Vol. 39 (2008) *ACTA PHYSICA POLONICA B* No 1

τ DECAYS TO FIVE MESONS IN TAUOLA

Johann H. Kühn

Institut für Theoretische Teilchenphysik, Universität Karlsruhe
76128 Karlsruhe, Germany

Z. Wąs

PH-TH Department, CERN, 1211 Geneva 23, Switzerland
and
The H. Niewodniczański Institute of Nuclear Physics PAN
Kraków, Radzikowskiego 152, Poland

(Received October 8, 2007)

The τ-decay library TAUOLA has gained popularity over the last decade. However, with the continuously increasing precision of the data, some of its functionality has become insufficient. One of the requirements is the implementation of decays into five mesons plus a neutrino with a realistic decay amplitude. This note describes a step into this direction. For the $2\pi^-\pi^+2\pi^0$ mode the three decay chains $\tau^- \to a_1^-\nu \to \rho^-(\to \pi^-\pi^0)\omega (\to \pi^-\pi^+\pi^0)\nu$, $\tau^- \to a_1^-\nu \to a_1^-(\to 2\pi^-\pi^+)f_0(\to 2\pi^0)\nu$, and $\tau^- \to a_1^-\nu \to a_1^-(\to \pi^- 2\pi^0)f_0(\to \pi^+\pi-)\nu$ are introduced with simple assumptions about the couplings and propagators of the various resonances. Similar amplitudes (without the $\rho\omega$ contributions) are adopted for the $\pi^-4\pi^0$ and $3\pi^-2\pi^+$ modes. The five-pion amplitude is thus based on a simple model, which, however, can be considered as a first realistic example. Phase-space generation includes the possibility of presampling the ω and a_1 resonances, in one channel only, however. This is probably sufficient for the time being, both for physics applications and for tests. The technical test of the new part of the generator is performed by comparing Monte Carlo and analytical results. To this end a non-realistic, but easy to calculate, purely scalar amplitude for the decay into five massless pions was used.

PACS numbers: 13.35.Dx, 12.40.Vv, 02.70.Uu

1. Introduction

Early studies of semileptonic tau decays have concentrated on final states with few mesons only. The recent advent of high-statistic samples in experiments at LEP and CESR, and the perspective of further increasing event rates at B-meson factories allow and require the study of relatively rare decay modes and thus of multibody final states. These configurations are of particular importance for a precise determination of the semileptonic branching ratio and for an improved limit on the mass of the tau neutrino. Final states with up to six pions have been observed up to now.

Complementary to the experimental studies, Monte Carlo simulations are required to determine the efficiencies of the detectors. The generator TAUOLA [1–3] has been specifically designed to simulate a wide variety of tau-decay modes and includes spin effects in the case of polarized τ decays. These simulations necessarily include form factors that model the resonant structure of intermediate hadronic states, such as ρ, ω or a_1 mesons. The combination of Monte Carlo simulation and experimental studies thus allows us to test the model input and leads to additional information about hadron physics at low energies.

For a few channels only, and in a limited kinematical range, the form factors can be predicted from a firm theoretical basis. In many cases additional input, such as vector dominance and phenomenological parametrizations are required. At present only final states with up to four pions can be simulated on the basis of realistic form factors [3], which have their basis in chiral Lagrangians and vector-dominance models. In the present paper we describe an extension of TAUOLA to five-pion final states. The model for the amplitude is based on the observation that the $2\pi^-\pi^+2\pi^0$ decay is dominated by the $\omega\pi^-\pi^0$ channel, with only about 20% left for the remainder, which does not exhibit sharp resonance structures. The amplitude is therefore constructed to accommodate this feature. It includes the dominant $W^* \to \omega 2\pi$ amplitude and an amplitude of the form $W^* \to a_1(\to \rho\pi) f_0(\to \pi\pi)$ with various charge assignments. The second amplitude is also used to describe the two remaining charge combinations, $\pi^- 4\pi^0$ and $3\pi^- 2\pi^+$. (The charge-conjugate combinations for the τ^+ decay are considered in parallel.) In the narrow-width approximation, important technical tests of the generator can be performed. The details of the new amplitude (effectively the transition matrix element of the hadronic current, often just denoted "current") and of the phase-space generation are described in Sec. 2. Tests of the program and results for some characteristic distributions are collected in Sec. 3. Sec. 4 contains our summary and conclusions.

2. Hadronic currents and phase-space generation

For each decay mode the basic ingredients for TAUOLA can be grouped into three parts: *(i)* the phase-space generator for a fixed number of particles in the final state, *(ii)* the algorithm for the calculation of the spin-dependent matrix element from the hadronic currents and from the properties of the electroweak interaction between the W boson, the τ lepton and its neutrino, and *(iii)* the hadronic current *per se*.

The structure of the program is decscribed in detail in Refs. [1, 3] (see also [4]) and there is no need to recall it here. The relation between the hadronic current and the matrix element remains the same as for the two-, three- and four-meson case presented before.

Some comments are required concerning the five-pion phase-space generator, although its structure is very similar to the well-documented case of four pions. The number of generated angles and invariant masses is, of course, increased. Since no extensive studies of amplitudes with narrow resonances are foreseen for the moment, the phase-space presampler is only prepared to compensate for possible sharp peaks in the invariant mass of the five-pion system as a whole and in one subsystem of the three-pion (single channel) only. To improve the efficiency, this mode of generation is merged with the flat phase-space subgenerator, again according to the rules described in detail in [1, 3].

The most interesting ingredient necessary for this generator is the matrix element of the hadronic current. For the decay into an n-pion state, it is given by

$$J_\mu(q_1, q_2, \ldots, q_n) \equiv \langle \pi(q_1), \pi(q_2), \ldots, \pi(q_n) | J_\mu(0) | 0 \rangle \,, \tag{1}$$

where the same letter J has been used for the operator and its matrix element. The decay into an odd number of pions proceeds through the axial part of the current only.

We now construct the amplitude as a sequence of (partly virtual) resonance decays and transitions, and concentrate, in a first step, on the most complicated channel $2\pi^-\pi^+2\pi^0$. From experiment we observe that the decay through ω seems to constitute the dominant mode in this case, with a branching ratio $\mathrm{Br}(\tau \to h^-\omega\pi^0) = (4.4 \pm 0.5) \times 10^{-3}$, to be compared with $\mathrm{Br}(\tau \to h^-h^-h^+2\pi^0(\mathrm{exl.}\ K^0, \omega, \eta)) = (1.1 \pm 0.4) \times 10^{-3}$ [5]. The first part will be implemented in a first step (current A), and the remainder (current B) afterwards. Normalizations will be appropriately set, as can be seen in Sec. 3. From isospin conservation — and the fact that the total hadronic system has isospin one, and the ω, however, isospin zero — it follows that the two pions are also in an isospin-one configuration, corresponding to a state with ρ-meson-like quantum numbers. We thus have to find a convenient description for the amplitude that specifies the transition $W^*(Q) \to \omega\rho$. Let us denote the polarization and momentum of the ρ meson by ε_ρ and p_ρ, and similarly for the ω meson. Three possible amplitudes describing the decay of a 1^{++} to two 1^{--} states can be constructed:

$$\begin{aligned} F^1_\mu &= (\varepsilon_\rho, \varepsilon_\omega, Q, \mu) \,, \\ F^2_\mu &= (\varepsilon_\rho, \varepsilon_\omega, p_\rho - p_\omega, \mu) - (\varepsilon_\rho, \varepsilon_\omega, p_\rho - p_\omega, Q)\frac{Q_\mu}{Q^2} \,, \end{aligned}$$

$$F_\mu^3 \;=\; (\varepsilon_\rho, \varepsilon_\omega, p_\rho, p_\omega)\left((p_\rho - p_\omega)_\mu - (p_\rho - p_\omega) Q \frac{Q_\mu}{Q^2}\right), \tag{2}$$

each of which is, of course, multiplied by a function of Q^2, where $Q = \sum_i q_i$. For simplicity we adopt amplitude F^1, which depends on the lowest power of the relative momentum $p_\rho - p_\omega$. Furthermore, we multiply this amplitude by a Breit–Wigner factor

$$c(Q^2) = c_0 BW_a(Q^2) \equiv c_0 \frac{m_a^2}{m_a^2 - Q^2 - i m_a \mathit{\Gamma}_a} \tag{3}$$

simulating the a_1 enhancement. For the constant c_0, which is determined by the product of $W^* a_1$- and $a_1 \omega \rho$-coupling we adopt the value $c_0 = 3$, so as to reproduce the desired branching ratio of 0.4%. Furthermore, we take $m_a = 1.26\,\mathrm{GeV}$ and $\mathit{\Gamma}_a = 0.4\,\mathrm{GeV}$. With the symbol $(\varepsilon_\rho, \varepsilon_\omega, Q, \mu)$ we denote the totally antisymmetric Levi–Civita symbol, contracted with three four-vectors ε_ρ, ε_ω (polarization vectors for ρ and ω), and Q; the last index μ remains open.

The amplitude for the "subsequent" decay of the virtual ρ is given by

$$\mathcal{M}_\rho^\mu = g_{\rho\pi\pi} (q_4 - q_5)^\mu , \tag{4}$$

which leads to the decay rate

$$\mathit{\Gamma}_\rho = \frac{|g_{\rho\pi\pi}|^2}{48\pi} \frac{(m_\rho^2 - 4 m_\pi^2)^{3/2}}{m_\rho^2} . \tag{5}$$

To reproduce the input parameters $m_\rho = 776\,\mathrm{MeV}$ and $\mathit{\Gamma}_\rho = 150\,\mathrm{MeV}$ the value $g_{\rho\pi\pi} = 6.0$ is adopted. For the ω we adopt the corresponding decay chain $\omega \to \pi\rho$, where all three π and ρ charges contribute with equal weight. We start from an ω–ρ–π coupling of the form

$$\mathcal{M}_{\omega\rho\pi} = \frac{1}{2} f_{\omega\rho\pi} (\varepsilon_\omega, p_\omega, \varepsilon_\rho, q_\pi) , \tag{6}$$

where the coupling $f_{\omega\rho\pi}$ has dimension $(\mathrm{mass})^{-1}$. Including the ρ decay according to Eq. (4), and taking the antisymmetric isopin wave function of the three points into account, which implies an antisymmetric momentum wave function, we finally arrive at

$$\mathcal{M}_{\omega 3\pi} = \frac{f_{\omega\rho\pi} g_{\rho\pi\pi}}{m_\rho^2} (\varepsilon_\omega, q_1, q_2, q_3) (BW_\rho(s_1) + BW_\rho(s_2) + BW_\rho(s_3)) , \tag{7}$$

where q_1, q_2, q_3 denote the momenta of π^+, π^-, π^0, and $s_1 = (q_2 + q_3)^2$ *etc.* This leads to the decay rate

$$\begin{aligned} \Gamma_\omega &= \frac{1}{3}\frac{1}{128}\frac{1}{(2\pi)^3}\frac{1}{m_\omega^3}\frac{(f_{\omega\rho\pi}g_{\rho\pi\pi})^2}{m_\rho^4}\int ds_1 ds_2 \Big| BW_\rho(s_1)+BW_\rho(s_2)+BW_\rho(s_3)\Big|^2 \\ &\quad \times \left(s_1 s_2 s_3 - m_\pi^2(Q^2 - m_\pi^2)^2\right) . \end{aligned} \tag{8}$$

If the sum of the three ρ Breit–Wigners is dropped (*i.e.* replaced by a constant taken to be 1) and the pion masses are set to zero, one finds

$$\Gamma_\omega = \frac{1}{3}\frac{1}{128}\frac{1}{(2\pi)^3}\, m_\omega^7\, \frac{(f_{\omega\rho\pi}g_{\rho\pi\pi})^2}{120 m_\rho^4} . \tag{9}$$

This formula will be useful for cross checks of the program. Whenever numerical values are required, we take $f_{\omega\rho\pi} = 0.07\,\mathrm{MeV}^{-1}$, that is only the first significant digit, $g_{\rho\pi\pi} = 6$ was defined earlier, $m_\omega = 782\,\mathrm{MeV}$ and $\Gamma_\omega = 8.5\,\mathrm{MeV}$.

The full five-pion amplitude is obtained by including the ρ and ω propagators

$$\begin{aligned} J_\mu^A(q_1,q_2,q_3,q_4,q_5) &= c_0\frac{f_{\omega\rho\pi}g_{\rho\pi\pi}^2}{m_\rho^4 m_\omega^2}\, BW_a(Q^2) BW_\rho\left((q_4+q_5)^2\right) \\ &\quad \times BW_\omega((q_1+q_2+q_3)^2)(\mu, q_4 - q_5, \alpha, Q)(\alpha, q_1, q_2, q_3) \\ &\quad \times (BW_\rho(s_1) + BW_\rho(s_2) + BW_\rho(s_3)) . \end{aligned} \tag{10}$$

The pictorial illustration of this decay amplitude is shown in Fig. 1(a).

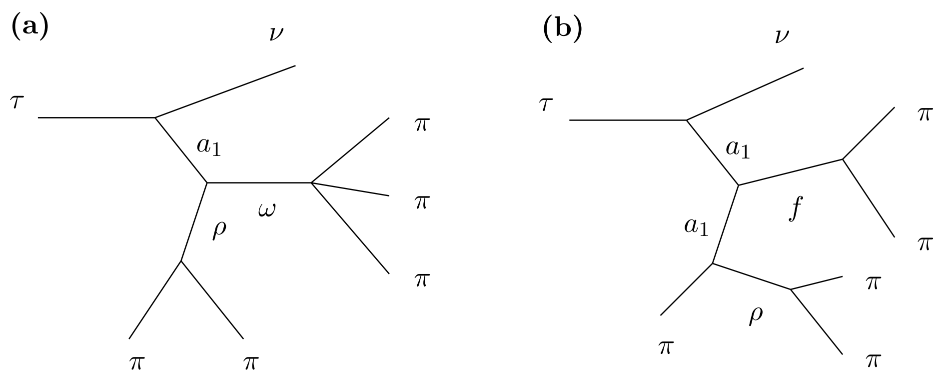

Fig. 1. Dominant decay amplitude for the decay of τ into five pions through an ω plus a ρ resonance (a) and through an f_0 plus $a_1(\to \rho\pi)$ (b).

This formula is directly applicable to the case of a narrow ω, if we identify, as stated above, q_1, q_2, q_3 with the momenta of π^+, π^- and π^0 from the ω decay, q_4 and q_5 with the momenta of the remaining π^- and π^0. Without this approximation the symmetrization between q_2 and q_4 on the one hand, and q_3 and q_5 on the other hand has to be performed

$$\begin{aligned} J_\mu^{A;\text{total}}(q_1,q_2,q_3,q_4,q_5) &= J_\mu^A(q_1,q_2,q_3,q_4,q_5) + J_\mu^A(q_1,q_4,q_3,q_2,q_5) \\ &\quad + J_\mu^A(q_1,q_2,q_5,q_4,q_3) + J_\mu^A(q_1,q_4,q_5,q_2,q_3) \end{aligned} \tag{11}$$

and the phase space has to be divided by 4 to take the identical particle statistical factor into account.

For a test of the proper implementation of the matrix element (Eq. (10)), the narrow-width approximation for a_1, ρ and ω is employed. (For this test $(m_\rho + m_\omega) < m_a$ must be assumed.) In this approximation the rate derived from the current J^A can be integrated analytically. (Again the sum of three ρ Breit–Wigner amplitudes in the ω decay is replaced by 1 and m_π is set to zero for this test.)

$$\begin{aligned} \frac{\Gamma_A}{\Gamma_{\ e}} &= \frac{1}{4}\frac{m_{a_1}^2}{m_\tau^2}\Big(1-\frac{m_{a_1}^2}{m_\tau^2}\Big)^2\Big(1+2\frac{m_{a_1}^2}{m_\tau^2}\Big)c_0^2\frac{m_{a_1}\pi}{\Gamma_{a_1}} f(m_{a_1}^2, m_\rho^2, m_\omega^2) R\,, \\ f(m_{a_1}^2, m_\rho^2, m_\omega^2) &\equiv 2\frac{2p}{m_{a_1}}\Big(\frac{p^2}{m_\rho^2}+\frac{p^2}{m_\omega^2}+3\Big)\,, \\ p &\equiv \frac{\lambda^{1/2}(m_{a_1}^2, m_\rho^2, m_\omega^2)}{2m_{a_1}}\,. \end{aligned} \tag{12}$$

The factor $R = \frac{\Gamma_\rho^p}{\Gamma_\rho}\frac{\Gamma_\omega^p}{\Gamma_\omega}$ consists of the product of the partial widths for ρ and ω as given by Eqs. (5) and (9), divided by total widths used as numerical inputs in the Breit–Wigner amplitudes, and the decay rate is normalized to $\Gamma_e \equiv \Gamma(\tau \to e\bar\nu_e\nu_\tau)$.

For a test of the generator, in particular of the phase-space integration, the following, totally unphysical form of the current

$$\begin{aligned} J_\mu^C &= c_1\ Q_\mu\,, \\ c_1 &= \frac{1}{m_\tau^3}\,4!\,(4\pi)^3\sqrt{20} \end{aligned} \tag{13}$$

was used, with the pion mass again set to zero[1]. The analytical result,

$$\frac{\Gamma^C}{\Gamma_e} = \cos^2\theta_{\text{Cabbibo}} = 0.950625\,, \tag{14}$$

[1] As an alternative, in principle we could also use $c_2 = 4!(4\pi)^3/Q^3$ and obtain the same result. In such a case, numerical-stability problems could appear as, for massless pions, $1/Q^3$ may approach integrable infinity within the allowed phase space.

is well reproduced by the generator. The numerical results of the second test, based on the narrow-width approximation of Eq. (12), will be discussed below.

The decay mode into $\omega\pi\pi$ is obviously only possible for the $2\pi^0 2\pi^- \pi^+$ final state. In contrast the amplitude introduced in the following will contribute to all three charge combinations $\pi^- 4\pi^0$, $2\pi^- 2\pi^0 \pi^+$ and $3\pi^- 2\pi^+$. For this second amplitude we use the transition of the virtual W into an a_1, with subsequent transition into a_1 plus two pions in an isospin zero, angular-momentum zero configuration, parametrized by the broad f_0 resonance. For the $a_1 a_1 f_0$ coupling the simplest Lorentz structure with minimal momentum dependence has been adopted. The a_1 subsequently decays into a ρ meson plus a pion, with equal amplitude for the two charge modes $\pi^- 2\pi^0$ and $\pi^+ 2\pi^-$, similar to the parametrization of the tau decay into three pions.

This leads to the following current:

$$
\begin{aligned}
& J^B_\mu(p_1,p_2,p_3,p_4,p_5) \equiv \frac{c}{m_a^4 m_f^2 m_\rho^2} f_{aaf} \;\; f_{f\pi\pi}\; g_{a\rho\pi}\; g_{\rho\pi\pi} \\
& \times BW_a((p_1+p_2+p_3+p_4+p_5)^2) BW_a((p_1+p_2+p_3)^2) BW_{f_2}((p_4+p_5)^2) \\
& \times \Big[\Big(\frac{Q_\mu Q_\nu}{Q^2} - g_{\mu\nu}\Big)\Big(\frac{p_2(p_1-p_3)}{(p_1+p_2+p_3)^2}(p_1+p_2+p_3)^\nu - (p_1-p_3)^\nu\Big) \\
& \times BW_\rho((p_1+p_3)^2) + (1 \leftrightarrow 2)\Big], \qquad (15)
\end{aligned}
$$

with the momentum assignments

$$a_1(\to \pi_-(p_1)\pi_-(p_2)\pi_+(p_3)) + f(\to \pi_0(p_4)\pi_0(p_5))\,.$$

The pictorial illustration is shown in Fig. 1(b). (In the program, we have used: $m_f = 0.8$ GeV, $\Gamma_f = 0.6$ GeV, $G_{a\rho\pi} = 6$, $f_{aaf} = 4$, $f_{f\pi\pi} = 5$ and $c = 4$.) The last constant was introduced to normalize the branching ratio for this channel to 0.11%. The amplitude is, by construction, symmetric under the exchange p_1 *versus* p_2 and p_4 *versus* p_5, as requested from Bose symmetry for the f_0 and a_1 decays. Alternatively, the two π^0 may originate from the a_1 with the f_0 then decaying into $\pi^-\pi^+$. In this case the symmetrization with respect to the momenta of the π^- has to be performed explicitly. For consistency we have to adopt the same momentum assignment as before: $\pi^+(q_1)\pi^-(q_2)\pi^0(q_3)\pi^-(q_4)\pi^0(q_5)$. The properly symmetrized amplitude thus reads:

$$
\begin{aligned}
J^{B;\mathrm{total}}_\mu(q_1,q_2,q_3,q_4,q_5) \;=\; & J^B_\mu(q_2,q_4,q_1,q_3,q_5) + J^B_\mu(q_3,q_5,q_2,q_1,q_4) \\
& + J^B_\mu(q_3,q_5,q_4,q_1,q_2)\,. \qquad (16)
\end{aligned}
$$

We include the same statistical factor of $\frac{1}{4}$ as before into the normalization of the phase space. The relative rate $[f(+-)+a(00)]:[f(00)+a(-+)]=2:1$ is recovered in the narrow width approximation for a_1 and f_0. At present there is no analytical benchmark for the overall normalization available for this channel.

The extension of this model to the description of the remaining charge configurations is straightforward. Let us start with $\pi^-4\pi^0$ and adopt the following momentum assignment: $\pi^-(q_3)\pi^0(q_1)\pi^0(q_2)\pi^0(q_4)\pi^0(q_5)$. Using the definition (15) for J^B as in Eq. (16), the amplitude has to be symmetrized with respect to the momenta of π^0. Since J^B is already symmetric with respect to the first two and the last two momenta, only 6 out of the 4! permutations have to be considered. This leads to the current

$$\begin{aligned} J_\mu^{00-00}(q_1,q_2,q_3,q_4,q_5) &= J_\mu^B(q_1,q_2,q_3,q_4,q_5)+J_\mu^B(q_5,q_2,q_3,q_4,q_1) \\ &\quad + J_\mu^B(q_2,q_4,q_3,q_1,q_5)+J_\mu^B(q_1,q_4,q_3,q_2,q_5) \\ &\quad + J_\mu^B(q_1,q_5,q_3,q_4,q_2)+J_\mu^B(q_4,q_5,q_3,q_1,q_2). \end{aligned} \tag{17}$$

For the evaluation of the rate the statistical factor of 1/4! must be included.

In complete analogy we obtain for the $3\pi^-2\pi^0$ mode

$$\begin{aligned} J_\mu^{--++-}(q_1,q_2,q_3,q_4,q_5) &= J_\mu^B(q_1,q_2,q_3,q_4,q_5)+J_\mu^B(q_5,q_2,q_3,q_4,q_1) \\ &\quad + J_\mu^B(q_1,q_5,q_3,q_4,q_2)+J_\mu^B(q_1,q_2,q_4,q_3,q_5) \\ &\quad + J_\mu^B(q_5,q_2,q_4,q_3,q_1)+J_\mu^B(q_1,q_5,q_4,q_3,q_2)\,, \end{aligned} \tag{18}$$

where the momentum assignment $\pi^-(q_1)\pi^-(q_2)\pi^+(q_3)\pi^+(q_4)\pi^-(q_5)$ has been adopted and a statistical factor $1/2!3! = 1/12$ is used in the evaluation of the rate.

For the "non-ω" decays and in the narrow-width approximation, this ansatz predicts the following abundances of the subchannels:

$$f(00)a(--+):f(-+)a(00-):f(00)a(00-):f(-+)a(--+)=1:2:1:2 \tag{19}$$

and the following relative rates

$$2\pi^0 2\pi^-\pi^+ : \pi^-4\pi^0 : 3\pi^-2\pi^+ = 3:1:2\,. \tag{20}$$

Note that this prediction is specific to the resonance and isospin structure of the model. Once more experimental information on the five-pion channel will be available, more elaborate possibilities can be considered.

3. Results from the Monte Carlo program

The new version of TAUOLA includes new channels, numbered 31 to 35, which will be discussed in turn. Channels 31, 32 and 33 refer to $2\pi^0 2\pi^- \pi^+$, channel 34 to $\pi^- 4\pi^0$ and channel 35 to $3\pi^- 2\pi^+$. Channels 31 and 32 are implemented for tests, channels 33, 34 and 35 for physics simulations.

Let us start with channel 31, which is based on current J^A, without symmetrization, as defined in Eq. (10). The order of the pions generated by TAUOLA is $(- - + 0\,0)$. This ordering is necessary for the test and the requirements of the presampler running in the narrow-resonance mode, which is only partly optimized. The result, based on the default parameter values listed after Eq. (3), is given in the first line of Table I. This channel is then used to test the program against Eq. (12), which is valid in the narrow-width approximation for ρ, ω, and a, using massless pions. This simulation was quite demanding on the phase-space generator. A precision of about 1% only was reached. Because of the complex structure of the resonances a_1, ρ and ω it was impossible to generate events efficiently. The ρ Breit–Wigner was only generated from a flat distribution. The variance of the generated raw sample was deteriorating quite fast with decreasing width, not only rendering the generator slow, but also risking to arrive at numerically unstable results. We nonetheless completed this test for several choices of masses and widths of a_1, ρ and ω. A typical result is listed in Table I, line 4, where $\Gamma_\omega = \Gamma_a = 1\,\text{MeV}$, $\Gamma_\rho = 5\,\text{MeV}$, $m_\rho = 373\,\text{MeV}$, was adopted for the parameters, and other parameters were left at their default values, in particular $m_\omega = 782\,\text{MeV}$ and $m_a = 1260\,\text{MeV}$ was kept. The remaining 2.5% difference between Monte Carlo and analytical calculation is due to contributions from the tails, which remain large, even for the extremely narrow widths adopted in this example. This illustrates the importance of non-resonant contributions and interferences, in particular for realistic values of the parameters.

Current J^C, which also serves to test the generator, is implemented in channel 32. In the case of massless pions, Eq. (14) is reproduced with a precision better than 0.1% (see line 5 of Table I). This result is stable

TABLE I

Test results of the generator for unphysical choices of parameters; see text.

Current	Γ_X/Γ_e (TAUOLA)	Γ_X/Γ_e (analytical)	Comment
A	$0.02404 \pm 3 \times 10^{-5}$	—	without symm.
B	$0.00928 \pm 9 \times 10^{-6}$	—	without symm.
C	$0.10690 \pm 2 \times 10^{-4}$	—	massive pions
A	$1.38 \times 10^7 \pm 1 \times 10^5$	1.41×10^7	massless pions
C	$0.95030 \pm 3 \times 10^{-5}$	0.950625	massless pions

and independent of the choice of options for our phase-space presampler, providing an important test of its correct operation. The result for J^C, taking the pion mass into account, is listed in line 3.

In the second line of Table I the result is listed for current J^B, as defined in Eq. (15), again without symmetrization.

The results for physical input parameters, and with properly symmetrized amplitudes, are listed in Table II, column 4. The first three lines refer again to $2\pi^0 2\pi^- \pi^+$, as implemented in channel 33, and are based on Eqs. (11) and (16). For the order of the pion momenta, in the program the convention[2] $(+--00)$ has been used. The first line is based on the $\omega\pi\pi$ channel ($J^{A;\mathrm{total}}$) alone, and the symmetrization evidently increases the result by about +5%. The second line is based on $J^{B;\mathrm{total}}$ alone, and the symmetrization decreases the result[3] by 11%. Adding coherently $J^{A;\mathrm{total}}$ and $J^{B;\mathrm{total}}$ one arrives at the result given in line 3. Keeping in mind our previous experiences with interferences, it is amazing that the result is exactly the sum of the two previous entries. This implies that the two amplitudes do not lead to interferences in the rate in any significant manner! But of course, perfect agreement for all significant digits is due to statistical fluctuations.

TABLE II

Test results of the generator for realistic choices of parameters; see text.

N_{channel}	Final state	Current	$\Gamma_X/\Gamma_e \times 10^3$ TAUOLA	$\Gamma_X/\Gamma_e \times 10^3$ experiment [5]	Γ_X/Γ_e narrow width
33	$2\pi^-\pi^+2\pi^0$	A; total	25.30 ±0.1%	25 ± 3	—
33	$2\pi^-\pi^+2\pi^0$	B; total	6.05 ±0.2%	6.2 ± 2	2192 ± 22
33	$2\pi^-\pi^+2\pi^0$	A+B; total	31.35 ±0.1%	31 ± 2	—
34	$\pi^-4\pi^0$	B; total	9.37 ±0.1%	$5.5^{+3.4}_{-2.8}$	788 ± 4
35	$3\pi^-2\pi^+$	B; total	11.03 ±0.1%	4.6 ± 0.3	1469 ± 7

The charge configurations $\pi^-4\pi^0$ and $3\pi^-2\pi^+$ (with the momentum ordered (-0000) and $(-++--)$ in the TAUOLA output, again differently as in Sec. 2) correspond to channels 34 and 35, respectively. The results are based on the amplitudes given in Eqs. (17) and (18) and the numerical predictions are listed in lines 4 and 5. These numbers are contrasted with the experimental results listed in Table II, column 5. The result for $2\pi^-\pi^+2\pi^0$ as obtained with $J^{A;\mathrm{total}}$ alone is compared with the mode $h^-\omega\pi^0$, the result for $J^{B;\mathrm{total}}$ alone with the mode $h^-h^-h^+\pi^0\pi^0$ (exl. K^0, ω, η). The $\pi^-4\pi^0$ mode is compared with $h^-4\pi^0$ (exl. K^0, η) and $3\pi^-2\pi^+$ with $h^-h^-h^-h^+h^+$. Let us emphasize again that the rates for the two submodes of channel 33

[2] Note the difference in order with respect to Sec. 2 and channel 31.

[3] This property is obscured by π^0, $\pi^\pm$ mass difference.

are fitted to the data, the rates for channels 34 and 35 are then the result of our ansatz.

The relative rates as displayed in lines 2, 5 and 6, column 4, are seemingly in contradiction with the expectations based on the narrow-width approximation for the intermediate states, Eq. (20). We, therefore, perform the simulation also in the narrow-width approximation, with the following parameters: $m_f = 0.45\,\mathrm{GeV}$, $\Gamma_f = 10\,\mathrm{MeV}$. For the a saturating intermediate 3-pion state, we took $m_a = 1.16\,\mathrm{GeV}$ and $\Gamma_a = 10\,\mathrm{MeV}$, while width and mass of the a_1 coupled directly to the τ lepton were kept at their standard value; this was also true for the mass and width of the ρ meson. Indeed, in this completely fictitious case, Eq. (20) is recovered within the statistical error (column 6).

Eq. (20) is only reproduced for very small widths of the resonances. For realistic parameters, the predictions for ratios of branching fractions are significantly affected by interferences, and this applies even more so for differential distributions. This observation has to be taken into account from the very beginning in the formulation of a realistic current and the construction of Monte Carlo programs where cascade decay chains must be implemented.

4. Summary

In this note, recent developments in TAUOLA for τ decays into five mesons are described. For the $2\pi^-\pi^+2\pi^0$ mode, two amplitudes were introduced. The first is motivated by the experimental observation of a dominant $\omega\pi\pi$ channel, the second corresponds to a non-ω contribution and is parametrized by the transition through an intermediate a_1+f_0 combination. This second amplitude was also used to predict the two remaining charge combinations for τ decays into five pions. Both amplitudes are implemented into TAUOLA. A third current, J^C, was introduced for tests of the phase-space generator. With the help of these currents basic technical tests of the algorithm were successfully completed. No sophisticated fits to the data were attempted. The model amplitude can, however, be used as a starting point for more detailed theoretical and experimental investigations.

We observed that effects due to interferences between different resonant amplitudes are often large. This is due to various threshold effects and may be specific for the choice of our amplitude. It may result, for example, from those various threshold effects or from simplifying assumptions adopted for our currents, such as the choice of constant instead of running, final-state mass-dependent widths. For different parametrizations, the influence of interferences on the total rates could be significantly smaller.

This issue is of great practical importance for the design of a generator used in fits to data during their analysis, when the general form for the currents often has to be changed several times. If the constraint of zero interference between various currents, standing for different subresonances, was imposed on the algorithm, as was the case *e.g.* with EURODEC [8,9], the fitting procedures would become unnecessarily difficult (even impossible), or might even not lead to a correct description of the amplitude. Therefore, distinct currents for each final state, which are labeled by stable decay products, will be also used in the future. Resonances such as ρ, a_1, ω are can only be used as intermediate states in the amplitudes.

Finally, let us point out that the extensions for TAUOLA as presented in this paper are available from the directory `tauola-BBB` of the standard distribution system for TAUOLA versions. The usage of the directory `tauola-BBB` is not documented yet; it is, however, identical to the one of the `tauola-F` subdirectory as explained in [4]. The most recent version of the code is available from the web page [7]. Test results for differential distribution, obtained with the help of MC-TESTER [10], are available from the web page [11].

Supported by grants BMBF/WTZ/POL01/103 and BMBF No. 05HT4 VKA/3. Supported in part by the EU grant MTKD-CT-2004-510126, in partnership with the CERN Physics Department, and by the Polish State Committee for Scientific Research (KBN) grant 2 P03B 091 27 for the years 2004–2006. One of us (Z.W.) acknowledges the hospitality of TTP Karlsruhe, where part of this work was done and the Polish–French collaboration within IN2P3 through LAPP Annecy.

REFERENCES

[1] S. Jadach, J.H. Kühn, Z. Wąs, *Comput. Phys. Commun.* **64**, 275 (1990).

[2] M. Jeżabek, Z. Wąs, S. Jadach, J.H. Kühn, *Comput. Phys. Commun.* **70**, 69 (1992).

[3] S. Jadach, Z. Wąs, R. Decker, J.H. Kühn, *Comput. Phys. Commun.* **76**, 361 (1993).

[4] P. Golonka, B. Kersevan, T. Pierzchala, E. Richter-Wąs, Z. Wąs, M. Worek, *Comput. Phys. Commun.* **174**, 818 (2006).

[5] S. Eidelman *et al.* [Particle Data Group Collab.], *Phys. Lett.* **B592**, 1 (2004).

[6] R. Decker, E. Mirkes, R. Sauer, Z. Wąs, *Z. Phys.* **C58**, 445 (1993).

[7] P. Golonka, Z. Wąs, see: `http://wasm.home.cern.ch/wasm/goodies.html` or `http://cern.ch/Piotr.Golonka/MC/PHOTOS-MCTESTER`

[8] B. van Eijk, EUROJET Reference Manual, CERN-DD Long Write up (1989), DELPHI report 89-39 PHYS 41.

[9] P.H. Eberhard *et al.*, CERN-EP/89-129, Proc. Workshop on Z Physics at LEP, Geneva, Switzerland, Feb. 20–21, May 8–9, 1989, ed. G. Altarelli, R. Kleiss, V. Verzegnassi, CERN 89-08 vols. 1–3.

[10] P. Golonka, T. Pierzchala, Z. Wąs, *Comput. Phys. Commun.* **157**, 39 (2004) [`hep-ph/0210252`].

[11] P. Golonka, Z. Wąs, `http://annapurna.ifj.edu.pl/~wasm/test-BBB.ps`