BETA BEAMS — AN ALTERNATIVE TO DOUBLE BETA DECAY?*

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(Received June 28, 2007)

It is shown that Majorana neutrino's absolute mass scale measurement can in principle be carried out using intense β beams. This could be achieved by counting the lepton number violating events in a two step process: the nuclear decay in flight and the subsequent neutrino induced interaction. The relativistic boost results in the gain in the content of Majorana neutrino helicities responsible for the lepton number violation. A simple formula to calculate this gain is presented. Specific examples of the two step processes are indicated and relevant cross sections are given.

PACS numbers: 23.40.–s, 14.60.Pq, 23.40.Bw

1. Introduction

The leading questions in neutrino physics concern the transformation properties of neutrinos under charge conjugation and the values of neutrino masses. The answers can be obtained if the neutrino-less double beta decay is observed and its rate is measured [1]. In this note we show how these answers can come from experiments using intense $\nu_e(\bar{\nu}_e)$ beams produced by radioactive ions decaying in flight (the "beta beams" [2]). Generally there are two kinds of nuclear decays in flight: three-body decays, *e.g.*

$$^{6}\mathrm{He} \rightarrow^{7}_{3} \mathrm{Li} + e^{-} + \bar{\nu}_e \,, \tag{1}$$

and two-body decays, *e.g.*

$$^{178}\mathrm{W}^{74+} \rightarrow^{178} \mathrm{Ta}^{73+} + \nu_e \,. \tag{2}$$

* Presented by L. Łukaszuk at The Final EURIDICE Meeting "Effective Theories of Colours and Flavours: from EURODAPHNE to EURIDICE", Kazimierz, Poland, 24–27 August, 2006.

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Let us consider now Majorana neutrinos *i.e.* $\bar{\nu}_e = \nu_e = \nu_e^{\mathrm{Majorana}}$. Then the total lepton number non-conservation could be observed, *e.g.*

$$^{6}\mathrm{He} \rightarrow^{7}_{3} \mathrm{Li} + e^{-} + \nu_e^{\mathrm{Majorana}} \tag{3}$$

$$\nu_e^{\mathrm{Majorana}} N \rightarrow e^{-} X \,, \tag{4}$$

or

$$^{178}\mathrm{W}^{74+} \rightarrow^{178} \mathrm{Ta}^{73+} + \nu_e^{\mathrm{Majorana}} \tag{5}$$

$$\nu_e^{\mathrm{Majorana}} N \rightarrow e^{+} X' \quad . \tag{6}$$

Similar two step processes have already been considered in the past by Langacker and Wang [3] for pion and reactor neutron decays. Authors found effects too small to be observed in foreseeable laboratory experiments [3]. In the case of intense beta beams we could expect to obtain less pessimistic results. The neutrinos produced during the flight will undergo a kinematical focusing and a relativistic boost. The former concentrates the neutrino beam at the detector (the "target"), the latter offers a dramatic increase in the detector efficiency. Moreover, rest frame ratio $\frac{E_\nu - p_\nu}{E_\nu + p_\nu} \approx (\frac{m_\nu}{2E_\nu})^2$ reflecting smallness of the effect [3] will be larger in our considerations; *e.g.* for W decay $E_\nu \approx 25$ KeV and for He decay E_ν is contained in the range $(m_\nu, 3.5$ MeV). Finally, amount of "wrong helicity" neutrinos should increase due to the effect of relativistic boost. Both nuclear decays and subsequent neutrino-induced reactions will be discussed under the assumption that the weak leptonic charge current is of (V–A) form and that three Majorana neutrinos are involved. We expect that the reaction rates of the type (3),(4) (or (5),(6)) will be low, proportional to m_ν^2. In the following we shall limit our discussion to the case of quasi degenerate (QD) mass spectrum [4]:

$$m_1 \approx m_2 \approx m_3 \approx m \,, \tag{7}$$

where

$$m_j^2 >> \mid \Delta m_{\mathrm{atm}} \mid^2 , \tag{8}$$

and

$$\mid \Delta m_{\mathrm{atm}} \mid^2 = 2.210^{-3} \ \mathrm{eV}^2 \,. \tag{9}$$

As a consequence it is sound to simplify our formulae using the following approximation:

$$m_1 = m_2 = m_3 = m \,. \tag{10}$$

We can safely assume that the source-detector distance and the detector size are small compared to the electron neutrino oscillation length. Our formalism differs from that of Ref. [3] and it is presented in Section 2. Section 3 contains discussion of neutrinos helicity basis in the laboratory frame. Results are presented in Section 4.

2. General formulae

The neutrino-target flux averaged cross section has the following form:

$$\langle \sigma^\rho \rangle = \int w(\vec{p}) \sigma^\rho(p) d^3p \,, \tag{11}$$

with

$$\sigma^\rho(p) = \rho^{a_1}(p)\sigma(a_1, E) + \rho^{a_2}(p)\sigma(a_2, E) \,, \tag{12}$$

where $w(\boldsymbol{p})$ is neutrino's momentum probability distribution in the laboratory frame, ρ^{a_i} are neutrino spin density matrix elements in the diagonal basis $\mid a_1\rangle$, $\mid a_2\rangle$. The basis states $\mid a_i\rangle$, can be always expressed as (momentum dependent) combination of helicity states. The lab distribution $w(\vec{p})$ can be calculated once distribution $w_{\rm CM}(\vec{p})$ in the rest frame of decaying nucleus is known (compare [5]):

$$w(\vec{p}) = \frac{w_{\rm CM}(\vec{p}_{\rm CM}) E_{\rm CM}}{E} \,, \tag{13}$$

where $(E_{\rm CM}, \vec{p}_{\rm CM})$ and $(E, \vec{p})$ are related by the boost transformation. The c.m. distributions are well known (see, *e.g.* [5]). For β decay

$$w^{\beta}_{\rm CM}(p_{\rm CM}) = \frac{f(E_{\rm CM})}{4\pi \int f(E) pE dE} \,, \tag{14}$$

with

$$f(E_{\rm CM}) = E_e \sqrt{E_e^2 - m_e^2} F(\pm Z, E_e) \Theta(E_e - m_e) \,, \tag{15}$$

where m_e is the electron mass and $E_e = Q - E_{\rm CM}$, Q being the Q value of the decay, F denotes Fermi's function. In the case of electron capture neutrino has well defined energy in c.m., E_0, and

$$w^{EC}_{\rm CM}(p_{\rm CM}) = \frac{\delta(E_{\rm CM} - E_0)}{4\pi p E_0} \,. \tag{16}$$

The diagonal spin density matrix ρ^a can be related, via Lorentz transformation, to the c.m. helicity density matrix [6]. In this note we shall deal with Gamow–Teller transitions. Then $\rho_{\rm CM}$ is diagonal and

$$\rho^{\rm CM}_{\pm 1/2, \pm 1/2} = \frac{1}{2} \left(1 \pm \frac{p_{\rm CM}}{E_{\rm CM}} \right) \tag{17}$$

for β^- decays and

$$\rho^{\rm CM}_{\pm 1/2, \pm 1/2} = \frac{1}{2} \left(1 \mp \frac{p_{\rm CM}}{E_{\rm CM}} \right) \tag{18}$$

for both β^+ decays and electron capture transitions.

For the diagonal $\rho_{\rm CM}$ the basis states $\mid a_i\rangle$ are the following combinations of lab helicity states $\mid p,\lambda\rangle$,

$$\mid a_1\rangle = \cos(\alpha/2)\mid p,\lambda=1/2\rangle - \sin(\alpha/2)\mid p,\lambda=-1/2\rangle\,, \tag{19}$$

$$\mid a_2\rangle = \sin(\alpha/2)\mid p,\lambda=1/2\rangle + \cos(\alpha/2)\mid p,\lambda=-1/2\rangle\,, \tag{20}$$

and

$$\rho^{a1} = \rho^{\rm CM}_{1/2,1/2} \quad , \quad \rho^{a2} = \rho^{\rm CM}_{-1/2,-1/2}\,, \tag{21}$$

where (compare Chapter 2.2.4 in Ref. [6])

$$\cos\alpha \;=\; \frac{\gamma}{p}(p_{\rm CM} + \beta E_{\rm CM}\cos\theta_{\rm CM})\,, \tag{22}$$

$$\sin\alpha \;=\; \frac{m_\nu\gamma\beta}{p}\sin\theta_{\rm CM}\,. \tag{23}$$

3. Gain factors and the helicity basis

We shall start this section with the discussion of boost enhancement of these helicities, which are produced with a smaller rate in c.m. of decay products. Let us take β^- decay and define gain factor R for helicity $h=-\frac{1}{2}$:

$$R(p) = \frac{\rho_{-1/2,-1/2}(p)}{\rho^{\rm CM}_{-1/2,-1/2}(p_{\rm CM}(p))} \tag{24}$$

$\rho^{\rm CM}$ is given in Eq. (17), while $\rho_{-1/2,-1/2}$ can be obtained from Eqs. (19)–(21):

$$\begin{aligned}\rho_{-1/2,-1/2} \;&=\; \rho^{a1}\mid\langle a_1\mid\lambda=-1/2\rangle\mid^2 + \rho^{a2}\mid\langle a_2\mid\lambda=-1/2\rangle\mid^2\\ &=\; \frac{1}{2}\left(1-\cos(\alpha)\,\frac{p_{\rm CM}}{E_{\rm CM}}\right)\,.\end{aligned} \tag{25}$$

Using Eq. (22), and the boost relation

$$E = \gamma(E_{\rm CM} + \beta p_{\rm CM}\cos\theta_{\rm CM}) \tag{26}$$

we can write $R(p)$ as

$$R(p) = \frac{1}{2}\left(1+\frac{p_{\rm CM}}{E_{\rm CM}}\right)\frac{E}{p}\left[\frac{2E_{\rm CM}\gamma}{E} - \frac{2E^2_{\rm CM}}{E(E+p)}\right]\,. \tag{27}$$

In this paper we shall work in the regime

$$E^2 \gg E^2_{\rm CM} \gg m^2_\nu\,. \tag{28}$$

Then

$$R(p) \approx \frac{E^{\max}}{E}\,, \tag{29}$$

where

$$E^{\max} = 2\gamma E_0 \tag{30}$$

is the largest energy of boosted neutrino's distribution (*e.g.* for $E = 20$ MeV, $E_{\mathrm{CM}} = 1$ MeV and $\gamma = 10^3$ the approximation works and we get $R = 100$). The largest values of $R(E)$ come from the backward c.m. cone. It is evident once we re-write Eq. (29) as

$$R(p) = \frac{2}{1 + \beta\frac{p_{\mathrm{CM}}}{E_{\mathrm{CM}}}\cos\theta_{\mathrm{CM}})}\,. \tag{31}$$

Let us add that quite large values of R can co-exist with small helicity mixing $\alpha \ll 1$ in Eqs. (19) and (20). In fact we can write Eq. (24) as

$$1 - \cos\alpha = (R-1)\left(\frac{E_{\mathrm{CM}}}{p_{\mathrm{CM}}} - 1\right) \approx \ (R-1)\frac{m_\nu^2}{2E_{\mathrm{CM}}^2}\,, \tag{32}$$

and $\alpha \ll 1$ even for

$$1 \ll R \ll \frac{2E_{\mathrm{CM}}^2}{m_\nu^2}\,. \tag{33}$$

Having this in mind, let us make transition from $\mid a_i\rangle$ basis, where $\sigma^\rho(p)$ was given by Eq. (12), to the helicity basis:

$$\sigma^\rho(p) = \rho_{1/2,1/2}(p)\sigma_+(E) + \rho_{-1/2,-1/2}(p)\sigma_-(E) + 2\rho_{-1/2,1/2}(p)\Delta\,, \tag{34}$$

where $\rho_{\lambda,\lambda'}(p)$ can be easily obtained from Eqs. (17),(18) and Eqs. (19),(20)

$$\rho_{\pm 1/2,\pm 1/2} = \frac{1}{2}\left(1 \pm \eta\frac{p_{\mathrm{CM}}}{E_{\mathrm{CM}}}\cos\alpha\right)\,, \tag{35}$$

$$\rho_{\pm 1/2,\mp 1/2} = -\frac{1}{2}\eta\frac{p_{\mathrm{CM}}}{E_{\mathrm{CM}}}\sin\alpha\,. \tag{36}$$

In Eqs. (35),(36) $\eta = +1$ for β^- decay and $\eta = -1$ for β^+ decay or electron capture transition. In Eq. (34) $\sigma_\pm(E)$ are cross sections for neutrino helicities $h = \pm 1/2$, while Δ contains mixed helicity terms. We shall not calculate Δ in detail; it can be shown, using bounds following from Schwartz inequality, that for neutrinos with energies exceeding 1 GeV this term can be safely neglected.

4. Results

We shall discuss two examples of $\Delta L = 2$ processes. First, neutrino from electron capture in the flight

$$^{178}\mathrm{W}^{74+} \rightarrow ^{178}\mathrm{Ta}^{73+} + \nu_e \tag{37}$$

reacts with nucleons in the nuclear target. The contribution from neutrino energies in the range (1 GeV, 10 GeV) to the averaged lepton number violating neutrino–nucleon cross section reaches $0.5 \times 10^{-47}((m_\nu)/\mathrm{eV})^2\mathrm{cm}^2$ for $\gamma = 10^4$; for $10^5 < \gamma < 5 \times 10^{13}$ it reaches $1.2 \times 10^{-47}((m_\nu)/\mathrm{eV})^2\mathrm{cm}^2$.There is no contribution for $\gamma < 2 \times 10^4$. As an example of 3-body decays in the flight we take ^{6}He nucleus. In this case the reactions considered are

$$\nu N \rightarrow e^- X \quad \text{for} \quad 1 \text{ GeV} < E_\nu < 10 \text{ GeV}\,, \tag{38}$$

and the cross sections are: $(1.2{\times}10^{-52}, 8{\times}10^{-51}, 10^{-50}, 10^{-50})((m_\nu)/\mathrm{eV})^2\mathrm{cm}^2$ for $\gamma = (10^3, 10^4, 10^5, 10^6)$. For $\gamma > 10^4$ the contributions come from neutrinos collimated along the direction of flight ($\theta < 10^{-3}$). For an iron target distributed along 1 km in a cone with ($\theta < 5 \times 10^{-4}$) and for $\gamma = 10^4$, 1 GeV $< E_\nu <$ 10 GeV) one needs 10^{21} decays of helium beam in order to observe $4((m_\nu)/\mathrm{eV})^2$ lepton number violating events. The corresponding number for $\gamma = 10^5$ and 1 GeV $< E_\nu <$ 100 GeV would be $40((m_\nu)/\mathrm{eV})^2$.

We would like to thank Piotr Chankowski, Milind Diwan, Tadeusz Kozlowski, Elliot Leader and Adam Para for their interest, comments and discussions. Partial support by the European Community's Human Potential Programme under contract HPRN-CT-2002-00311 EURIDICE and by Polish–French Scientific Agreement Polonium is acknowledged.

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