qquad (15)$$

where, for instance, $P(K_{\rm S},*) \equiv P(K_{\rm S},K^0) + P(K_{\rm S},\bar{K}^0)$. Note that each one of the two inequalities follows from the other by just inverting left and right measurements on the left–right asymmetric state (14).

By substituting the QM predictions in the inequalities (15), one finds:

$$\frac{2 - {\rm Re}\,R + \frac{1}{4}|R|^2}{2+|R|^2} \le 1\,, \quad \frac{2 + {\rm Re}\,R + \frac{1}{4}|R|^2}{2+|R|^2} \le 1\,, \qquad (16)$$

whose only difference is the sign affecting the linear term in ${\rm Re}\,R$. According to this sign, one of these two inequalities is violated if $|{\rm Re}\,R| \ge 3|R|^2/4$. The greatest violation occurs for a purely real value of R, $|R| \simeq 0.56$, for which one of the two ratios in Eq. (16) reaches the value 1.14. This 14 % violating effect predicted by QM opens up the possibility of refuting LR models modulo the fair sampling hypothesis.

We conclude our review by discussing a proposal that does not assume auxiliary hypotheses going beyond the reality and locality requirements. In our opinion, it represent an interesting attempt for a loophole-free test of LR *vs* QM with neutral kaons. It is based on Hardy's proof of Bell's theorem [29] without inequalities and it has been applied in Ref. [30] to the non-maximally entangled state (14). This considerably improves the analysis of Ref. [5]. Indeed, Hardy's non-locality proof can be translated into a Bell inequality [26] which could discriminate between LR and QM if the detection efficiencies for strangeness and lifetime measurements at disposal are high enough.

Let us first concentrate on the "non-locality without inequalities" proof of Ref. [30]. Neglecting CP violation and $K_{\rm L}$–$K_{\rm S}$ misidentifications, from state (14) with $R = -1$ (Hardy's state) one obtains the QM predictions:

$$P_{\rm QM}(K^0,\bar{K}^0) \;=\; \frac{\eta\bar{\eta}}{12}\,, \tag{17}$$
$$P_{\rm QM}(K^0,K_{\rm L}) \;=\; 0\,, \tag{18}$$
$$P_{\rm QM}(K_{\rm L},\bar{K}^0) \;=\; 0\,, \tag{19}$$
$$P_{\rm QM}(K_{\rm S},K_{\rm S}) \;=\; 0\,, \tag{20}$$

where η ($\bar{\eta}$) is the overall efficiency for K^0 ($\bar{K}^0$) detection. It is found that the necessity to reproduce, under LR, equalities (17)–(19) requires $P_{\rm LR}(K_{\rm S},K_{\rm S}) \geq P_{\rm LR}(K^0,\bar{K}^0) = \eta\,\bar{\eta}/12 > 0$, which contradicts Eq. (20). In principle, this allows for a test of LR *vs* QM without inequalities. However, since $K_{\rm L}$–$K_{\rm S}$ misidentifications (due to the finite value of $\Gamma_{\rm S}/\Gamma_{\rm L} \simeq 579$) preclude an ideal lifetime measurement even when the detection efficiency η_τ for the kaon decay products is 100%, the above proposal must be reanalised paying attention to these inefficiencies [26].

Retaining the $K_{\rm S}$–$K_{\rm L}$ misidentifications effects, the results (17)–(20) are replaced by (see the Appendix of Ref. [26] for details):

$$P_{\rm QM}(K^0,\bar{K}^0) \;=\; \frac{\eta\bar{\eta}}{12}\,, \tag{21}$$
$$P_{\rm QM}(K^0,K_{\rm L}) \;=\; 6.77\times 10^{-4}\,\eta\,\eta_\tau\,, \tag{22}$$
$$P_{\rm QM}(K_{\rm L},\bar{K}^0) \;=\; 6.77\times 10^{-4}\,\bar{\eta}\,\eta_\tau\,, \tag{23}$$
$$P_{\rm QM}(K_{\rm S},K_{\rm S}) \;=\; 1.19\times 10^{-5}\,\eta_\tau^2\,. \tag{24}$$

In the standard Hardy's proof [29], the probabilities corresponding to our (22)–(24) are perfectly vanishing. In our case they are very small but not zero. Nevertheless, this does not prevent from deriving a contradiction between LR and QM. One has to use the Eberhard inequality [26, 31]:

$$H \equiv \frac{P(K^0,\bar{K}^0)}{P(K^0,K_{\rm L})+P(K_{\rm S},K_{\rm S})+P(K_{\rm L},\bar{K}^0)+P(K^0,F)+P(F,\bar{K}^0)} \leq 1\,, \tag{25}$$

where the argument F refers to failures in lifetime detection, and, in QM:

$$P_{\rm QM}(K^0,F) = \frac{1}{6}\eta\,(1-\eta_\tau)\;, \qquad P_{\rm QM}(F,\bar{K}^0) = \frac{1}{6}\bar{\eta}\,(1-\eta_\tau)\;. \tag{26}$$

Note that the use of an inequality also allows for deviations, existing in real experiments, in the value of R required to prepare Hardy's state. It is important to stress that the inequality (25) has been obtained *without invoking supplementary assumptions* on undetected events. From this inequality one obtains the restrictions on the efficiencies η, $\bar{\eta}$ and η_τ required for a loophole-free experiment.

To discuss the feasibility of such an experiment, let us start considering a few ideal cases. Assume first that perfect discrimination between $K_{\rm S}$ and $K_{\rm L}$ were possible; one could then make a conclusive test of LR for any nonvanishing values of η and $\bar{\eta}$: $H_{\rm QM} \to \infty$, $\forall\, \eta, \bar{\eta} \neq 0$. In a second ideal case with no undetected events, *i.e.*, $\eta = \bar{\eta} = \eta_\tau = 1$, the inequality is strongly violated by QM, $H_{\rm QM} \simeq 60.0$, even if one allows for unavoidable $K_{\rm S}$ and $K_{\rm L}$ misidentifications (see Ref. [32] for a criticism on this point). Finally, assuming that only the detection efficiency of kaon decay products is ideal ($\eta_\tau = 1$), for $\eta = \bar{\eta}$ ($\eta = \bar{\eta}/2$) Eberhard inequality is contradicted by QM for $\eta > 0.023$ ($\eta > 0.017$).

More realistic situations, with small and possibly achievable values of η and $\bar{\eta}$, must be considered. According to the results of Ref. [26], to have a loophole-free test, this implies that we have to consider large decay-product detection efficiencies such as $\eta_\tau \simeq 0.98$, for which η and $\bar{\eta}$ can be lowered to about 0.06 (see Fig. 1 of Ref. [26]). The values of η_τ, η and $\bar{\eta}$ required by the test proposed in Ref. [26] seem to be not far from the present experimental capabilities.

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