STATUS OF $(g_\mu - 2)/2$ IN STANDARD MODEL*

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The current status of the muon anomalous magnetic moment is discussed. The leading order hadronic contribution is reevaluated basing on the new data on e^+e^- annihilation. The experimental value is about 3.3 standard deviations higher than the Standard Model prediction.

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1. Anomalous magnetic moment

The muon anomalous magnetic moment, a_μ, is one of the most accurately known physical quantities recently measured by E821 [1] with a 5×10^{-7} relative accuracy. Although for electron it is known much better (a_e is measured with a 4.9×10^{-10} accuracy [2]), a_μ is much more sensitive to new physics effects: the gain is usually $\sim (m_\mu/m_e)^2 \approx 4.3 \times 10^4$. Any significant difference of $a_\mu^{\rm exp}$ from $a_\mu^{\rm th}$ indicates new physics beyond the Standard Model (SM). It is conventional to write a_μ as

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}\,. \tag{1}$$

Taking into account recent progress with the calculation of the $4^{\rm th}$ and $5^{\rm th}$ order terms [3–5] one obtains

$$a_\mu^{\rm QED} \;=\; (116584719.4 \pm 1.4) \times 10^{-11}\,. \tag{2}$$

With the value of α from the latest result for a_e [6] α^{-1}=137.035999710(96), one obtains [7]:

$$a_\mu^{\rm QED} \;=\; (116584718.09 \pm 0.14 \pm 0.08) \times 10^{-11}\,. \tag{3}$$

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Here the errors are due to the uncertainties of the $\mathcal{O}(\alpha^5)$ term and α.

The electroweak term is known rather accurately [8]:

$$a_\mu^{\rm EW} \;=\; (15.4 \pm 0.1 \pm 0.2) \times 10^{-10}\,, \tag{4}$$

where the 1st uncertainty is due to hadronic loops while the 2nd is caused by the errors of M_H, M_t and 3-loop effects.

The hadronic contribution can also be written as a sum:

$$a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm had,LBL}\,. \tag{5}$$

The dominant contribution comes from the leading order term

$$a_\mu^{\rm had,LO} = \Big(\frac{\alpha m_\mu}{3\pi}\Big)^2 \int\limits_{4m_\pi^2}^{\infty} ds\, \frac{R(s)\,\hat{K}(s)}{s^2}\,, \tag{6}$$

where

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}\,, \tag{7}$$

and the kernel $\hat{K}(s)$ grows from 0.63 at $s = 4m_\pi^2$ to 1 at $s \to \infty$, $1/s^2$ emphasizes the role of low energies. Particularly important is the reaction $e^+e^- \to \pi^+\pi^-$ with a large cross section below 1 GeV.

Our new estimate takes into account the recent progress in the low energy e^+e^- annihilation and includes the data not yet available previously [9–11].

In addition to the previously published ρ meson data [12], CMD-2 reported their final results on the pion form factor F_π from 370 to 1380 MeV [13, 14]. The new ρ meson sample has one order of magnitude larger statistics and a systematic error of 0.8%. SND measured F_π from 390 to 970 MeV with a systematic error of 1.3% [15]. KLOE studied F_π using the method of radiative return or ISR [16, 17] at $590 < \sqrt{s} < 970$ MeV with a sample of 1.5×10^6 events and systematic error of 1.3% [18]. BaBar also used ISR and achieved impressive results on various final states with more than two hadrons [19–21].

2. New data

In Fig. 1 we show the pion form factor data from CMD-2, KLOE and SND. The $|F_\pi|$ values from CMD-2 and SND are in good agreement. The KLOE data are consistent with them near the ρ meson peak, but exhibit a somewhat different energy dependence: they are higher to the left and lower to the right of the ρ meson peak. However, the contributions to a_μ from all three experiments are consistent.

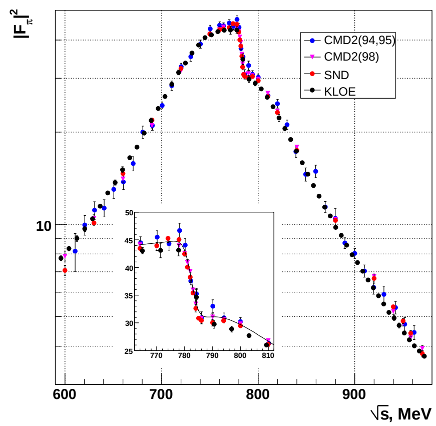

Fig. 1. $|F_\pi|$ from CMD-2, KLOE and SND.

3. Results

Using the new data below 1.8 GeV discussed above in addition to the whole data set of [9, 10] for old experiments, and assuming that for the hadronic continuum above 1.8 GeV one can already use the predictions of perturbative QCD [22], we can reevaluate the leading order hadronic contribution to a_μ. The results for different energy ranges are shown in Table I.

TABLE I

Updated $a_\mu^{\mathrm{had,LO}}$.

$\sqrt{s}$, GeV	$a_\mu^{\mathrm{had,LO}}, 10^{-10}$
2π	$504.6 \pm 3.1 \pm 1.0$
ω	$38.0 \pm 1.0 \pm 0.3$
ϕ	$35.7 \pm 0.8 \pm 0.2$
0.6–1.8	$54.2 \pm 1.9 \pm 0.4$
1.8–5.0	$41.1 \pm 0.6 \pm 0.0$
$J/\psi, \psi'$	$7.4 \pm 0.4 \pm 0.0$
> 5.0	$9.9 \pm 0.2 \pm 0.0$
Total	$690.9 \pm 3.9_{\mathrm{exp}} \pm 2.0_{\mathrm{th}}$

The theoretical error consists of 1.9×10^{-10} due to uncertainties of radiative corrections in old measurements and 0.7×10^{-10} related to mentioned above use of perturbative QCD. It can be seen that due to a higher accuracy of e^+e^- data the uncertainty of $a_\mu^{\rm had,LO}$ is now 4.4 (0.63%) compared to 15.3 of Ref. [9] and 7.2 of Ref. [11]. We move now to the higher order hadronic contributions. Their most recent estimate performed in [23] gives $(-9.8\pm0.1)\times10^{-10}$ and has negligible error compared to that of the leading order one.

The most difficult situation is with the light-by-light hadronic contribution, which is estimated only theoretically. The older predictions based on the chiral model [24, 25] were compatible and much lower than that using short-distance QCD constraints [26] (see also [27]). Their approximate averaging in [28] gives $(120 \pm 35) \times 10^{-11}$.

Adding all hadronic contributions we obtain $a_\mu^{\rm had} = (693.1\pm5.6)\times10^{-10}$. This result agrees with other recent estimations, *e.g.* [11, 23, 29, 30] and has better accuracy due to the new e^+e^- data. All separate contributions are collected in Table II. The improved precision of the leading order hadronic contribution allows to confirm previously observed excess of the experimental value over the SM prediction with a higher than before significance of more than three standard deviations. For the first time during last years the accuracy of the SM prediction is slightly better than the experimental one.

TABLE II

Experiment *versus* theory.

Contribution	$a_\mu, 10^{-10}$
Experiment	11659208.0 ± 6.3
QED	11658471.94 ± 0.14
Electroweak	$15.4 \pm 0.1 \pm 0.2$
Hadronic	693.1 ± 5.6
Theory	11659180.5 ± 5.6
Exp.–Theory	27.5 ± 8.4 (3.3σ)

What is the future of this SM test? From the experimental side there are suggestions to improve the accuracy by a factor of 2.5 at E969 (BNL) or even by an order of magnitude at JPARC. It is clear that it will be extremely difficult to improve the accuracy of the SM prediction significantly. One can optimistically expect that by 2008 new high-statistics ISR measurements at KLOE, BaBar and Belle together with the more precise R below 4.3 GeV from CLEO-c will decrease the error of $a_\mu^{\rm had,LO}$ from 4.4 to 2.8×10^{-10}. Experiments are planned at the new machine VEPP-2000 (VEPP-2M upgrade) with 2 detectors (CMD-3 and SND) up to $\sqrt{s}$=2 GeV with $L_{\rm max} = 10^{32}$ cm^{-2}s^{-1}. A similar machine (DAΦNE-II) is discussed in Frascati. New R measurements below 5 GeV will be done at the $\tau - c$

factory now under construction in Beijing. We can estimate that by 2010 the accuracy of $a_\mu^{\rm had,LO}$ will be improved from 2.8 to 2.2×10^{-10} and the total error of 4.1 will be limited by the LBL term (3.5) and still higher than the expected 2.5 in E969.

There is still no explanation for the observed discrepancy between the predictions based on τ lepton and e^+e^- data [11]. For this reason we are not using τ data in this update. More light on the problem should be shed by the high-statistics measurement of the two-pion spectral function by Belle which preliminary results indicate to better agreement with e^+e^- data than before [31].

Let us hope that progress of theory will allow a calculation of $a_\mu^{\rm had}$ from 1st principles (QCD, Lattice). One can mention here a new approach in the QCD instanton model [32] or calculations on the lattice, where there are encouraging estimates of $a_\mu^{\rm had,LO}$, *e.g.* [33] $(667 \pm 20) \times 10^{-10}$ or attempts to estimate $a_\mu^{\rm had,LBL}$ [34].

In conclusion, I would like to emphasize once again that BNL success stimulated significant progress of e^+e^- experiments and related theory. Improvement of e^+e^- data led to substantial decrease of an error of $a_\mu^{\rm had,LO}$, which now matches the experimental accuracy. Future experiments as well as development of theory should clarify whether the observed difference between $a_\mu^{\rm exp}$ and $a_\mu^{\rm th}$ is real and what consequences for the Standard Model it implies.

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