FANO RESONANCES WITH HADRONIC ATOMS*

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The formation of hadronic atoms in multichannel systems is discussed. These atoms created in intermediate states produce different signals in the inelastic and elastic transitions. In the first case the scattering amplitude is dominated by a simple pole. In the second case an accompanying zero arises. Such structures may be quite distinct in two channel cases but disappear with the increasing number of open channels.

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1. Introduction

Systems composed of two mesons are of great interest for theorists. First, due to the apparent simplicity of the related quark structure and second due to the applicability of effective theories in the low energy region. The latter aspect, makes scattering lengths to be the quantities of prime interest. In principle, the easiest way to measure the length is a low energy scattering experiment. Unfortunately, mesonic targets are not available and a comparable information has to be extracted from mesonic atoms. The atomic level shifts and broadenings, due to nuclear interactions, are to a good approximation proportional to the scattering lengths and to learn atomic levels one has to detect the X-ray transitions. This has been so far successful only in the meson–baryon systems. Measurements of $\pi\pi$ or $K\bar{K}$ scattering lengths have to be done in a different way. Experiments have been contemplated, and few have been attempted. One experiment with a 70 KeV energy resolution

$$p + d \rightarrow {}^3\mathrm{He} + (\pi^+\pi^-)_{\mathrm{atom}} \rightarrow {}^3\mathrm{He} + (\pi^0\pi^0) \qquad (1)$$

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was performed at the Indiana UCL storage ring [1]. Although charged pions have been produced the atoms have not been detected. Another experiment performed with high energy K-mesons at CERN [2]

$$K^{+,-} \to \pi^{+,-} + (\pi^0\pi^0)\,, \tag{2}$$

studied the spectrum of the $(\pi^0\pi^0)$ pair close to the $(\pi^+\pi^-)$ threshold. Here, the intention was to detect the cusp at the threshold and extract the relevant scattering length in a way suggested in Ref. [3]. However, the energy resolution in this experiment is about 1 MeV. Thus the energy bins reflecting the strongest cusp effect overlap the $\pi^+\pi^-$ atomic states.

The purpose of this note is to discuss semi-quantitatively the signals of such atoms produced in intermediate states. More quantitative analysis related to the actual experiment will be performed elsewhere.

2. Atoms in intermediate states

Consider an S-wave scattering in two channel systems. Let c denote the charged meson pair and o the neutral pair. For many systems the neutral channel has a lower threshold and the region of our special interest is a situation when channel c is closed and channel o is open. For phenomenological applications, it is convenient to describe the scattering matrix $\hat{T}$ in terms of a real and symmetric reaction matrix $\hat{K}$. These two matrices are related by the Heitler equation $\hat{T} = \hat{K} - i\hat{K}\hat{q}\hat{T}$ where $\hat{q}$ is a diagonal matrix of the center of mass momenta. In a single channel case this equation leads to low energy parametrization $1/T = 1/a + iq$, where $a = K$ is a scattering length related to the phase shift by $a = -\tan\delta/q$. A generalization of the scattering length to a many channel situation is provided by the $\hat{K}$-matrix. For two channels, the relation of T and K matrices which follows from the Heitler equation is more involved

$$\hat{K} = \begin{pmatrix} K_{oo} & K_{co} \\ K_{oc} & K_{cc} \end{pmatrix}\,, \tag{3}$$

and

$$\hat{T} = \begin{pmatrix} T_{oo} & T_{co} \\ T_{oc} & T_{cc} \end{pmatrix} = \begin{pmatrix} \frac{A_{oo}}{1+iq_oA_{oo}} & \frac{A_{co}}{1+iq_cA_{cc}} \\ \frac{A_{oc}}{1+iq_cA_{cc}} & \frac{A_{cc}}{1+iq_cA_{cc}} \end{pmatrix}\,, \tag{4}$$

where $q_{o,c}$ are the momenta in the two channels o, c and the channel scattering lengths A_{ij} are expressed in terms of the K-matrix elements by

$$\begin{aligned} A_{cc} &= K_{cc} - iK_{co}^2 q_o/(1 + iq_oK_{oo})\,, \\ A_{co} &= K_{co}/(1 + iq_oK_{oo})\,, \\ A_{oo} &= K_{oo} - iK_{co}^2 q_c/(1 + iq_cK_{cc})\,. \end{aligned} \tag{5}$$

These equations form a basis for the description of two channel scattering which guarantees the unitarity condition $\mathrm{Im}A_{cc} = -|A_{co}|^2 q_o$. There are three parameters of K-matrix but these may be reduced to two lengths under the isospin invariance.

The isospin symmetry is broken in the first instance by Coulomb interactions and meson mass splittings. The first allow for atomic binding in channel c while the second induce decays of atomic states. To describe the atoms in collisions, such effects have to be built into Eq. (4) and Eq. (5). The standard way is to put the Coulomb interactions into propagators $\bar{G}^c$. Solving any type of Lippman–Schwinger equation for the scattering matrix $\bar{T} = V + V\bar{G}^c\bar{T}$ allows to separate long range effects and include the short range effects into "Coulomb corrected" scattering lengths $\bar{A}_{ij}$. This separation follows the general form of the propagator $\bar{G}$ that describes both the Coulomb and the short ranged interactions $\bar{G} = \bar{G}^c + \bar{G}^c\bar{T}\bar{G}^c$. As a consequence, the scattering is described by amplitudes of the form

$$f_{ij} = f^c\delta_{ij} + e^{i\sigma_i}C_i\bar{T}_{ij}C_j e^{i\sigma_j}\,, \tag{6}$$

where f^c, σ and C^2 are the Coulomb scattering amplitudes, phases and penetration factors. All these arise as effects of long range Coulomb force and may be expressed by two functions that should replace iq in Eqs. (4),(5). Thus, in channel c:

$$iq \to f = iqC^2 + 2\gamma h\,, \tag{7}$$

where $C^2 = 2\pi\eta/[\exp(2\pi\eta) - 1]$ and $h = \frac{1}{2}[\psi(i\eta) + \psi(-i\eta)] - \frac{1}{2}\ln\eta^2$ with $\gamma = ZZ'\alpha\mu = 1/B$ and $\eta = \gamma/q$. The μ denotes reduced mass and B the Bohr radius. In the neutral channel o one has $f = iq$. Coulomb corrections to A arise as a consequence of short range behavior of the Coulomb propagator G^c. The corrected $\bar{A}$ are introduced to $\bar{K}, \bar{T}$ by formulas (4),(5). A way to calculate the transition from A to $\bar{A}$ may be found in Ref. [4].

Atomic effects that occur in the intermediate states of nuclear reactions are generated by singularities in f. Eq. (7) allows to expand f around an atomic level ε_0 to obtain the pole term

$$f \simeq \frac{R}{E - \varepsilon_0}, \quad \text{with} \quad R = -\frac{2\pi}{\mu}|\psi(0)|^2 = -2\mu^2\alpha^3\,. \tag{8}$$

Here, ε_0 is the pure Coulomb energy of an s-level, E is the energy relative to the $\pi^+\pi^-$ threshold and ψ is the pure Coulomb atomic wave function. From Eqs. (7) and (8) one finds the position of the atomic pole in T_{cc} of Eq. (4) *i.e.* the complex energy levels

$$\varepsilon - i\Gamma/2 = \varepsilon_0 + R\bar{A}_{cc}\,. \tag{9}$$

Eq. (9) is a well known result relating atomic level shifts and widths to the scattering length in the leading order of $\bar{A}_{cc}/B$. On the other hand the atoms observable in the scattering experiments are given by $\bar{T}_{oo}$ which in the vicinity of the atomic level has a different structure

$$\bar{T}_{oo} \approx \frac{\bar{K}_{oo}}{1+iq_o\bar{K}_{oo}} \frac{E-\varepsilon_o+R(\bar{K}_{cc}-\bar{K}_{co}^2/\bar{K}_{oo})}{E-\varepsilon_o+R\bar{A}_{cc}} . \tag{10}$$

In addition to the pole at an atomic level this matrix element has also a zero which is typical for Fano resonances. This zero is a consequence of a pole which occurs in $\bar{A}_{oo}$ close to the atomic level. Calculations indicate that in $\pi\pi$ system the pole is far away from zero and the peak profile is almost symmetric [5]. On the other hand, in the $K\bar{K}$ system these two points almost coincide and generate a strongly asymmetric profile. In realistic cases more channels may be open. Each additional channel m enlarges the dimension of K and changes $K_{ij} \to K_{ij} - iq_mK_{im}K_{mj}/(1+iq_mK_{mm})$. This adds absorptive parts to all matrix elements in Eq. (10). The Fano zero is shifted to the complex energy plane, and disappears from the physical $\bar{T}_{oo}$. However, the asymmetry of the atomic peak is quite stable. Weakly coupled channels *e.g.* those involving real photons introduce very limited asymmetries.

These two structures one of Breit–Wigner shape involved in $\bar{T}_{cc}$ and the other of Fano shape involved in $\bar{T}_{oo}$ arise when atoms are formed in the final states of a reaction.

3. Atoms in final states

Let $F_o(r)$ denote the source of neutral pair and $F_c(r)$ the corresponding source of charged pair. The leading formation amplitude is $F_o = \int F_o(r)\varphi_o(r)$ where φ_o is the wave function for non-interacting mesons in channel o. Final state interactions induce two additional amplitudes. The elastic one is described by $F_{oo} = F_oG_o\bar{T}_{oo}\varphi_o$ where a proper off-shell extrapolation and integrations are understood. The other amplitude, due to charge exchange scattering, is given by $F_{co} = F_c\bar{G}_c\bar{T}_{co}\varphi_o$. Close to the c threshold these amplitudes describe the cusp and the atomic states. Here, we discuss only atoms in a zero range approximation for the scattering operator $T(\boldsymbol{r}) = 2\pi/\mu\delta(\boldsymbol{r})\bar{T}$. Close to the "$n$"-th atomic level $\varepsilon_{o,n}$ the propagator $\bar{G}_c$ is dominated by $\psi_n(r)\psi_n(r')/(E-\varepsilon_{o,n})$ and formulas of the previous section generate the charge exchange amplitude

$$F_{co} = \int F_c(r)\psi_n(r)\frac{2\pi}{\mu}\frac{\bar{A}_{co}}{E-\varepsilon_{o,n}+R\bar{A}_{cc}}\psi_n(0) , \tag{11}$$

(note: in Ref. [5] the sign of F_{oc} is mistaken).

The elastic amplitude is also dominated by the atomic pole

$$F_{oo} = -\int F_o(r)\frac{\exp(iq_o r)}{r}\,\frac{\bar{K}_{oo}}{1+iq_o\bar{K}_{oo}}\,\frac{E-\varepsilon_{o,n}+R(\bar{K}_{cc}-\bar{K}_{co}^2/\bar{K}_{oo})}{E-\varepsilon_{o,n}+R\bar{A}_{cc}}\,, \tag{12}$$

which, because of the Fano zero, is expected to be a smaller one. An additional suppression arises if the value of K_{oo} is smaller than the source radius. That happens in the case of $\pi\pi$ but not the $K\bar{K}$ formation. At this point, an additional information is needed on the source radius in Eqs. (11),(12). Another radius is involved in the meson–meson interactions but this one is weighted by the large Bohr radius and introduces small and calculable effects. The ratio $\lambda = F_c/F_o$ is expected to be given by the isospin symmetry or by the experimental conditions. For reaction (1) one has $\lambda \simeq \sqrt{3}$ [5] and for the decay (2) the isospin invariance gives $\lambda = 2$ [3].

The probability to form a $1S$ state atom is given by integration of $|F_{co}|^2$ over the whole final phase space. It includes the integral over atomic line which may be calculated with the help of unitarity condition. In this way one recovers a branching ratio for the atom formation

$$R_{\rm atom} = P(\text{atom})/P(\text{total}) = \psi_{1S}(0)^2/(2\mu)\,L^{N-1}/L^N$$

given by the coalescence model. In the last relation L^{N-1} is the phase space for the atom and residual particles, L^N is the phase space for two mesons and residual particles. The atomic wave functions scale with the main quantum number n as $\psi_n(0)^2 \sim 1/n^3$ and higher levels contribute a correction of 22%. For decay (2) the result is $R_{\rm atom} = 0.9\times 10^{-5}$ [6], but the experimental best fit indicates a bigger number $R_{\rm atom} = 1.61(0.66)\times 10^{-5}$ [2].

The coalescence model gives only a part of the atomic effect. One correction comes from the interference of direct F_o and charge exchange F_{co} amplitudes. For pionium it is small, as the relevant parameter $2q_o{\rm Im}(A_{co}/\bar{\lambda})$ is small [5]. On the other hand the "atomic" contributions to F_{co} and F_{oo} interfere constructively and corrections may be sizable. These depend on the radius of the source. With a tentative value $\langle 1/r\rangle = 2$ fm we obtain a 30% increase of $R_{\rm atom}$. This number is closer to the best fit experimental estimate. However, a complete analysis including the whole near threshold region and a more subtle understanding of the formation range is still pending.

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