NNLO QCD CORRECTIONS TO $\bar{B} \to X_s \gamma$*

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(Received June 28, 2007)

Current status of the $\mathcal{O}(\alpha_s^2)$ calculations of $\mathcal{B}(\bar{B} \to X_s \gamma)$ is summarized.

PACS numbers: 12.38.Bx, 13.20.He

1. Introduction

Calculating the Next-to-Next-to-Leading Order (NNLO) QCD corrections to the inclusive $\bar{B} \to X_s \gamma$ decay was declared by Greub and myself at the EURIDICE Start-Off Collaboration Meeting in Frascati [1] as one of our tasks within the network activity. I am happy to be able to present the final results at the final meeting. Many of the co-authors of our main publication [2] have performed their research at the network nodes.

The weak radiative B-meson decay is known to be a powerful mean for constraining extensions of the Standard Model (SM). Its branching ratio is most precisely determined from the product of three factors: the measured semileptonic branching ratio, the Leading Order (LO) ratio of the perturbative decay rates $\Gamma(b \to s\gamma)/\Gamma(b \to ce\bar{\nu})$, and the leading-logarithmic QCD factor that depends on $\alpha_s(M_W)/\alpha_s(m_b)$. All the other contributions can be considered as corrections only. The $\mathcal{O}(\alpha_s)$ Next-to-Leading Order (NLO) perturbative QCD corrections reach over 30%, while the $\mathcal{O}(\alpha_s^2)$ NNLO ones amount to around 10%. Non-leading electroweak contributions affect the branching ratio by around 4%. The non-perturbative corrections include both the known ones ($\sim 3\%$) that scale like Λ^2/m_b^2 or Λ^2/m_c^2 (where $\Lambda \sim \Lambda_{\rm QCD}$), as well as the unknown $\mathcal{O}(\alpha_s \Lambda/m_b)$ ones which arise in the presence of at least one gluon that is not soft.

* Presented at The Final EURIDICE Meeting "Effective Theories of Colours and Flavours: from EURODAPHNE to EURIDICE", Kazimierz, Poland, 24–27 August, 2006.

Methods for arguing that the non-perturbative effects are just corrections and for calculating them are based on the optical theorem, operator product expansion, and the Heavy Quark Effective Theory (see, *e.g.*, Refs. [3–6]). The calculations are straightforward for the Λ^2/m_b^2 corrections, somewhat more complex for the Λ^2/m_c^2 ones, and very hard for the α_s-suppressed ones. The latter contributions remain largely unknown, and cause the dominant ($\sim 5\%$) uncertainty at present.

The current experimental world average [7] for the branching ratio amounts to

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm exp} = \left(3.55 \pm 0.24\ {}^{+0.09}_{-0.10} \pm 0.03\right) \times 10^{-4}, \tag{1}$$

with the photon energy cut $E_\gamma > 1.6\,\text{GeV}$ in the $\bar{B}$ meson rest frame. The combined error in this result does not exceed 8%, *i.e.* it is similar in magnitude to the $\mathcal{O}(\alpha_s^2)$ corrections, which gives a strong motivation for performing the NNLO QCD calculation.

2. Outline of the calculation

Examples of LO diagrams are shown in Fig. 1. The (top-quark)–(W-boson) loop is one of the dominant contributions in the SM. In the Two-Higgs-Doublet Model (THDM), there is an additional contribution from the (top-quark)–(charged Higgs) boson loop. In the Minimal Supersymmetric Standard Model (MSSM), there are many other contributions, for instance, the chargino-stop loops. All these contributions arise at the same order in the electroweak interactions as the SM ones, and they become suppressed only if the new particle masses are much larger than the top quark mass. This is the reason why the $b \to s\gamma$ constraints on new physics are so stringent.

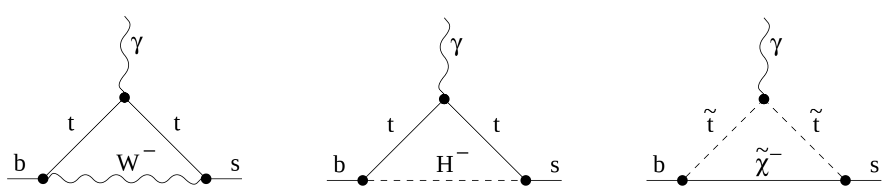

Fig. 1. Sample LO diagrams in the SM (left), THDM (middle) and MSSM (right).

Two-loop diagrams obtained from Fig. 1 by adding a virtual gluon contain large logarithms $\ln M_W^2/m_b^2$. In the SM, they enhance the branching ratio by more than a factor of 2. Such large logarithms need to be resummed at each order of the perturbation series in α_s by means of renormalization group techniques. Thus, logarithmically-enhanced parts of two-loop diagrams count as LO effects. Since two loops are necessary at the LO, four loops are necessary at the NNLO, which explains why the NNLO calculation is so complex.

To resum the large logarithms, one employs a low-energy effective theory that arises after decoupling the top quark and the heavy electroweak bosons. Weak interaction vertices (operators) in this theory are either of dipole type ($\bar{s}\sigma^{\mu\nu}bF_{\mu\nu},\ \ \bar{s}\sigma^{\mu\nu}T^a bG^a_{\mu\nu}$) or contain four quarks ($[\bar{s}\Gamma b][\bar{q}\Gamma' q]$). Coupling constants at these vertices (Wilson coefficients) are first evaluated at the electroweak renormalization scale $\mu_0 \sim m_t, M_W$ by solving the so-called *matching* conditions. Next, they are evolved down to the low-energy scale $\mu_b \sim m_b$ according to the effective theory Renormalization Group Equations (RGE). The RGE are governed by anomalous dimensions that originate from the operator *mixing* under renormalization. Finally, one computes the *matrix elements* of the operators, which in our case amounts to calculating on-shell diagrams with single insertions of the effective theory vertices.

At the NNLO level, the dipole and the four-quark operators need to be matched up to three and two loops, respectively. Renormalization constants up to four loops must be found for $b \to s\gamma$ and $b \to sg$ diagrams with four-quark operator insertions, while three-loop mixing is sufficient in the remaining cases. Two-loop matrix elements of the dipole operators and three-loop matrix elements of the four-quark operators must be evaluated in the last step.

The two- and three-loop NNLO matching conditions were found in Refs. [8,9], respectively. The necessary three-loop mixing was calculated in Refs. [10,11]. The four-loop mixing was evaluated in Ref. [12]. Two-loop matrix element of the photonic dipole operator together with the corresponding bremsstrahlung was found in Refs. [13,14]. Recently, these matrix element results were confirmed and extended to the case of arbitrary charm quark mass m_c [15–17]. Three-loop matrix elements of the four-quark operators were found in Ref. [18] within the so-called large-β_0 approximation. A calculation that goes beyond this approximation by employing an interpolation in m_c was performed in Ref. [19].

3. Results

Our final result for the branching ratio has been tested by studying its dependence on the renormalization scales: the matching scale μ_0, the low-energy scale μ_b and the charm-mass renormalization scale μ_c. The dotted, dashed and solid lines in Fig. 2 show the LO, NLO and NNLO results, respectively. Once one of the scales is varied, the remaining ones are fixed

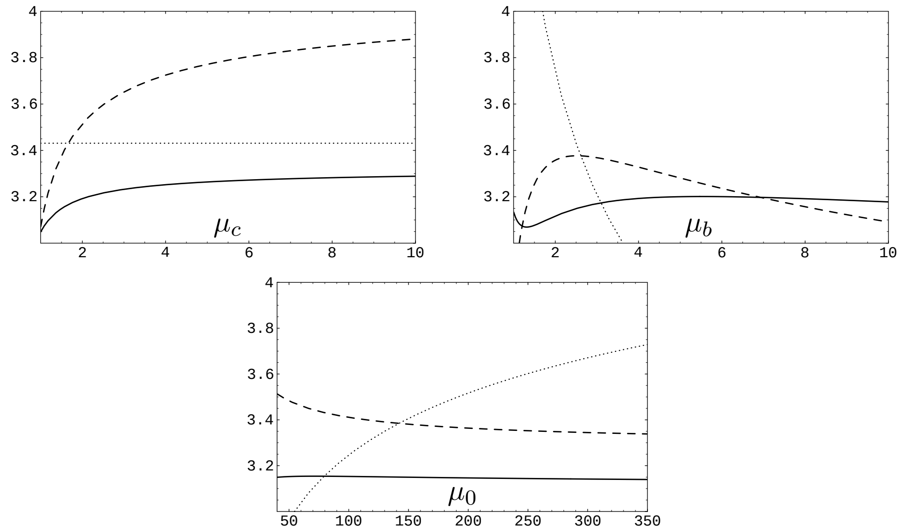

Fig. 2. Renormalization scale dependence of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in units 10^{-4} at the LO (dotted lines), NLO (dashed lines) and NNLO (solid lines). The upper-left, upper-right and lower plots describe the dependence on μ_c, μ_b and μ_0 [GeV], respectively.

at their default values that have been chosen to be 160 GeV, 2.5 GeV and 1.5 GeV for μ_0, μ_b and μ_c, respectively. As expected, higher-order corrections stabilize the scale-dependence. The stabilization is perfect in the case of μ_0 because $\alpha_s(\mu_0)$ is small, and the matching is known completely. The low-energy scale dependence is non-negligible even at the NNLO when the scales are very low — between 1 and 2 GeV. The most pronounced effect occurs for μ_c that was the main source of uncertainty at the NLO. This plot also explains why the previous NLO estimates for the branching ratio were significantly higher than the current NNLO one.

The final result of the current analysis is

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm SM} = (3.15 \pm 0.23) \times 10^{-4} \tag{2}$$

for the branching ratio with $E_\gamma > 1.6\,\mathrm{GeV}$. The uncertainty has been obtained by combining four types of uncertainties in quadrature:

- The 5% non-perturbative uncertainty due to the $\alpha_s \Lambda/m_b$ effects that have been mentioned in the beginning of my talk. Here, a dedicated analysis is necessary. A step in this direction has already been made in Ref. [6].

- The 3% parametric uncertainty that is dominated by $\alpha_s(M_Z)$, the measured semileptonic branching ratio, and the charm quark mass m_c.

The error due to m_c would have been much larger if no normalization to the measured semileptonic rate was applied. Once it is applied, a fortunate cancellation of uncertainties occurs.

- The m_c-interpolation ambiguity is estimated [19] to be around 3%. A complete three-loop matrix element calculation even in the m_c=0 case would help a lot. Then the m_c-interpolation would become a real interpolation, not an "interpolation with an assumption" as it is now.

- Finally the 3% of the higher-order QCD uncertainty is estimated by studying the renormalization scale dependence of the current results. This uncertainty is going to stay with us for a long time.

The SM result in Eq. (2) is around 1.2σ below the experimental one (1). This should be compared to the relatively recent situation when the NLO SM result was significantly above the measurement, though within the 1σ error. Now the experimental results have moved up and, at the same time, the NNLO corrections have pushed the SM result down. Although no serious disagreement between theory and experiment arises, such a change has a visible impact on beyond-SM physics constraints. For instance, the lower bound on the charged Higgs boson mass in the THDM II goes down to around 300 GeV, while for $M_{H^\pm} \simeq 650$ GeV, the THDM II fits the data even slightly better than the SM (see Ref. [2] for details). More generally, any beyond-SM model that predicts a suppression of $b \to s\gamma$ becomes disfavored, while models predicting a small enhancement survive more easily.

4. Summary

The NNLO QCD calculations of the $b \to s\gamma$ matching conditions and anomalous dimensions are now complete. The same refers to the perturbative matrix element of the dominant photonic dipole operator. As far as the three-loop matrix elements of the four-quark operators are concerned, they have been found only either in the so-called large-β_0 approximation or the $m_c \gg m_b/2$ limit. An interpolation in m_c is the basis for the current NNLO estimate $\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm SM} = (3.15 \pm 0.23) \times 10^{-4}$.

I am greatly indebted to the authors of Refs. [8–19] for their contributions to the NNLO $\bar{B} \to X_s\gamma$ calculations. I would like to thank the organizers of the Kazimierz meeting for their efforts and hospitality. Financial support from the EU Contracts HPRN-CT-2002-00311, "EURIDICE", MRTN-CT-2006-035482 "FLAVIAnet", as well as from the Polish State Committee for Scientific Research under the grant 2 P03B 078 26 is gratefully acknowledged.

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