A PROBLEM OF FINAL STATE RADIATION IN THE PROCESS $e^+e^- \to \pi^+\pi^-\gamma$ NEAR THE THRESHOLD*

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A possibility to study final state radiation near the $\pi^+\pi^-$ threshold region at the Φ-factory DAΦNE is discussed. The dependence on the final state radiation model is tested by a Monte Carlo event generator that includes the contribution of the direct $\phi \to \pi^+\pi^-\gamma$ decay, the double-resonance $\phi \to \rho\gamma \to \pi^+\pi^-\gamma$ contribution and "pure" final state radiation both in the framework of the sQED model and in the Resonance Perturbation Theory. Finally, a model-independent way to study final state radiation is proposed.

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Final state radiation (FSR) is an irreducible background in radiative return measurements of the hadronic e^+e^- cross section [1]. Up to now the hadronic cross section is a main source of theoretical uncertainty in the prediction of the anomalous magnetic moment of the muon, a_μ [2]. The biggest contribution (about 80%) to the hadronic part of a_μ comes from the $\pi^+\pi^-$ final state and to be compatible with experimental data on a_μ the corresponding contribution $a_\mu^{(\pi^+\pi^-)}$ should be known with accuracy of at least 1%. That means that the corresponding cross section of the process

$$e^+(p_1) + e^-(p_2) \to \pi^+(p_+)\pi^-(p_-) \tag{1}$$

or, more exactly, radiative corrections (RCs) to it should be calculated with accuracy higher than 1%. In principle, RC caused by initial state radiation (ISR) can be calculated by QED, although the accuracy is limited by technical problems. For the calculation of the FSR cross section, instead, the situation is much more difficult. The pQCD model can be used to estimate

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FSR RC for the high energy region, $s=(p_1+p_2)^2 \leq 2\ \mathrm{GeV}^2$. The low energy region, that dominates in the evaluation of $a_\mu^{(\pi^+\pi^-)}$, cannot be described by pQCD. Therefore calculation of FSR RC relies on models describing pion–photon interaction in the low energy region.

Usually the combined sQED$*$VMD model is assumed as a model to calculate FSR RC [3]. In this case the pions are treated as point-like particles (the sQED model) and the total FSR amplitude is multiplied by the pion form factor, estimated in the VMD model. But, the sQED$*$VMD model is an approximation that is valid for relatively soft photons and it can fail for high energy photons, *i.e.* near the $\pi^+\pi^-$ threshold. In this energy region the contributions to FSR, beyond the sQED$*$VMD model, can become important.

Radiative return experiments allow to measure the cross section for the process (1) using the radiative process

$$e^+(p_1) + e^-(p_2) \to \pi^+(p_+)\pi^-(p_-)\gamma(k)\,, \tag{2}$$

where the photon is radiated by leptons. As mentioned, FSR process is the main irreducible background for (2). The cross section of FSR process is written as

$$d\sigma_{\mathrm{F}} = \frac{1}{2s(2\pi)^5}\int \delta^4(Q-p_+-p_--k)\frac{d^3p_+d^3p_-d^3k}{8E_+E_-\omega}|M^{(\mathrm{FSR})}|^2\,, \tag{3}$$

where $Q=p_1+p_2$, $s=Q^2$ and

$$M^{(\mathrm{FSR})} = \frac{e}{s}M^{\mu\nu}\bar{u}(-p_1)\gamma_\mu u(p_2)\varepsilon^\star_\nu\,. \tag{4}$$

In the general case the tensor $M^{\mu\nu}$ describing the process

$$\gamma^*(Q) \to \pi^+(p_+)\pi^-(p_-)\gamma(k)$$

can be rewritten in the terms of three gauge invariant tensors (see [4] and Refs. [23,24] in it):

$$\begin{aligned}
&M^{\mu\nu}(Q,k,l) \equiv -ie^2M_{\mathrm{F}}^{\mu\nu}(Q,k,l) = -ie^2(\tau_1^{\mu\nu}f_1+\tau_2^{\mu\nu}f_2+\tau_3^{\mu\nu}f_3)\,,\\
&\tau_1^{\mu\nu} = k^\mu Q^\nu - g^{\mu\nu}k\cdot Q, \qquad l=p_+-p_-\,,\\
&\tau_2^{\mu\nu} = k\cdot l(l^\mu Q^\nu - g^{\mu\nu}k\cdot l) + l^\nu(k^\mu k\cdot l - l^\mu k\cdot Q)\,,\\
&\tau_3^{\mu\nu} = Q^2(g^{\mu\nu}k\cdot l - k^\mu l^\nu) + Q^\mu(l^\nu k\cdot Q - Q^\nu k\cdot l)\,.
\end{aligned}$$

We would like to stress that this expansion is totally model-independent. The model dependence is related only with the implicit form of the scalar functions f_i. There are several different contributions to FSR depending on the photon radiation mechanism.

First, there is "pure" FSR. It is a contribution that is described by the sQED$*$VMD model in the soft photon limit. Second, in the case of the accelerator DAΦNE ($s = m_\phi^2$) considered in this paper, there are contributions to FSR related to the intermediate ϕ meson state: at least, the ϕ direct decay ($\gamma^* \to \phi \to \pi^+\pi^-\gamma$) and the double resonance contributions ($\gamma^* \to \phi \to \rho^\pm\pi^\mp \to \pi^+\pi^-\gamma$).

That means that for DAΦNE energy region the total cross section $d\sigma_{\rm T}$ of the process (2), when the photon is radiated either by leptons or by final pions, can be written as

$$\begin{aligned} d\sigma_{\rm T} &\sim |M_{\rm ISR} + M_{\rm FSR}|^2 = d\sigma_{\rm I} + d\sigma_{\rm F} + d\sigma_{\rm IF}\,, \\ d\sigma_{\rm I} &\sim |M_{\rm ISR}|^2,\ d\sigma_{\rm IF} \sim 2{\rm Re}\{M_{\rm ISR}\cdot(M_{\rm RPT}+M_\phi)^*\}\,, \\ d\sigma_{\rm F} &\sim |M_{\rm RPT}|^2 + |M_\phi|^2 + 2{\rm Re}\{M_{\rm RPT}\cdot M_\phi^*\}\,, \end{aligned} \tag{5}$$

where $d\sigma_{\rm I}$ corresponds to ISR, $d\sigma_{\rm F}$ to FSR and $d\sigma_{\rm IF}$ to their interference.

The part caused by the ϕ decay (it corresponds to M_ϕ in (5)) consists of two parts: the ϕ direct decay and the double resonance contributions. FSR contribution that is related with the ϕ direct decay is described by Achasov's four quark model [5] and corresponds to the following: $\gamma^* \to \phi \to\ (f_0;\ f_0+\sigma) \to \pi^+\pi^-\gamma$. The numerical values of parameters (like mass and width of the scalar mesons, the numerical value of the constants describing $\gamma\phi f_0$, $\gamma\phi\sigma$ and $f_0\pi\pi$, $\sigma\pi\pi$ vertexes) are taken from the KLOE fit for the neutral channel $\pi^0\pi^0\gamma$ [6]. The last version of KLOE parametrization uses the mixed $(f_0+\sigma)$ intermediate state [6] whereas only the f_0 meson was included in previous version [7]. The dependence on the choice of the intermediate scalar state is shown in Fig. 1. As we can see the inclusion of σ meson of the experimental spectrum and decreases enough the value of $|M_\phi|^2$ near the threshold and describes the data better.

The double resonance contribution $\gamma^* \to \phi \to \rho^\pm\pi^\mp \to \pi^+\pi^-\gamma$ is described in [8] and its value is compatible with the value of the ϕ direct decay. Presently, we used PDG values for the experimental values $\Gamma(\rho\to\pi\gamma)$ and $\Gamma(\phi\to e^+e^-)$ [9] to calculate the $\phi\rho\pi$ and $\rho\pi\gamma$ coupling constants.

"Pure" FSR (a part of FSR that is not related with the intermediate ϕ meson state) is described in the framework of both sQED$*$VMD and Resonance Perturbation Theory[1] (RPT) [4]. Following the KLOE experiment results we consider only the case of destructive interference between the ϕ direct amplitude and "pure" FSR one.

The results $d\sigma_{\rm T}/d\sigma_{\rm I}$ for two different initial energies possible at DAΦNE ($s = 1$ GeV^2 and $s = m_\phi^2$) are shown in Fig. 2 with and without contributions from RPT and ϕ decay. The case of the KLOE large angle analysis $50^\circ \le \theta_\gamma \le 130^\circ$, $50^\circ \le \theta_\pi \le 130^\circ$ is considered.

[1] RPT is assumed to be an appropriate theory to describe pion–photon interaction about 1 GeV. See, for example, [10].

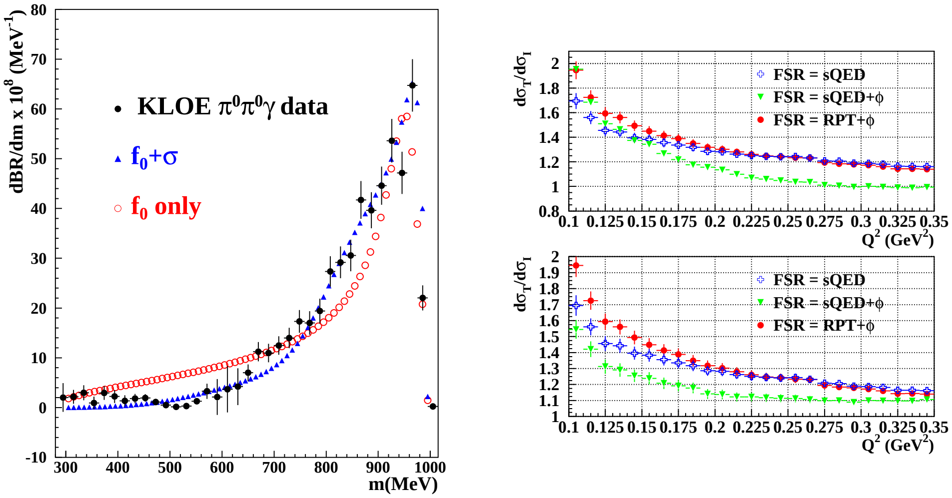

Fig. 1. Left: The dependence of the branching ratio of the ϕ direct decay on the intermediate scalar states. Right: The ratio $d\sigma_{\rm T}/d\sigma_{\rm I}$ as function of the invariant mass of the two pions, in the region $50^\circ \leq \theta_\gamma \leq 130^\circ$, $50^\circ \leq \theta_\pi \leq 130^\circ$ $s = m_\phi^2$ when only f_0 (down) and $f_0 + \sigma$ (above) are included.

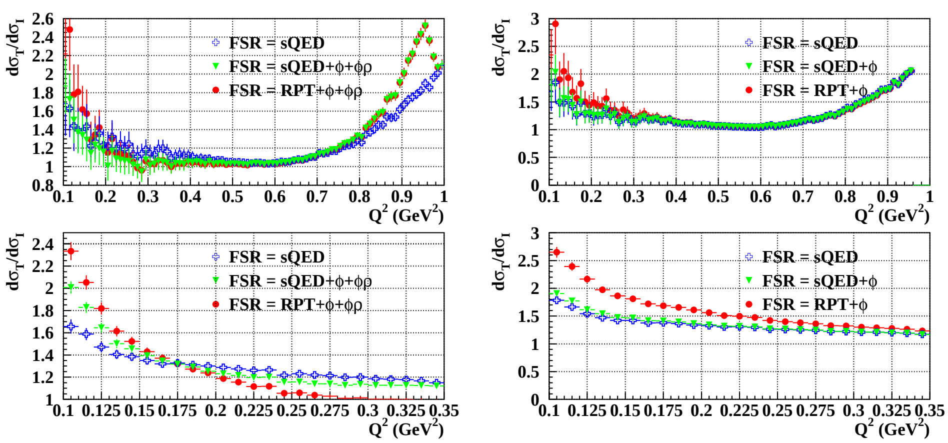

Fig. 2. The ratio $d\sigma_{\rm T}/d\sigma_{\rm I}$ as function of the invariant mass of the two pions, in the region $50^\circ \leq \theta_\gamma \leq 130^\circ$, $50^\circ \leq \theta_\pi \leq 130^\circ$, for different models of FSR. Left corresponds to $s = m_\phi^2$, right to $s = 1$ GeV^2.

Now let us dicuss the results. To start with, we consider the case $s = m_\phi^2$. First, the peak near 1 GeV^2 corresponds to the f_0 meson intermediate state. Second, as one can see, the destructive interference between ϕ decay and "pure" FSR decreases the visible cross section in the whole Q^2 region. Last but not least, all contributions are large enough near the threshold to make the analysis difficult.

In the case of $s = 1$ GeV^2 the ϕ resonant contribution is suppressed and the main contribution beyond of sQED comes from RPT (see Fig. 2, (right), where the value of $d\sigma_{\rm T}$ with and without the ϕ direct decay almost coincides).

We think that a possibility to test FSR models near the $\pi^+\pi^-$ threshold, using only the cross section and other variables (like mass spectrum, asymmetries, *etc.*) for one value of the initial energy (one value of the variable s), is a difficult task due to large numbers of parameters. (In fact, in our case we have about ten different parameters describing FSR.) The situation can be simplified a bit if we extract all possible information for the ϕ decay from the neutral $\pi^0\pi^0$ channel. In order to describe "pure" FSR we have used a combined fit for different values of the initial energy. Also we propose to consider a physical quantity which very well determined in the framework of sQED and then study its deviation from sQED behaviour in the data. In our opinion this quantity can be determined as [11]

$$Y_s(Q^2) = \frac{\left(\frac{d\sigma_{\rm T}}{dQ^2}\right)_s - \left(\frac{d\sigma_{\rm sQED+\phi}}{dQ^2}\right)_s}{H_s(Q^2)} \equiv |F_\pi(Q^2)|^2 + \Delta F_s(Q^2)\,. \tag{6}$$

The quantity $\dfrac{d\sigma_{\rm sQED+\phi}}{dQ^2}$ is the differential FSR cross section in the framework of sQED and includes ϕ direct decay. As one can see the value of $Y(Q^2)$ coincides with the square of the pion form factor in the framework of sQED$*$VMD ($F_\pi(Q^2)$) and does not depend on the initial energy. Then we can determine a quantity

$$\Delta Y(Q^2) = Y_{s_1}(Q^2) - Y_{s_2}(Q^2)\,,$$

where s_1 and s_2 are two different c.m. energy values for e^+e^-, for KLOE setup $s_1 = 1$ GeV^2 and $s_2 = m_\phi^2$. In the case of the sQED$*$VMD model, the value $\Delta Y(Q^2) = 0$. This means that any deviation from zero will be in favour of some contribution beyond sQED.

Fig. 3 shows the quantity $Y_s(Q^2)$ at $s_1 = 1$ GeV^2 and at $s_2 = m_\phi^2$ and the value $\Delta Y(Q^2)$, when FSR is described by sQED. As expected, each of the quantities Y_{s_1} and Y_{s_2} coincides with the square of the pion form factor $|F_\pi(Q^2)|^2$, shown by solid line. The value of ΔY is consistent with zero.

A combined fit of Y_{s_1} and Y_{s_2} to the pion form factor is also possible as

$$F_\pi(Q^2) \simeq 1 + p_1 * Q^2 + p_2 * q^4 \,. \tag{7}$$

It gives the following values: $p_1 = 1.4 \pm 0.186$ GeV^{-2}, $p_2 = 8.8 \pm 0.73$ GeV^{-4}, $\chi^2/\nu = 0.25$.

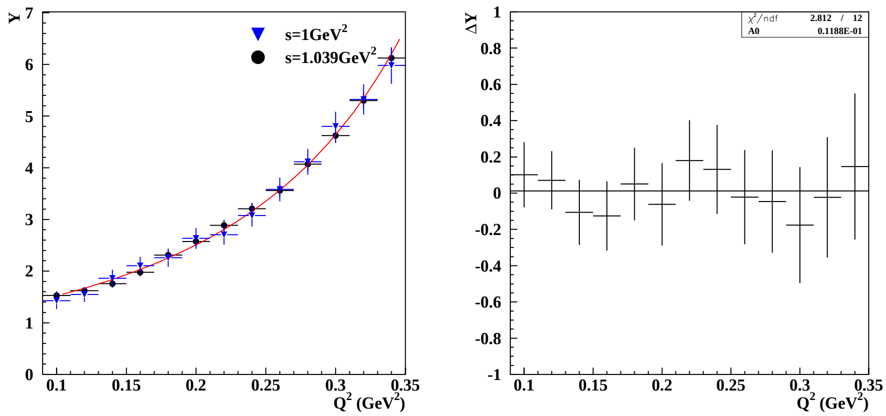

Fig. 3. Left: $Y_s(Q^2)$ at $s = 1$ GeV^2 (triangles), and at $s = m_\phi^2$ (circles), when FSR includes only sQED and ϕ contribution. The pion form factor $|F_\pi(Q^2)|^2$ is shown by the solid line. Right: The difference $\Delta Y(Q^2)$.

A different situation appears if FSR emission from pions is modeled by RPT. In this case, as shown in Fig. 4, the difference $\Delta Y(Q^2) \neq 0$ and the quantities $Y_s(Q^2)$ cannot be anymore identified with $|F_\pi(Q^2)|^2$. A combined fit of Y_{s_1} and Y_{s_2} is no longer possible.

To conclude this paper we would like to summarize the main results. We have shown that the $\pi^+\pi^-$ threshold energy region is very sensitive to all FSR contributions. To test the FSR model one can use two different ways. In the first method one constructs a general amplitude for the $\pi^+\pi^-\gamma$ final state according to some underlying theory and then determines the free parameters of that theory by constrained fit on specific variables (like mass spectrum, asymmetries, *etc.*). In the second method, that was proposed here, one tries to find a physical quantity that has a very well described behaviour in the framework of sQED and then compare the experimental spectra at two different energies of the initial particles. In this way one can test any deviation from the behaviour predicted by sQED for that physical quantity. In our opinion these two methods are mostly complementary and should be used together to study effective FSR.

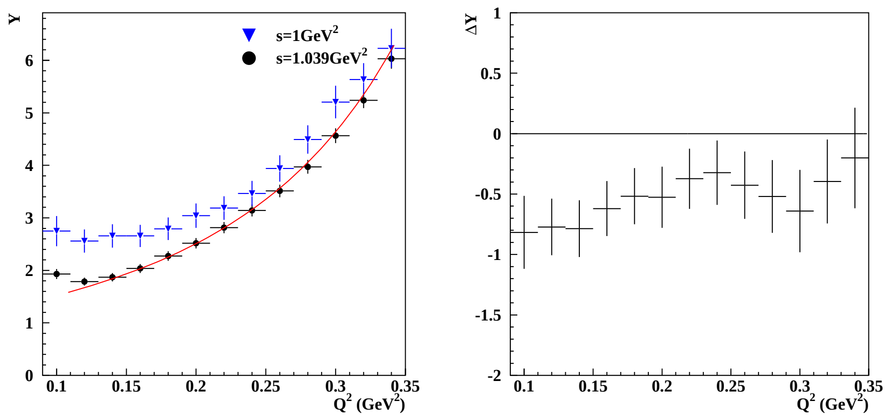

Fig. 4. Left: $Y_s(Q^2)$ at $s = 1$ GeV^2 (triangles), and at $s = m_\phi^2$ (circles), when FSR includes RPT and ϕ contribution. The pion form factor $|F_\pi(Q^2)|^2$ is shown by the solid line. Right: The difference $\Delta Y(Q^2)$.

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