# MERGING FLAVOUR SYMMETRIES <br> WITH QCD FACTORISATION FOR $B \rightarrow K K$ DECAYS* 

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The interplay between flavour symmetries connecting $B_{s} \rightarrow K K$ decays with the recently measured $B_{d} \rightarrow K^{0} \bar{K}^{0}$ decay and QCD Factorisation opens new strategies to describe the decays $B_{s} \rightarrow K^{0} \bar{K}^{0}$ and $B_{s} \rightarrow K^{+} K^{-}$ in the SM and in supersymmetry. A new relation, emerging from the sumrule for the $B_{s} \rightarrow K^{0} \bar{K}^{0}$ decay mode, is presented offering a new way to determine the weak mixing angle $\phi_{s}$ of the $B_{s}$ system.

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## 1. Introduction

The huge effort on the experimental side at present $B$ facilities (BaBar, Belle and CDF) to increase the precision on data measurements force us to revise the strategies on the theory side to produce more accurate predictions. Non leptonic $B$ decays offer different strategies to determine the Unitarity Triangle, to search for New Physics (NP) [1] but also to rule out models [2]. While a lot of attention has been devoted to the $B \rightarrow \pi K[3-7]$ decay modes, here we will focus on $B \rightarrow K K$ decays that have been observed at CDF [8] $\left(B_{s} \rightarrow K^{+} K^{-}\right)$, BaBar [9] and Belle [10] $\left(B_{d} \rightarrow K^{0} \bar{K}^{0}\right)$.

There are two main approaches in the literature to describe $B \rightarrow K K$ decays: flavour symmetries and $1 / m_{b}$-expansion methods (QCD Factorisation [11,12], soft collinear effective theories [13] or PQCD [14]). Each of those methods has pros and cons, that we will discuss in turn. Flavour symmetries, like U-spin symmetry that relates $B_{s} \rightarrow K^{+} K^{-}$with $B_{d} \rightarrow$ $\pi^{+} \pi^{-}$[15-18], provide a model independent analysis and extract most of the

[^0]needed hadronic parameters from data. However, this method has the disadvantage that relies strongly on the accuracy of data, and, at present, there is still not full agreement between BaBar and Belle data on the CP asymmetries of the $B_{d} \rightarrow \pi^{+} \pi^{-}$mode. As a consequence, error bars are still quite large, see for instance, the prediction $\mathrm{BR}\left(B_{s} \rightarrow K^{+} K^{-}\right)=\left(35_{-20}^{+73}\right) \times 10^{-6}[7]$ or, more recently, $4.2 \times 10^{-6} \leq \operatorname{BR}\left(B_{s} \rightarrow K^{+} K^{-}\right) \leq 60.7 \times 10^{-6}$ [18]. Also when relating $B_{d} \rightarrow \pi^{+} \pi^{-}$with $B_{s} \rightarrow K^{+} K^{-}$some of the needed U-spin parameters can only be roughly estimated and they are usually taken to be of the order of $20 \%$.

Concerning $1 / m_{b}$-expansion methods, here we will focus on QCD Factorisation (QCDF) [11, 12, 19]. The main idea is to exploit the existence of a large scale $m_{b} \gg \Lambda_{\mathrm{QCD}}$ together with colour transparency, that applies when the outgoing meson that does not contain the spectator quark is very energetic. At leading power in $\Lambda / m_{b}$ all long distance contributions can be parametrized in terms of form factors and light cone distribution amplitudes, while the contribution from energetic gluons comes in a perturbative series in $\alpha_{\mathrm{s}}$ and is incorporated into the hard scattering kernels. QCDF predicts some of the hadronic parameters reducing the error bars, however in the computation one has to face chirally enhanced IR divergences. They are formally suppressed by a power of $1 / m_{b}$, but can be numerically significant. They are modelled and induce an important uncertainty in the predictions.

However, there is a third possibility and it is the proposal presented in [20] that combines QCDF and Flavour symmetries giving rise to rather accurate predictions for the branching ratios of the above mentioned $B_{s} \rightarrow$ $K K$ decays in SM and in supersymmetry. Moreover, the method predicts some of the $\mathrm{SU}(3)$ breaking parameters which can be useful for other flavour approaches and, at the same time, deals with the problem of the chirally enhanced IR divergences coming at order $\Lambda / m_{b}$ (see also [21]).

Since IR divergences play a central role in this discussion, it is worth to mention the two sources of IR divergences in QCDF:

- Hard spectator-scattering: Hard gluons exchange between spectator quark and the outgoing energetic meson gives rise to integrals of the following type (see [12] for definitions):

$$
H_{i}\left(M_{1} M_{2}\right)=C \int_{0}^{1} d x \int_{0}^{1} d y\left[\frac{\Phi_{M_{2}}(x) \Phi_{M_{1}}(y)}{\bar{x} \bar{y}}+r_{\chi}^{M_{1}} \frac{\Phi_{M_{2}}(x) \Phi_{m_{1}}(y)}{x \bar{y}}\right]
$$

where the second term (formally of order $\Lambda / m_{b}$ ) diverges when $y \rightarrow 1$.

- Weak annihilation: These type of diagrams also exhibit endpoint IR divergences as it is explicit in the corresponding integrals:

$$
\begin{aligned}
A_{1}^{i}= & \pi \alpha_{\mathrm{s}} \int_{0}^{1} d x d y\left\{\Phi _ { M _ { 2 } } ( x ) \Phi _ { M _ { 1 } } ( y ) \left[\frac{1}{y(1-x \bar{y})}\right.\right. \\
& \left.\left.+\frac{1}{\bar{x}^{2} y}\right]+r_{\chi}^{M_{1}} r_{\chi}^{M_{2}} \Phi_{m_{2}}(x) \Phi_{m_{1}}(y) \frac{2}{\bar{x} y}\right\}
\end{aligned}
$$

Both divergences are modelled in the same way [12]:

$$
\int_{0}^{1} \frac{d y}{\bar{y}} \Phi_{m_{1}}(y)=\Phi_{m_{1}}(1) \int_{0}^{1} \frac{d y}{\bar{y}}+\int_{0}^{1} \frac{d y}{\bar{y}}\left[\Phi_{m_{1}}(y)-\Phi_{m_{1}}(1)\right] \equiv \Phi_{m_{1}}(1) X_{H, A}^{M_{1}}+r
$$

where $r$ is a finite piece and the divergent piece is cut-off by a physical scale of order $\Lambda_{\mathrm{QCD}}$ with an arbitrary complex coefficient to take into account possible multiple soft scattering: $X_{H, A}=\left(1+\rho_{H, A}\right) \ln \left(m_{b} / \Lambda\right)$.

## 2. Sum-rules: $\alpha$ and $\phi_{s}$

The SM amplitude for a $B$-decay into two mesons can be split into tree and penguin contributions $[19]^{1}: \bar{A} \equiv A\left(\bar{B}_{q} \rightarrow M \bar{M}\right)=\lambda_{u}^{(q)} T_{M}^{q C}+\lambda_{c}^{(q)} P_{M}^{q C}$, with $C$ denoting the charge of the decay products, and the products of CKM factors $\lambda_{p}^{(q)}=V_{p b} V_{p q}^{*}$. Tree and penguin contributions in $\bar{B}_{d} \rightarrow K^{0} \bar{K}^{0}$ in QCDF are:

$$
\begin{align*}
& \hat{T}^{d 0}=\alpha_{4}^{u}-\frac{1}{2} \alpha_{4 \mathrm{EW}}^{u}+\beta_{3}^{u}+2 \beta_{4}^{u}-\frac{1}{2} \beta_{3 \mathrm{EW}}^{u}-\beta_{4 \mathrm{EW}}^{u} \\
& \hat{P}^{d 0}=\alpha_{4}^{c}-\frac{1}{2} \alpha_{4 \mathrm{EW}}^{c}+\beta_{3}^{c}+2 \beta_{4}^{c}-\frac{1}{2} \beta_{3 \mathrm{EW}}^{c}-\beta_{4 \mathrm{EW}}^{c} \tag{1}
\end{align*}
$$

where $\hat{P}^{d 0}=P^{d 0} / A_{K K}^{d}, \hat{T}^{d 0}=T^{d 0} / A_{K K}^{d}$, the super-scripts identify the channel, the normalisation is $A_{K K}^{q}=M_{B_{q}}^{2} F_{0}^{\bar{B}_{q} \rightarrow K}(0) f_{K} G_{F} / \sqrt{2}$ (see [12,20] for the corresponding expression of the $B_{s} \rightarrow K K$ channels). Following the observation in [20] that the structure of the IR divergences is the same, independently of the charm or up quark running in the loop, we identified an IR-safe quantity at NLO in QCDF that we called $\Delta_{d} \equiv T^{d 0}-P^{d 0}$. All chirally enhanced IR divergences cancel exactly in this quantity at this order. Its explicit expression in terms of the coefficients in Eq. (1) is:

$$
\Delta_{d}=A_{k k}^{d}\left[\alpha_{4}^{u}-\alpha_{4}^{c}+\beta_{3}^{u}-\beta_{3}^{c}+2 \beta_{4}^{u}-2 \beta_{4}^{c}\right]
$$

[^1]where electroweak contributions are neglected. This quantity can be safely evaluated in QCDF and the result found in [20] was: $\Delta_{d}=(1.09 \pm 0.43) \times$ $10^{-7}+i(-3.02 \pm 0.97) \times 10^{-7} \mathrm{GeV}$. The largest uncertainty entering $\Delta_{d}$ comes from the ratio $m_{c} / m_{b}$ and the scale dependence. Interestingly, this quantity can be expressed in terms of observables, providing a relation between the direct induced CP-asymmetry $\left(A_{\text {dir }}^{d 0}\right)$, the mixing induced CP-asymmetry ( $A_{\text {mix }}^{d 0}$ ) and the branching ratio $\left(\mathrm{BR}^{d 0}\right)$ of $\bar{B}_{d} \rightarrow K^{0} \bar{K}^{0}$ :
$\left|\Delta_{d}\right|^{2}=\frac{\mathrm{BR}^{d 0}}{L_{d}}\left\{x_{1}+\left[x_{2} \sin \phi_{d}-x_{3} \cos \phi_{d}\right] A_{\text {mix }}^{d 0}-\left[x_{2} \cos \phi_{d}+x_{3} \sin \phi_{d}\right] A_{\Delta}^{d 0}\right\}$,
where $\left|A_{\Delta}^{d 0}\right|^{2}+\left|A_{\text {dir }}^{d 0}\right|^{2}+\left|A_{\text {mix }}^{d 0}\right|^{2}=1, \phi_{d}$ is the weak mixing angle for the $B_{d}$ system, $L_{d}=\tau_{d} \sqrt{M_{B d}^{2}-4 M_{K}^{2}} /\left(32 \pi M_{B d}^{2}\right)$ and $x_{i}$ are functions of $\gamma$ and CKM elements. All SM inputs are taken as in [20] following [22].

Moreover, it was found in [23] that this sum-rule encodes also a very interesting information. It provides a new way of measuring $\sin \alpha$ :

$$
\sin ^{2} \alpha=\frac{\mathrm{BR}^{d 0}}{4 L_{d}\left|\lambda_{u}^{(d)}\right|^{2}\left|\Delta_{d}\right|^{2}}\left(1-\sqrt{1-{A_{\mathrm{dir}}^{d 0}}^{2}-{A_{\mathrm{mix}}^{d 0}}^{2}}\right) .
$$

(See [24] for a recent review on the extraction of $\alpha$ ). In a similar way, it was found in [20] a corresponding IR safe quantity $\Delta_{s} \equiv T^{s 0}-P^{s 0}$ and sum-rule for the decay $\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0}$ :

$$
\left|\Delta_{s}\right|^{2}=\frac{\mathrm{BR}^{s 0}}{L_{s}}\left\{y_{1}+\left[y_{2} \sin \phi_{s}-y_{3} \cos \phi_{s}\right] A_{\mathrm{mix}}^{s 0}-\left[y_{2} \cos \phi_{s}+y_{3} \sin \phi_{s}\right] A_{\Delta}^{s 0}\right\},
$$

which provides a completely new way to determine the weak mixing angle $\phi_{s}$ that we present here (see [20] for definitions):

$$
\sin ^{2} \frac{\phi_{s}}{2}=\frac{\mathrm{BR}^{s 0}}{4 L_{s}\left|\lambda_{c}^{(s)}\right|^{2}\left|\Delta_{s}\right|^{2}}\left(1-\sqrt{1-{A_{\mathrm{dir}}^{s 0}}^{2}-{A_{\mathrm{mix}}^{s 0}}^{2}}\right) .
$$

This implies that a measurement of the branching ratio, direct CP asymmetry and mixing induced CP asymmetry of the decay $B_{s} \rightarrow K^{0} \bar{K}^{0}$ automatically translates into a value for $\sin ^{2} \phi_{s} / 2$. Finally, given the relation $\Delta_{s}=f \Delta_{d}$, where $f=A_{K K}^{s} / A_{K K}^{d}$ a new relation between $\sin \alpha$ and $\sin \phi_{s} / 2$ immediately emerges.

## 3. Description of the method: Flavour Symmetries and QCDF

$$
\text { 3.1. } B_{s} \rightarrow K^{0} \bar{K}^{0}
$$

The SM amplitude of this $b \rightarrow s$ penguin decay is given by:

$$
\bar{A} \equiv A\left(\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0}\right)=\lambda_{u}^{(s)} T^{s 0}+\lambda_{c}^{(s)} P^{s 0}
$$

Its dynamics is described in terms of three parameters: $\left|T^{s 0}\right|,\left|P^{s 0}\right|$ and the relative strong phase $\arg \left(P^{s 0} / T^{s 0}\right)$ (remember that $T^{s 0}$ stands for the piece proportional to $\lambda_{u}^{(s)}$ but it is not due to an actual tree diagram in this case). Its hadronic parameters can be related via U-spin with the hadronic parameters $\left(\mid T^{d 0}\right\rfloor,\left|P^{d 0}\right|$ and $\left.\arg \left(P^{d 0} / T^{d 0}\right)\right)$ of the also penguin governed mode $B_{d} \rightarrow K^{0} K^{0}$. This has several advantages: first, we can expect similar final state interactions (although not equal), second, the sources of U-spin breaking can be better controlled using QCDF. These sources are: (i) the factorisable ratio $f=A_{K K}^{s} / A_{K K}^{d}$ (extrapolated from the lattice) (ii) U-spin breaking $1 / m_{b}$ suppressed terms $\delta \alpha_{i}$ and $\delta \beta_{i}$ : sensitive to the difference of $B_{d}$ and $B_{s}$ distribution amplitudes and spectator quark dependent contributions coming from a gluon emitted from the $d$ or $s$ quark. This leads to the relations $\left|P^{s 0} /\left(f P^{d 0}\right)-1\right| \leq 3 \%$ and $\left|T^{s 0} /\left(f T^{d 0}\right)-1\right| \leq 3 \%$.

The next step is to determine the hadronic parameters $\left(T^{d 0}, P^{d 0}\right)$ of the decay $\bar{B}_{d} \rightarrow K^{0} \bar{K}^{0}$. This is done using as inputs the $\operatorname{BR}\left(\bar{B}_{d} \rightarrow K^{0} \bar{K}^{0}\right)$ and $\Delta_{d}$ (from QCDF). The direct CP asymmetry $A_{\operatorname{dir}}\left(\bar{B}_{d} \rightarrow K^{0} \overline{K^{0}}\right)$ (denoted by $\left.A_{\text {dir }}^{d 0}\right)$ will be taken as a free parameter. The combination of those constraints gives rise to a set of non-linear equations (see definitions in [20]):

$$
\begin{align*}
x_{C}+i y_{C} & =-\Delta_{d}(1-\cos \gamma / R) / a \\
r^{2} & =\rho_{0}^{2} /\left[a\left|\lambda_{u}^{(d)}\right|^{2}\right]-\left[\sin \gamma\left|\Delta_{d}\right| /(a R)\right]^{2} \\
y_{P} x_{\Delta} & =y_{\Delta} x_{P}-\rho_{0}^{2} A_{\mathrm{dir}}^{d 0} /\left(2\left|\lambda_{u}^{(d)} \lambda_{c}^{(d)}\right| \sin \gamma\right), \tag{2}
\end{align*}
$$

that determines $P^{d 0}=x_{P}+i y_{P}$, then using $\Delta_{d}$ one gets $T^{d 0}$. Two remarks are important here: first, there is a twofold ambiguity in the sign of $\operatorname{Im} P^{d 0}$ (solved in the next subsection). Second, current data together with our knowledge on $\Delta_{d}$ limits the $A_{\text {dir }}^{d 0}$ asymmetry (by means of Eqs. (2)) within a restricted range between $-0.2 \leq A_{\text {dir }}^{d 0} \leq 0.2$.

Finally, our SM predictions for the branching ratio and CP asymmetries of $\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0}$ are obtained using the relations between $P^{s 0} \leftrightarrow P^{d 0}$ and $T^{s 0} \leftrightarrow T^{d 0}$ mentioned above and including all U-spin breaking sources together with the QCDF uncertainties in $\Delta_{d}$. The resulting predictions are $\operatorname{BR}\left(B_{s} \rightarrow K^{0} \bar{K}^{0}\right)=(18 \pm 7 \pm 4 \pm(2)) \times 10^{-6},\left|A_{\text {dir }}\left(B_{s} \rightarrow K^{0} \bar{K}^{0}\right)\right| \leq 1.1 \times 10^{-2}$ and $\left|A_{\text {mix }}\left(B_{s} \rightarrow K^{0} \bar{K}^{0}\right)\right| \leq 1.5 \times 10^{-2}$. The sign of these asymmetries can be fixed once $A_{\text {dir }}^{d 0}$ will be measured with enough accuracy.

$$
\text { 3.2. } B_{s} \rightarrow K^{+} K^{-}
$$

The analysis of the decay mode $B_{s} \rightarrow K^{+} K^{-}$follows similar steps, but with some important differences: (i) we will use U-spin and isospin to connect $B_{d} \rightarrow K^{0} \bar{K}^{0}$ with $B_{s} \rightarrow K^{+} K^{-}$, (ii) $B_{s} \rightarrow K^{+} K^{-}$contains a tree (denoted by $\bar{\alpha}_{1}$ in QCDF) with no counterpart in $B_{d} \rightarrow K^{0} \bar{K}^{0}$, (iii) $\operatorname{BR}\left(B_{s} \rightarrow K^{+} K^{-}\right)$has been measured with excellent precision at CDF [8] and (iv) we will use the information only on the sign predictions (not the absolute value) for the CP-asymmetries of $B_{s} \rightarrow K^{+} K^{-}$from the strategy that uses U-spin to relate $B_{s} \rightarrow K^{+} K^{-}$with $B_{d} \rightarrow \pi^{+} \pi^{-}$.

The hadronic parameters describing the amplitude:

$$
A\left(\bar{B}_{s} \rightarrow K^{+} K^{-}\right)=\lambda_{u}^{(s)} T^{s \pm}+\lambda_{c}^{(s)} P^{s \pm}
$$

are obtained from the relations containing the sub-leading $1 / m_{b}$ U-spin breaking: $\left|P^{s \pm} /\left(f P^{d 0}\right)-1\right| \leq 2 \%,\left|T^{s \pm} /\left(A_{k k}^{s} \bar{\alpha}_{1}\right)-1-T^{d 0} /\left(A_{k k}^{d} \bar{\alpha}_{1}\right)\right| \leq 4 \%$. Those errors, estimated within QCDF, are stretched roughly by a factor two to be conservative.

The two-fold ambiguity on the sign of $\operatorname{Im} P^{d 0}$ is lifted here using U-spin arguments based on the $B_{d} \rightarrow \pi^{+} \pi^{-}$strategy. While the signs of $\operatorname{Im} P^{d 0}$ and $A_{\text {dir }}^{s \pm}$ are correlated (being both positive or negative), the prediction based on the $B_{d} \rightarrow \pi^{+} \pi^{-}$strategy points towards a positive sign for $A_{\mathrm{dir}}^{s \pm}$ [18], discarding then the solution with $\operatorname{Im} P^{d 0}<0$.

Our result in the SM for the branching ratio of $B_{s} \rightarrow K^{+} K^{-}$averaging over all values of $A_{\text {dir }}^{d 0}$ is $\operatorname{BR}\left(B_{s} \rightarrow K^{+} K^{-}\right)=(20 \pm 8 \pm 4 \pm(2)) \times 10^{-6}$, where the last error in parenthesis stands for a rough estimate of finite, non-enhanced $\Lambda / m_{b}$ corrections. Finally, confronting our predictions for $B_{s} \rightarrow K^{+} K^{-}$with the data on $B_{d} \rightarrow \pi^{+} \pi^{-}[25]:\left|T_{\pi \pi}^{d \pm}\right|=(5.48 \pm 0.42) \times 10^{-6}$ and $\left|P_{\pi \pi}^{d \pm} / T_{\pi \pi}^{d \pm}\right|=0.13 \pm 0.05, \arg \left(P_{\pi \pi}^{d \pm} / T_{\pi \pi}^{d \pm}\right)=(131 \pm 18)^{\circ}$, provides a double information. First, we can give predictions for the U-spin breaking parameters: $\mathcal{R}_{\mathcal{C}}=\left|T^{s \pm} / T_{\pi \pi}^{d \pm}\right|=2.0 \pm 0.6$ and $\xi=\left|P^{s \pm} / T^{s \pm}\right| /\left|P_{\pi \pi}^{d \pm} / T_{\pi \pi}^{d \pm}\right|=$ $0.8 \pm 0.3$ connecting $B_{s} \rightarrow K^{+} K^{-}$with $B_{d} \rightarrow \pi^{+} \pi^{-}$. These parameters can be compared with the QCD sum rules predictions in Ref. [26]. Notice that while QCD sum rules give only the factorizable part, our predictions include, in principle, the full contribution. Second, a comparison between the two relative strong phases $\arg \left(P^{s \pm} / T^{s \pm}\right)$ and $\arg \left(P_{\pi \pi}^{d \pm} / T_{\pi \pi}^{d \pm}\right)$ selects $A_{\text {dir }}^{d 0} \geq 0$. Then, if we restricts only to positive values of $A_{\text {dir }}^{d 0}$ according to the previous arguments, our SM predictions turn out to be [20,27]:

$$
\begin{array}{r}
\mathrm{BR}\left(B_{s} \rightarrow K^{+} K^{-}\right) \quad=\quad(17 \pm 6 \pm 3 \pm(2)) \times 10^{-6} \\
-0.22 \leq A_{\mathrm{dir}}^{s \pm} \leq 0.49 \text { and }-0.55 \leq A_{\mathrm{mix}}^{s \pm} \leq 0.02 \tag{3}
\end{array}
$$

Another argument in favour of $A_{\text {dir }}^{d 0} \geq 0$ comes from the preference of $A_{\text {mix }}^{s \pm}<0$ of the U-spin based $B_{d} \rightarrow \pi^{+} \pi^{-}$strategy [17] (see the anticorrelation between $A_{\text {dir }}^{d 0}$ and $A_{\text {mix }}^{s \pm}$ in Table 1 of [20]).

The accuracy on these CP-asymmetries will be substantially improved once a precise measurement on $A_{\text {dir }}^{d 0}$ will be available or the error of $\mathrm{BR}\left(B_{d} \rightarrow\right.$ $K^{0} \bar{K}^{0}$ ) and the QCD uncertainties on $\Delta_{d}$ (mainly $m_{c} / m_{b}$ and scale dependence) will be reduced.

### 3.3. Supersymmetry

The leading gluino-squark box and penguin contributions [3] to $B_{s} \rightarrow$ $K^{0} \bar{K}^{0}$ and $B_{s} \rightarrow K^{+} K^{-}$were evaluated first in [18] using the U-spin flavour strategy with $B_{d} \rightarrow \pi^{+} \pi^{-}$and, afterwards, in [27] using the new method combining flavour and QCDF [20]. The relative size of this contribution compared to the SM penguin is $\left(\alpha_{\mathrm{s}} / M_{\text {susy }}^{2}\right) /\left(\alpha / M_{W}^{2}\right) \sim 1$. The amplitude of these decays in presence of NP contains an extra contribution: $\mathcal{A}\left(B_{s}^{0} \rightarrow\right.$ $\left.K^{+} K^{-}\right)=\mathcal{A}_{\mathrm{SM}}^{s \pm}+\mathcal{A}^{u} e^{i \Phi_{u}}, \mathcal{A}\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right)=\mathcal{A}_{\mathrm{SM}}^{s 0}+\mathcal{A}^{d} e^{i \Phi_{d}}$. These NP amplitudes $\mathcal{A}^{u} e^{\Phi_{u}}$ and $\mathcal{A}^{d} e^{\Phi_{d}}$ in terms of Wilson coefficients are:
$\mathcal{A}^{q} e^{i \Phi_{q}}=\frac{G_{F}}{\sqrt{2}}\left[-\chi\left(\frac{1}{3} \bar{c}_{1}^{q}+\bar{c}_{2}^{q}\right)-\frac{1}{3}\left(\bar{c}_{3}^{q}-\bar{c}_{6}^{q}\right)-\left(\bar{c}_{4}^{q}-\bar{c}_{5}^{q}\right)-\lambda_{t} \frac{2 \alpha_{\mathrm{s}}}{3 \pi} \bar{C}_{8 g}^{\mathrm{eff}}\left(1+\frac{\chi}{3}\right)\right] A$,
with $q=u, d, A=i\left(m_{B}^{2}-m_{K}^{2}\right) f_{K} F^{B_{s} \rightarrow K}$ and $\chi=1.18$ (see [27] for definitions). These Wilson coefficients are sensitive to the $\tilde{s}-\tilde{b}$ mass splitting. After including the constraints coming from $\operatorname{BR}\left(B \rightarrow X_{s} \gamma\right), B \rightarrow \pi K$ and $\Delta M_{s}$ we found that a large isospin violation controlled by the mass splitting $\widetilde{u}_{R}-\widetilde{d}_{R}$ is possible between the NP amplitudes $\mathcal{A}^{u} e^{i \Phi_{u}}$ and $\mathcal{A}^{d} e^{i \Phi_{d}}$. For the region of parameters considered in this supersymmetric scenario, $\mathcal{A}^{u} e^{i \Phi_{u}}$ can be up to a factor three larger than $\mathcal{A}^{d} e^{i \Phi_{d}}$. The specific results in supersymmetry for each decay mode are [27]:

- $B_{s}^{0} \rightarrow K^{+} K^{-}$: The branching ratio is very little affected by SUSY. At most, the SM prediction can be increased by $15 \%$ for $A_{\text {dir }}^{d 0}=0.1$, increasing a bit the already good agreement with the new CDF data [8]. The direct CP asymmetry within SUSY falls inside the range $-0.1 \lesssim$ $A_{\mathrm{dir}}\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)^{\mathrm{SUSY}} \lesssim 0.7$ for $-0.1 \leq A_{\mathrm{dir}}^{d 0} \leq 0.1$. The deviation depends on the relative size of the competing SM tree versus the NP amplitude. $A_{\text {mix }}\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)^{\text {SUSY }}$ can take any value from $[-1,1]$.
- $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}$ : The impact of SUSY on $\operatorname{BR}\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right)$ is even smaller, reflecting the reduced allowed region for $\mathcal{A}^{d} e^{i \Phi_{d}}$ as compared to $\mathcal{A}^{u} e^{i \Phi_{u}}$. The situation is very different for the CP asymmetries, that are particularly promising, due to the very small size of their SM
prediction [20]. The direct CP asymmetry in SUSY can be 10 times larger than the SM one. $A_{\text {mix }}\left(B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}\right)^{\text {SUSY }}$ covers the entire range, and so this asymmetry can be large in the presence of SUSY, contrary to the SM prediction.

The method discussed here is being applied to other non-leptonic $B$-decays.

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[^1]:    ${ }^{1}$ Conventionally, we will call "tree" the piece proportional to $\lambda_{u}^{(q)}$ and "penguin" the piece proportional to $\lambda_{c}^{(q)}$, even if applied to decays with no actual tree diagram.

