Vol. 39 (2008) ACTA PHYSICA POLONICA B No 1

# RESIDUAL BOSE–EINSTEIN CORRELATIONS AND THE SÖDING MODEL

Ayda Beddall, Andrew Beddall, Ahmet Bingül

Department of Engineering Physics, University of Gaziantep
Gaziantep 27310, Turkey

(Received May 30, 2007; revised version received September 11, 2007)

Bose–Einstein correlations between identical pions close in phase-space is thought to be responsible for the observed distortion in mass spectra of non-identical pions. For example in the decays $\rho^0 \to \pi^+\pi^-$ and $\rho^\pm \to \pi^\pm\pi^0$, such distortions are a residual effect where the pions from the $\rho$ decay interact with other identical pions that are close in phase-space. Such interactions can be significant in, for example, hadronic decays of $Z$ bosons where pion multiplicities are high, and resonances such as $\rho$ mesons decay with a very short lifetime thereby creating pions that are close to prompt pions created directly. We present the Söding model and show that it has been used successfully to model distortions in $\pi^\pm\pi^0$ mass spectra in hadronic $Z$ decays recorded by ALEPH.

PACS numbers: 29.85.+c

## 1. Introduction

Bose–Einstein correlations (BECs) are an attraction in phase-space between two-identical bosons, and therefore cause a change in their kinematics. BECs are significant in hadronic $Z$ decays since most of the particles generated in hadronic events are pions obeying Bose statistics. Studies of BECs have helped in the understanding of QCD studies at LEP1 [1,2], and in the determination of the $W^\pm$ mass at LEP2 [3]. Various attempts have been made to include BECs in Monte Carlo simulations, including both local and global approaches [3]. These BEC algorithms can be applied to pions in the fragmentation state, and/or to final state pions for which the 4-momenta difference is calculated for each pair of identical pions.

In this study, distortions due to residual BECs in $\pi^\pm\pi^0$ mass spectra in the region of the $\rho^\pm$ resonance are studied in hadronic decays of the $Z$ recorded by ALEPH. The $\rho^\pm$ meson decays with a very short lifetime thereby creating pions that are close ($\sim$1 fm) to prompt pions created directly; therefore pions from the $\rho^{\pm}$ signal have the opportunity to interact with pions that form a combinatorial background.

To measure production rates of a resonance such as the $\rho^{\pm}$ meson, it is necessary to model all contributions to the mass spectra in the region of the signal; this includes signal and background functions, reflections, and distortions due to residual BECs. In this study, we present the results of applying the Söding Model for the modelling of the distortions due to residual BECs.

## 2. Data

Using hadronic event selection criteria defined in [4], approximately 3.2 million hadronic $Z$ decays around the center-of-mass energy of 91.2 GeV recorded by ALEPH at LEP in the period between 1991 and 1995 are selected for the analysis. Charged tracks passing basic quality cuts are selected if their impact parameters are consistent with an origin close to the interaction point, and if the ionisation rate ($dE/dx$) is consistent with the $\pi^{\pm}$ hypothesis (where available). Neutral pions are built by combining pairs of photons from the decay channel $\pi^0 \to 2\gamma$. Candidates are accepted if the invariant mass of the photon pair is within $\pm 2\sigma$ around the expected mass, where $\sigma$ is the mass resolution of the $\pi^0$ signal. The momentum resolution is improved by constraining the mass of the $\pi^0$ candidates to the nominal mass. The poor $\pi^0$ signal purity is improved by a 'ranking' method [5]. To compare the modelling of the real data with data that does not contain BEC effects, the same number of Monte Carlo events are selected. These events are generated with the Jetset [6] program and passed through a full detector simulation and reconstruction program.

## 3. Two-pion mass spectra

The invariant mass distribution of $\pi^{\pm}\pi^0$ pairs reveals a wide $\rho^{\pm}$ resonance peak (nominal mass $\approx 0.776$ GeV/$c^2$, nominal width $\approx 0.152$ GeV/$c^2$) on top of a background that contains large reflections from other decays, a large combinatorial background, and a significant distortion due to residual BECs. All these contributions are modelled in terms of functions which contribute to a total function $F(m)$ that is fitted to the mass spectra using the method of least-squares with a total of seven free parameters. This total fit function is as follows:

$$F(m) = f_s(m) + f_b(m) + f_\omega(m) + f_\eta(m) + f_{K^*}(m) + f_i(m)\,. \qquad (1)$$

Here, $f_s(m)$ is a relativistic $p$-wave Breit–Wigner function representing the $\rho^{\pm}$ signal with resonance parameters taken from Monte Carlo predictions.

This signal function also includes a slightly modified component corresponding to partially reconstructed $\rho^{\pm}$ mesons where a $\pi^0$ is reconstructed from one photon originating from the $\rho^{\pm}$ signal and one that is not. The term $f_b(m)$ is an appropriate smooth function representing the combinatorial background, and the functions $f_\omega(m)$, $f_\eta(m)$ and $f_{K^*}(m)$ represent reflections from the decays $\omega \to \pi^0\pi^+\pi^-$, $\eta \to \pi^0\pi^+\pi^-$ and $K^{*\pm} \to \pi^0 K^{\pm}$ respectively. Finally the function $f_i(m)$ is introduced to account for the distortions (interference) in the mass spectra due to residual BECs. The details of the functions $f_s(m)$ and $f_i(m)$ are discussed in Section 4.

The data is analysed in nine intervals of scaled energy $x_E = E_\rho/E_{\rm beam}$ where $E_\rho$ is the energy of the reconstructed $\rho^{\pm}$, and $E_{\rm beam}$ is the beam energy (45.6 GeV). The result of fitting one such interval is shown in Fig. 1. In this figure all contributions are shown except for the interference term

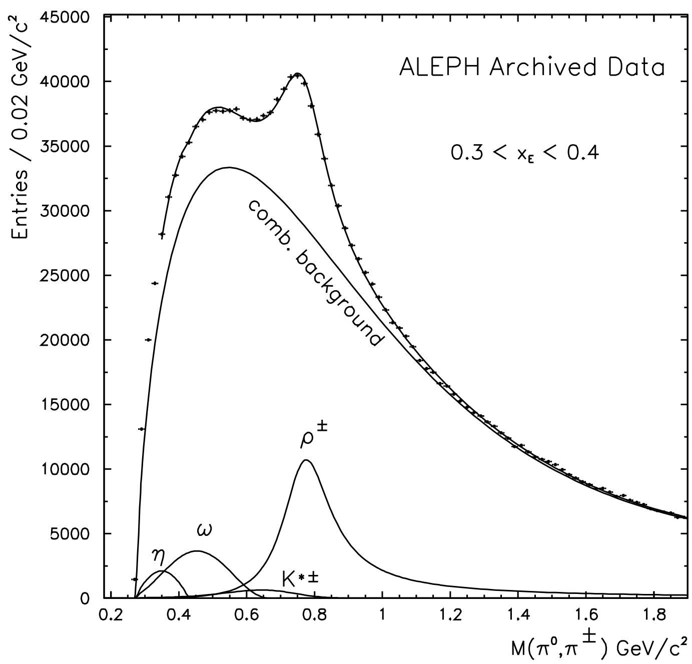


Fig. 1. Fit to a two-pion invariant mass spectra in the region of the $\rho^{\pm}$ peak. The data points are well described by the fit ($\chi^2/\mathrm{NDF} = 2.2$). The contributions from the signal, combinatorial background and reflections are shown.

$f_i(m)$ which instead is shown in Fig. 2 where the non-signal functions are subtracted from the data, leaving only the $\rho^{\pm}$ signal. The data in this plot displays the expected relativistic Breit–Wigner shape (with a noticeable high mass tail). From this figure is clear that the interference function $f_i(m)$

(dashed curve) contributes significantly to the mass spectra in the region of the $\rho^{\pm}$ resonance. Similar results are found for each $x_E$ interval though with the significance of the distortion reducing with increasing $x_E$. These distortions are discussed in more detail in the next section.

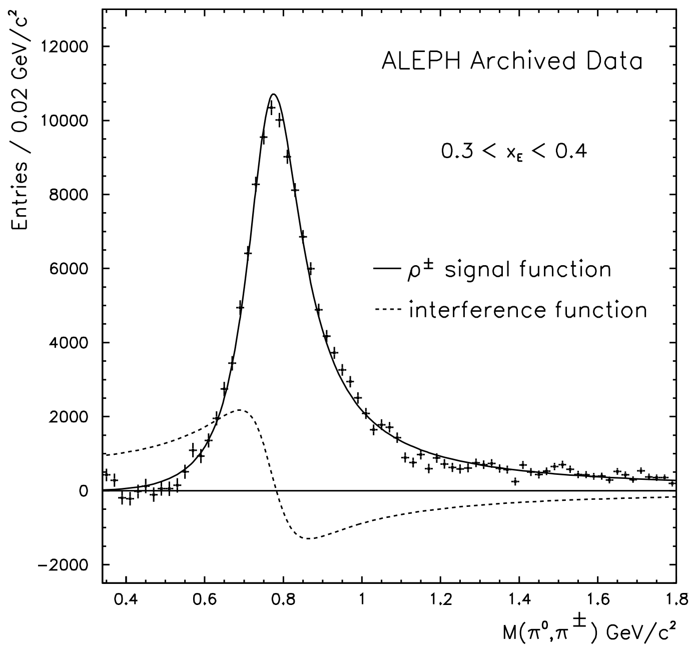


Fig. 2. The same data as shown in Fig. 1 after all contributions, except for the signal function, are subtracted. The $\chi^2$/NDF of the fit is 2.2. The data shows the expected relativistic Breit–Wigner shape. The interference function $f_i(m)$ (dashed curve) is also shown.

## 4. Residual BECs, and the Söding model

Experimental studies reveal that BECs affect the distribution of invariant masses of identical $\pi^{\pm}\pi^{\pm}$ pairs in hadronic decays of $Z$ bosons [1, 2]. Also, it is shown in [1] that residual BECs affect significantly the kinematics of very short lived resonances with decay lengths of $\approx 1$ fm. This is the case for the $\rho^0$ meson [4] where BECs exist between pions from the decay $\rho^0 \to \pi^+\pi^-$ and prompt pions emitted directly. Residual BECs are understood to affect both the shape of resonant signals, and the shape of the combinatorial background; this is the interpretation of [7] and the present study.

It has been pointed out in Reference [1] that the background interference mechanisms studied by [8] can have similar phenomenological effects as residual BECs. These interference mechanisms have been treated successfully by the Söding model, and has led the OPAL group [7] in the analysis of $\rho^{\pm}$ production, and the OMEGA group [8] in the analysis of $\rho^0$ production, to employ the Söding model to account for the distortions seen under the $\rho$ peaks in their respective two-pion mass spectra.

In the present study, taken from the Söding model, the term $f_i(m)$ in Eq. (1) is given by:

$$f_i(m) = C\left(\frac{m_0^2 - m^2}{m\,\Gamma(m)}\right) f_s(m)\,. \tag{2}$$

Here, $C$ is a free parameter describing the strength of the interference, $m_0$ is the nominal mass of the $\rho^{\pm}$ signal[1], $m$ is the two-pion invariant mass, and $\Gamma(m)$ is the mass-dependent width of the resonance. Various parameterisations for $\Gamma(m)$ have been suggested [1], the form chosen in this study is:

$$\Gamma(m) = \Gamma_0\left(\frac{q}{q_0}\right)^3 \frac{2q_0^2}{q_0^2 + q^2}\,, \tag{3}$$

where $\Gamma_0$ is the natural width of the $\rho^{\pm}$ signal[2], $q$ is the momentum of the decay products in the rest frame of the parent given by:

$$q^2 = \frac{m^2 - m_{\pi^\pm}^2 + m_{\pi^0}^2}{4m^2} - m_{\pi^0}^2 \tag{4}$$

and $q_0$ is the momentum when $m = m_0$ [4].

The inclusion of the factor $f_s(m)$ in Eq. (2) illustrates that the effect can be considered to be due to interference between the amplitudes of the $\rho^{\pm}$ and the non-resonant $\pi\pi$-background. The form of this signal function is therefore important, and in this study this is chosen to be the relativistic $p$-wave Breit–Wigner:

$$f_s(m) = \frac{m\,m_0\,\Gamma(m)}{(m^2 - m_0^2)^2 + m_0^2\,\Gamma^2(m)}\,, \tag{5}$$

where the parameters $m$, $m_0$ and $\Gamma(m)$ are already defined above.

The value of parameter $C$ in Eq. (2) may depend on a number of factors, such as pion multiplicity and momenta. The effect of the value of $C$ on the shape of the invariant mass spectra can be illustrated by subtracting $f_i(m)$ from the data. This is shown in Fig. 3 where the value of $C$ is varied from 0.0 to 0.5 for a medium $x_E$ interval. The data is compared to the Monte

[1] The value of $m_0$ is in fact calibrated, using Monte Carlo, to take into account detector resolution effects.

[2] The value of $\Gamma_0$ is in fact calibrated, using Monte Carlo, to take into account detector resolution effects.

Carlo data that contains no BEC distortions. The first plot ($C = 0.0$) corresponds to the original spectra with no correction; the spectra appear to be shifted toward the left with respect to the Monte Carlo with a depleted region to the right of the $\rho^{\pm}$ peak, and enhanced region to the left of the peak. If BECs exist, then the expected result of correlated particle momenta is a shift to lower mass, as seen in the first plot of Fig. 3. Introducing a small correction ($C = 0.1$) improves the agreement between the data and Monte Carlo, a larger correction ($C = 0.3$) yields a very good agreement. If the value of $C$ is increased further ($C = 0.5$), the agreement becomes poor again but with an opposite distortion. Note that the form of $f_i(m)$ seen in Fig. 2 suitably modifies the depleted and enhanced regions at either side of the resonance peak, with a zero contribution at the peak mass. The value of $C$ in Eq. (2) is a free parameter in the fit of $F(m)$ to the data, results for the values of $C$ from fits to each energy interval are shown in Fig. 4. The exponential function $e^{-4x_E}$ (solid line), gives a good fit to the data.

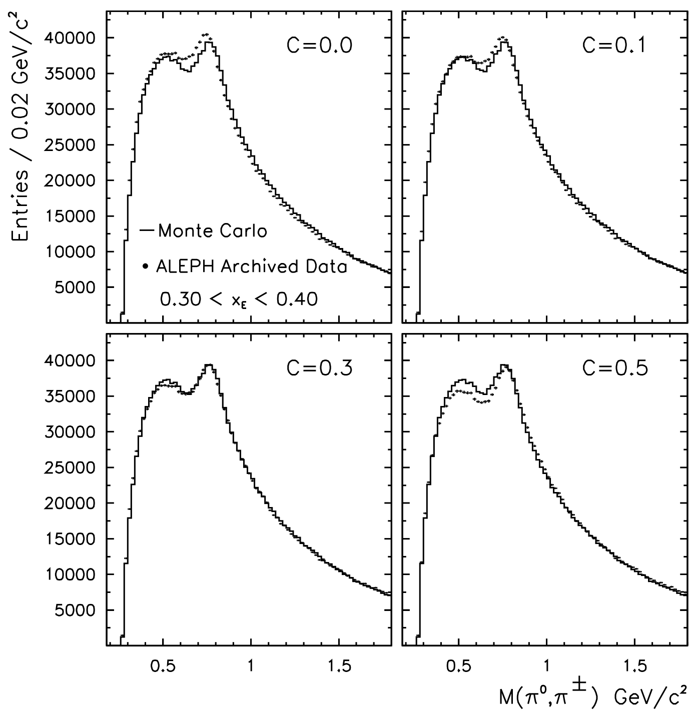


Fig. 3. Two-pion invariant mass spectra for the Monte Carlo (solid line), and the interference term-subtracted real data mass spectra (data points). The value of $C$, in Eq. (2), is set to 0.0, 0.1, 0.3 and 0.5 for each plot, respectively.

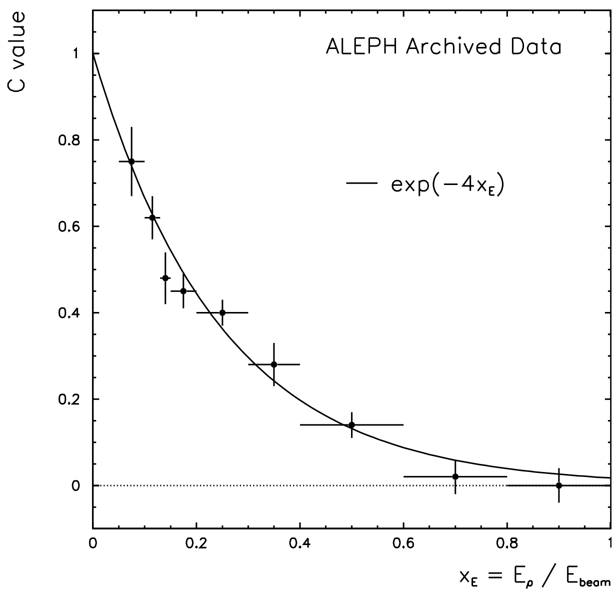


Fig. 4. Distribution of fitted $C$ values as a function of $x_E$. The error bars represent the quadratic sum of statistical and systematic errors. The exponential function $e^{-4x_E}$ (solid line) yields a $\chi^2$/NDF of 0.9.

## 5. Summary and conclusion

In hadronic decays of the $Z$ boson, the $\pi^{\pm}\pi^0$ mass spectra reveals a significant distortion around the $\rho^{\pm}$ resonance. It is commonly accepted that the distortion is due to Bose–Einstein correlations causing interference between pions from the $\rho^{\pm}$ decay and other identical pions that are close in phase space. Such interference only affects resonances that have sufficiently short lifetimes to produce decay products close to ($\sim$ 1 fm) prompt pions created directly. In particle production measurements it is important to model correctly all significant contributions to a mass spectra. While the physics of the Söding model applies only to elastic photo-production of $\rho^0$ mesons, the model provides a convenient way to parameterise distortions of the $\rho^{\pm}$ line shape in hadronic $Z$ decays. This model has also been use successfully elsewhere for both the $\rho^0$ and $\rho^{\pm}$ decays. The strength of the distortion, represented by the value of $C$ in Eq.(2), is seen to fall exponentially with $x_E$; this is consistent with the observations of [7] and [8].

We wish to thank the ALEPH collaboration for access to the archived data since the closure of the collaboration [9]. We also wish to thank the CERN accelerator divisions for the successful operation of LEP, and we are indebted to the researchers, engineers and technicians for their contribution to the excellent performance of ALEPH.

## REFERENCES

[1] G.D. Lafferty, *Z. Phys.* **C60**, 659 (1993).

[2] A. Heister *et al.* [ALEPH Collaboration], *Eur. Phys. J.* **C36**, 147 (2004).

[3] L. Lönnblad, *Eur. Phys. J.* **C2**, 165 (1998).

[4] D. Buskulic *et al.* [ALEPH Collaboration], *Z. Phys.* **C69**, 379 (1996).

[5] A. Beddall *et al.*, *Nucl. Instrum. Methods Phys. Res.* **A482**, 520 (2002).

[6] T. Sjöstrand, *Comput. Phys. Commun.* **82**, 74 (1994).

[7] K. Ackerstaff *et al.* [OPAL Collaboration], *Eur. Phys. J.* **C5**, 411 (1998).

[8] R.J. Apsimon *et al.* [OMEGA Photon Collaboration], *Z. Phys.* **C53**, 581 (1992).

[9] Statement on use of ALEPH data for long-term analyses `http://aleph.web.cern.ch/aleph/alpub/archive_data.pdf`