THE SCALAR MESONS, SYMMETRY BREAKING, THREE COLORS AND CONFINEMENT*

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The same, well known, $\det \Sigma + \det \Sigma^\dagger$ term in effective theories, which 't Hooft showed is generated by instantons in QCD and which resolves the $\mathrm{U_A}(1)$ problem giving mass, in particular to the η' is for three light flavors shown to give three classical minima along the $\mathrm{U_A}(1)$ circle. The three minima are related to the center Z(3) of SU(3). The term also contributes, in a similar way as the diquark model of Jaffe, to an inverted scalar mass spectrum for the light scalars. The three vacua suggests a connection to the strong CP problem and confinement.

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It is widely believed that QCD with three nearly massless light quark flavors explain the well-known approximate $\mathrm{SU(3)_L\times SU(3)_R}$ chiral symmetry seen in the light meson mass spectrum. Our present understanding of the symmetry breaking involves three basic mechanisms:

(i) Spontaneous symmetry breaking in the QCD vacuum, which gives rise to a near flavor symmetric $\langle \bar{q}q \rangle$ condensate and an octet of (would be massless) Goldstone pseudoscalars.

(ii) A contribution from the gluon anomaly, which explicitly breaks the axial symmetry $\mathrm{U_A}(1)$ in $\mathrm{U(3)_L\times U(3)_R}$, and which gives in, particular, mass to the η' [1].

(iii) Small chiral quark masses m_u, m_d, m_s from the electro-weak sector, which give the pseudoscalar octet states a small mass and break flavor symmetry. A large m_s/m_d mass ratio together with the anomaly term *(ii)* also splits the η from the pion, which saves isospin symmetry in spite of the large m_d/m_u chiral quark mass ratio.

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In effective theories for scalar and pseudoscalar mesons one models the global $\mathrm{U}(3)_{\mathrm{L}}\times \mathrm{U}(3)_{\mathrm{R}}$ symmetry by potential terms. Including up to dimension four terms one writes[1]

$$V_{\mathrm{U3U3}} = \frac{\mu^2}{2}\mathrm{Tr}[\Sigma\Sigma^\dagger] + \lambda\mathrm{Tr}[\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger] + \lambda'(\mathrm{Tr}[\Sigma\Sigma^\dagger])^2\,, \tag{1}$$

where Σ is the usual 3×3 matrix containing the scalar (s) and pseudoscalar (p) nonets. (Denoting the nonet members by s_k and p_k for $k=0$ to 8, one has $\Sigma = \sum_k (s_k + ip_k)\lambda_k$, where λ_k are Gell–Mann matrices).

If μ^2 has the "wrong sign" $\mu^2 < 0$ Eq. (1) predicts the often quoted spontaneous symmetry breaking with a nonet of massless pseudoscalars. But, in this case the U(1) problem [2] arises. There is "too much symmetry", the axial $\mathrm{U_A}(1)$ problem appears and the η and η' become massless.

To have a realistic zeroth order $\mathrm{SU}(3)_{\mathrm{L}}\times \mathrm{SU}(3)_{\mathrm{R}}$ model, one must follow the step *(ii)* above and break the axial $\mathrm{U_A}(1)$ symmetry explicitly in the strong interactions. The simplest way to do it [1] is by adding a determinant term to the Lagrangian,

$$V_{\mathrm{SU3SU3}} = V_{\mathrm{U3U3}} + \beta[\det(e^{i\theta}\Sigma) + \det(e^{i\theta}\Sigma)^*]\ . \tag{2}$$

The addition of the complex conjugate term is required by parity, and also by C parity, since a trilinear coupling of three $C=+$ mesons must by Bose statistics be symmetric under interchange of two mesons. We have included a $\mathrm{U_A}(1)$ phase factor given by the angle θ. To give the pseudoscalar octet members mass (and the η' a small extra mass) one conventionally adds a term $\propto (\mathrm{Tr}[\Sigma M_q] + \mathrm{h.c.})$ where M_q is a diagonal matrix containing the chiral light quark masses.

Thereby one obtains an instructive and simple effective tree level model for scalar and pseudoscalar mesons, essentially the SU(3) version of the linear sigma model, by which one can model the basic global symmetries of QCD and their zeroth order breaking with the nonperturbative instanton term. Eq. (2) is the simplest model for the lightest mesons, which is consistent with the symmetries of QCD. In its first formulations it has been with us for almost 50 years [3, 5] and remain as a first understanding of the symmetries involved in strong interactions.

It is the main point of this paper to show that determinant term can give rise to three classical minima, and to show how color symmetry enters for the lightest scalar mesons, although in an almost hidden form.

[1] We recall that by putting the pseudoscalars (p) into the anti-Hermitian part of $\Sigma = s + ip$ the γ_5 in $\bar{u}(s+i\gamma_5 p)u$ disappears since we can write it as $\bar{u}[\frac{1}{2}(1-\gamma_5)\Sigma + \frac{1}{2}(1+\gamma_5)\Sigma^\dagger]u{=}\bar{u}_{\mathrm{L}}\Sigma u_{\mathrm{R}} + \bar{u}_{\mathrm{R}}\Sigma^\dagger u_{\mathrm{L}}$. The parity transformation of γ_5 is thus just complex conjugation, $\Sigma\to\Sigma^*$, while CP is represented by $\Sigma\to\Sigma^\dagger$. From this it is also clear that Σ transforms as $\Sigma\to \mathrm{U_L}\Sigma\ \mathrm{U_R}$, and $\Sigma^\dagger \to \mathrm{U_R}\Sigma^\dagger\ \mathrm{U_L}$ under $\mathrm{U}(3)_{\mathrm{L}}\times \mathrm{U}(3)_{\mathrm{R}}$, from which the invariance of the potential (1) follows.

There are well known mathematical identities for the determinant, which are useful for our purpose, and which we give in Eqs.(3)–(5) below. The first[2] is (for $N_f = 3$),

$$6 \det \Sigma = (\mathrm{Tr}\Sigma)^3 + 2\mathrm{Tr}(\Sigma^3) - 3\mathrm{Tr}(\Sigma^2)\mathrm{Tr}(\Sigma)\,. \tag{3}$$

In this expression each term has less symmetry ($\mathrm{SU}(3)_F$) than the sum $\mathrm{SU(3)_L} \times \mathrm{SU(3)_R}$. In fact, each term when evaluated in terms of the 18 meson fields has many more terms than the determinant, where most terms cancel against each other.

Another identity for a determinant[3] $\det \Sigma_{ij} = \det(\bar{q}_i q_j)$ comes directly from its basic definition

$$\begin{aligned} \det \Sigma = \det(\bar{q}_i q_j) &= \varepsilon_{ijk} \bar{q}_1 q_i \ \bar{q}_2 q_j \ \bar{q}_3 q_k \\ &= \frac{1}{3!} \delta^{lmn}_{ijk} \bar{q}_l q_i \ \bar{q}_m q_j \ \bar{q}_n q_k \,. \end{aligned} \tag{4}$$

The second expression is written in a way which is clearly frame independent [6].

Perhaps the simplest expression is obtained when the flavor sum in Eq. (4) is written out explicitly:

$$\begin{aligned} \det \Sigma = \det(\bar{q}_i q_j) = \ &+\bar{u}u \ \bar{d}d \ \bar{s}s - \bar{u}u \ \bar{d}s \ \bar{s}d + \bar{u}d \ \bar{d}s \ \bar{s}u \\ &-\bar{u}d \ \bar{d}u \ \bar{s}s + \bar{u}s \ \bar{d}u \ \bar{s}d - \bar{u}s \ \bar{d}d \ \bar{s}u \,. \end{aligned} \tag{5}$$

The most important physics properties of these determinant forms are

(a) The determinant is completely antisymmetric with respect to flavor.

(b) In each term one has 3 quarks and 3 antiquarks, and any quark flavor occurs only once, and similarly any antiquark flavor occurs only once.

(c) It is a flavor singlet both in the three quarks and in the three antiquarks, and as already noted invariant under an SU(3) transformation from both the left as well as from the right of Σ.

(d) A $\mathrm{U_A}(1)$ transformation is just a simple phase transformation $e^{i\varphi}$ from the left and from the right, whereby only the phase of Σ changes by $e^{2i\varphi}$. Because of this we have the freedom in choosing θ in Eq. (2).

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[2] This identity is easily derived after diagonalisation. In terms of the eigenvalues a, b, c one has $6 \det \Sigma = 6abc = (a+b+c)^3 + 2(a^3+b^3+c^3) - 3(a^2+b^2+c^2)(a+b+c)$.

[3] Here the matrix element $\bar{q}_i q_j$ stands for the weight of the left handed antiquark — right handed quark component in a meson, or in a superposition of mesons, in a rather obvious way. Thus the dimension of $\bar{q}q$ is as for a meson, GeV.

It is of interest to note that in Eq. (5) the first term is contained only in the first term, $(\mathrm{Tr}\Sigma)^3$, of Eq. (3). The three negative terms are contained in the third term, $-3\mathrm{Tr}(\Sigma^2)\mathrm{Tr}(\Sigma)$, of Eq. (3), while the two remaining positive terms in the above equation are contained in, $2\mathrm{Tr}(\Sigma^3)$, of Eq. (3).

These equations (2)–(5) show that the determinant term involves a remarkably symmetric but entangled quantum system. In particular, note that because the three quarks or three antiquarks involved form a flavor singlet, any diquark subsystem must be in the $\bar{\mathbf{3}}_F$ representation of $\mathrm{SU}(3)_F$.

In fact, many years ago Jaffe [7] found that in the bag model the strongest bound diquarks are those, which are in the antisymmetric $\bar{\mathbf{3}}_F$ $\mathrm{SU}(3)_F$ representation, have antisymmetric spin $S = 0$, symmetric space (S-wave) and are antisymmetric in color $\bar{\mathbf{3}}_C$. Therefore he suggested a diquark model for the lightest scalar nonet, which would have an "inverted" mass spectrum (compared to the vector mesons), where the $\sigma(600)$ is the lightest, followed by a κ near 800 MeV and the $a_0(980), f_0(980)$. In fact, the model described by Eq. (2) predicts a very broad, light sigma and the determinant term (when including $s - d$ quark mass splitting) shifts the κ down from the a_0 by the same amount as the K is shifted up from the π [4].

The light and broad sigma [8], the $\sigma(600)$, is now accepted as a true resonance also by the chiral perturbation theory experts [9]. Also the expected extremely broad κ pole, which has been claimed in experiments [10], has very recently [11] been determined to a remarkable accuracy by Roy–Steiner constraints involving crossing symmetry, analyticity and unitarity.

The connection between Jaffe's diquark model and the determinant term is clear. It is natural to expect the lowest diquarks to have spin 0 and to be in an S-wave. Since the determinant requires any diquark to be in the $\bar{\mathbf{3}}_F$ they must also be in the antisymmetric $\bar{\mathbf{3}}_C$ by spin-statistics. Thus if one wants to include color, then the determinant term should be multiplied by a similar factor, but now with color replacing flavor in the indices.

This shows the flavor-color connection through Fermi–Dirac statistics within the scalar mesons, in a analogous way as the color factor is needed for the proton wave function. There is, however, one clear difference compared to Jaffe's model. The determinant term does not describe diquark–diquark bound states but a transition from $\bar{q}q$ to $\bar{q}q\ \bar{q}q$. Similarly, because it describes such a transition, and not a qqq state, it is not in conflict with the fact that a flavor singlet, color singlet, S-wave, spin $\frac{1}{2}$ spectroscopic qqq state is forbidden by Fermi–Dirac statistics (because of only two degrees of freedom for spin).

[4] The contribution to $m_\kappa^2 - m_{a_0}^2 = -(m_K^2 - m_\pi^2) = 2\beta(v_{\bar{s}s} - v_{\bar{d}d})$.

Now the physical states are of course not the $\bar{u}u$, $\bar{d}d$, $\bar{s}s$ appearing in Eqs. (4), (5), but superpositions of these (and because of the preceding discussion also mixings with four-quark meson–meson states, with same quantum numbers). In particular, the pure SU(3) singlet states are equal superposition of $\bar{u}u$, $\bar{d}d$, $\bar{s}s$. They are thus represented by the complex matrix $\Phi = \phi \cdot \mathbf{1}/\sqrt{3}$, where $\phi = (s_0 + ip_0)$ and where $\mathbf{1}$ is the 3×3 unit matrix. It is of some interest that these singlet terms appear only in the first term of Eqs. (3), (5).

First neglect the phase angle θ in Eq. (2). There is then a real minimum of the potential Eq. (2) *i.e.* a non zero vacuum value. (For $\mu = 0$ this is $v = \frac{1}{\sqrt{3}}\phi^{\min} = -\beta/(2\lambda + 6\lambda')$.) Note that the usual positivity condition for a minimum, $v > 0$, chooses the sign of $\beta < 0$ when the sign in front of β in Eq. (6) is chosen positive.

But, in fact, there are three minima in the effective potential defining 3 vacuum expectation values! Substituting Φ into Σ of Eq. (6) one finds, now including the phase θ in Eq. (2):

$$\begin{aligned} V(\phi) &= \frac{\mu^2}{2}|\phi|^2 + \frac{\lambda + 3\lambda'}{3}|\phi|^4 + \frac{\beta}{3\sqrt{3}}\left[(e^{i\theta}\phi)^3 + (e^{-i\theta}\phi^*)^3\right] \\ &= \frac{\mu^2}{2}|\phi|^2 + \frac{\lambda + 3\lambda'}{3}|\phi|^4 + \frac{2\beta}{3\sqrt{3}}|\phi|^3 \cos\left[3\theta + 3\arg(\phi)\right] . \end{aligned} \tag{6}$$

The cosine factor in the β term makes this potential different from the usual "Mexican hat" potential. As an illustrative example it is shown in Fig. 1 as a contour plot in the complex ϕ plane near parameter values found in Ref. [4]. It has three "hills" in the directions $\arg(\phi) = \pi - \theta$, $\pi - \theta + 2\pi/3$ and $\pi - \theta + 4\pi/3$, and three valleys in between. Most importantly, provided μ^2 is not too large and positive, it has three minima defining three vacuum expectation values in the downhill directions of the steepest hills

$$\begin{aligned} v_1 &= \phi_1^{\min}/\sqrt{3} = ve^{-i\theta} , \\ v_2 &= \phi_2^{\min}/\sqrt{3} = ve^{-i\theta + i2\pi/3} , \\ v_3 &= \phi_3^{\min}/\sqrt{3} = ve^{-i\theta + i4\pi/3} . \end{aligned} \tag{7}$$

One should expect that instantons in QCD can tunnel between these vacua and, in fact, 't Hooft [1] motivated the determinant term because of instantons. The inclusion of the θ angle shows that the three minima are all on the same footing. Although the term (6) can resolve a continuous ambiguity in θ there remains a threefold ambiguity. In the $\mathrm{SU}(3)_F$ limit, *i.e.* if one neglects weak interactions and chiral quark masses, one has the freedom to chose this chiral angle θ to be a multiple of $2\pi/3$, such that this choice ($\theta = 0, 2\pi/3$ or $4\pi/3$) makes any of the three minima real and > 0. Reality of $v_i e^{i\theta}$ is required by CP, at least as long as weak interactions are neglected. Expanding the meson fields around any of these vacua $\Sigma \to \Sigma + v_i\mathbf{1}$ one finds a singlet η' mass, $m^2_{p_0} = -6\beta|v|{=}12(\lambda+3\lambda')|v|^2$, from the second derivative in the angular variable ($\arg(\phi) \propto p_0$) of the potential (6).

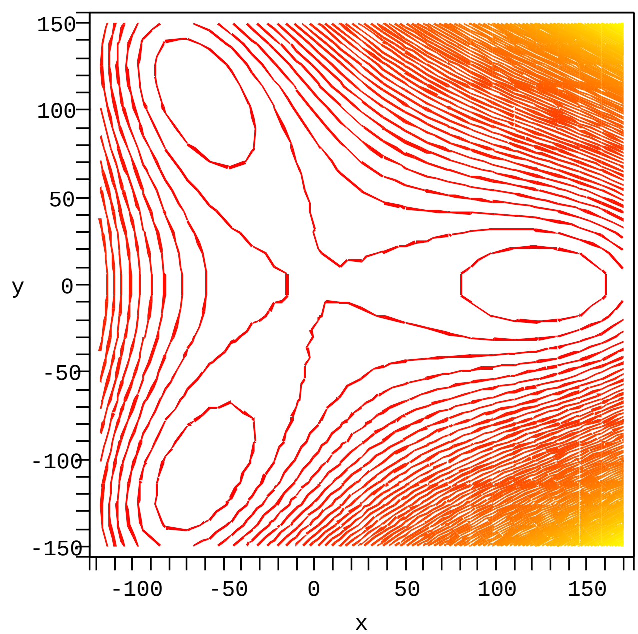

Fig. 1. An illustrative example of the potential $V(\phi)$ of Eq. (6) as a contour plot in the complex ϕ plane. The three minima are here at $|\phi_{\rm min}| \approx 130$ MeV. (This corresponds to an average f_π and f_K decay constant of $130\sqrt{(}2/3)$ MeV$\approx$ 106 MeV.) The parameters in Eq. (6) are chosen in this illustration as $\mu = 0$, $\beta = -1700$ MeV and $\lambda+3\lambda' = 11.5$. The masses of the SU(3) singlet pseudoscalar and singlet scalar states are given by the second derivatives at any of the three minima.

The scalar singlet mass is similarly obtained $m^2_{s_0} = 4(\lambda+3\lambda')v^2{=}m^2_{p_0}/3$, or 553 MeV for a 958 MeV p_0, from the second derivative in the radial direction $|\phi|$ of the same potential (6). The scalar octet mass is given by $m^2_{s_{1..8}}{=}\,16(\lambda{+}3/2\lambda')v^2{=}4/3m^2_{p_0}{-}8\lambda'v^2$, which means in the region of 1 GeV. The 0^{-+} Goldstone octet remain in this $\rm SU(3)_L\times SU(3)_R$ limit, as expected massless.

Why 3 minima? The threefold symmetry, together with CP, is related to the center Z(3) of the axial SU(3) symmetry in $\mathrm{SU_L}(3)\times\,\mathrm{SU_R}(3)$. Above we showed how the determinant connects flavor and three colors because of Fermi–Dirac statistics. This makes three flavors special for scalar mesons, and $N_f = 3$ is also special because $\mathrm{SU}(3)_F$ remains approximate after symmetry breaking from the small chiral quark masses. The symmetry breaking is small compared to Λ_{QCD} or, here perhaps better, compared to the η' or proton mass.

Thus for the meson spectrum it does not matter which of the three v_i's is chosen in the shift, $\Sigma \to \Sigma + v_i\mathbf{1}$. The meson masses remain the same since they depend on $|v_i|^2 = v^2$, but for fermions a problem appears because of the possible phase of v_i. A constituent quark can get mass, $m_q^{\mathrm{const}} = gv_i$, through Yukawa couplings to the vacuum as in the original linear sigma model [3]: $g\bar{q}_{\mathrm{L}}\Sigma q_{\mathrm{R}} + \mathrm{h.c.} \to gv_i\bar{q}_{\mathrm{L}}q_{\mathrm{R}} + \mathrm{h.c.}$, where g is a pion quark coupling. Here v_i must be chosen real for each quark. A phase of v_i like Eq. (7) could violate parity and charge conjugation, by which one could argue that such single free quarks are forbidden not only by color but also CP.

The three minima in Fig. 1 are puzzling, Are these just a curiosity of the effective model studied, or are they connected to the longstanding strong CP problem [13] and perhaps confinement? The axial $\mathrm{U_A}(1)$ current is, of course, well known not to be conserved, because of the triangle quark graph and the gluon chiral anomaly. In the strong CP problem one also derives from the anomaly many different vacua connected by "large" gauge transformations and winding numbers in the same $\mathrm{U_A}(1)$ degree of freedom as discussed here. The situation in our model seems similar although the model fixes the number of vacua to three in a cyclic fashion. Should the true vacuum be a superposition of the three vacua as for the θ vacuum [13] $(\sum_n e^{in\theta}|v_i>)$, and should flavor symmetry be broken in a way that maintains the permutation Z(3) symmetry of the three vacua, because of its possible connection to color?

To get a finite pseudoscalar octet mass one can, instead of a conventional term $\propto m_q\mathrm{Tr}\Sigma + \mathrm{h.c.}$, introduce a small term $\propto m_q(\mathrm{Tr}\Sigma)^3 + \mathrm{h.c.}$, which retains the Z(3) symmetry (like the terms on the r.h.s. of Eq. (3)). Similarly, instead of a conventional term $\propto \mathrm{Tr}(\Sigma M_q) + \mathrm{h.c.}$, which breaks $\mathrm{SU}(3)_F$, one can introduce *e.g.* a term $\propto (\mathrm{Tr}\Sigma)^2\mathrm{Tr}(\Sigma M_q) + \mathrm{h.c.}$, which also retains the Z(3) symmetry, *i.e.* one still has the three equal minima as in Fig. 1. These alternative forms for the symmetry breaking results in only minor modifications for the predictions to the mass spectrum [14], since these depend only on the second derivatives of the Lagrangian at the chosen minimum v_i, as the singlet η' and σ masses in the demonstration of Fig. 1.

Fig. 1 suggests that the three vacua v_i and Z(3) play a role in the confinement mechanism (see Huang [15]). Instantons can tunnel between these minima, and the wave function of the proton may be a superposition of states in each of the three v_i. For a proton or baryon mass term $m_p \bar{p}_{\rm L} p_{\rm R}$, where p stands for qqq it would be natural that it should transform under $\mathrm{U_A}(1)$ like the determinant term, which also involves 3 quarks and 3 antiquarks. Now, for a three quark system imagine a qqq wave function in the tricyclic potential of Fig. 1, with three probability maxima at the three minima and let the phase of $\bar{p}_{\rm L} p_{\rm R}$ wind 3 times that of ϕ, *i.e.* like $\det \Sigma$. *E.g.* a "trial wave function" (for $\bar{p}_{\rm L}$ or $p_{\rm R}$) along the chiral circle $\propto \cos(3\varphi) e^{3i\varphi}$, where $\varphi = \arg(\phi)/2$, would do. This gives for $\bar{p}_{\rm L} p_{\rm R}$: $\propto [\cos(3\varphi) e^{3i\varphi}]^2$, which transforms as $\det \Sigma$ under a chiral rotation ($\det \Sigma \to e^{6i\varphi} \det \Sigma$, when $\Sigma \to e^{2i\varphi} \Sigma$). Then for baryons, as for mesons, it does not matter which of the three minima is chosen as real, but for a quark it would.

A perhaps better approach is to assume a quark to be a soliton, which interpolates between two of the vacua as in the Sine–Gordon equation. A baryon is then composed of a three soliton solution which interpolates through all three vacua starting and ending at the same v_i, which is chosen real and which remain the true minimum after symmetry breaking.

This may open the door for a simple understanding of the confinement mechanism. As a self-consistency check, such a threefold rotation in the chiral angle for the baryon mass term is consistent with the fact that baryons have integer baryon number, but quarks have fractional baryon number of $\frac{1}{3}$. The number three is then of topological nature as a winding number, which is conserved although $\mathrm{SU}(3)_F$ is broken.

In conclusion the puzzling fact that this well known effective model has three vacua, which are illustrated in Fig. 1, opens many interesting questions. A better understanding should illuminate the long standing strong CP and confinement problems.

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