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MATHISSON EQUATIONS: NON-OSCILLATORY SOLUTIONS IN A SCHWARZSCHILD FIELD

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The Mathisson equations under the Frenkel–Mathisson supplementary condition are studied in a Schwarzschild field. The choice of solutions, which describe the motions of the proper center of mass of a spinning test particle, is discussed, and the calculation procedure for highly relativistic motions is proposed. The very motions are important for astrophysics while investigating possible effects of the gravitational spin-orbit interaction on the particle's world line and trajectory.

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1. Introduction

70 years ago Myron Mathisson has presented the equations describing the motions of a spinning test particle in a gravitational field [1]

$$\frac{D}{ds}\left(mu^\lambda + u_\mu\frac{DS^{\lambda\mu}}{ds}\right) = -\frac{1}{2}u^\pi S^{\rho\sigma}R^\lambda_{\pi\rho\sigma}\,, \tag{1}$$

$$\frac{DS^{\mu\nu}}{ds} + u^\mu u_\sigma\frac{DS^{\nu\sigma}}{ds} - u^\nu u_\sigma\frac{DS^{\mu\sigma}}{ds} = 0\,, \tag{2}$$

where u^λ is the 4-velocity of a spinning particle, $S^{\mu\nu}$ is the antisymmetric tensor of spin, m and D/ds are, respectively, the mass and the covariant derivative with respect to the proper time s; $R^\lambda_{\pi\rho\sigma}$ is the Riemann curvature tensor of the spacetime. (Throughout this paper we use units $c = G = 1$. Greek indices run 1, 2, 3, 4 and Latin indices 1, 2, 3; the signature of the metric $(-,-,-,+)$ is chosen.) Eqs. (1), (2) were supplemented by the condition [1]

$$S^{\mu\nu}u_\nu = 0\,. \tag{3}$$

(In special relativity condition (3) was introduced by Frenkel [2].) It is known [3,4] that in the Minkowski spacetime the Mathisson equations (1)–(3) have, in addition to usual solutions describing the straight worldlines, a family of solutions describing the oscillatory (helical) worldlines (as a partial case, this family contains the circular solutions).

In [3] the oscillatory solutions of equations (1)–(3) were connected with *"Zitterbewegung"*. Möller proposed another interpretation of these solutions: he pointed out that 1. in relativity the position of the center of mass of a rotating body depends on the frame of reference, and 2. condition (3) is common for the so-called proper and non-proper centers of mass [5]. The usual solutions describe the motion of the proper center of mass of a spinning body (particle), and the helical solutions describe the motions of the family of the non-proper centers of mass [5].

Later Papapetrou derived equations (1), (2) by the method which differs from Mathisson's one [6], and, instead of (3), the non-covariant condition

$$S^{i4} = 0 \tag{4}$$

was used [7] in concrete calculations.

To avoid the superfluous solutions of equations (1), (2), Tulczyjew and Dixon introduced the covariant condition [8]

$$S^{\mu\nu} P_\nu = 0\,, \tag{5}$$

where

$$P^\nu = m u^\nu + u_\mu \frac{DS^{\nu\mu}}{ds} \tag{6}$$

is the particle's 4-momentum. In contrast to relation (3) the Tulczyjew–Dixon condition (5) picks out the unique worldline of a spinning test particle in the gravitational field. That is, equations (1), (2) under (5) do not admit the oscillatory solutions. However, the question arises: is this worldline close, in the certain sense, to the usual (non-helical) worldline of equations (1), (2) under condition (3), for example, in a Schwarzschild field? It is simple to answer this question when the relation

$$m|u^\nu| \gg \left| u_\mu \frac{DS^{\nu\mu}}{ds} \right| \tag{7}$$

takes place, because in this case condition (5) practically coincides with (3). For example, these conditions are equivalent for post-Newtonian expansions [9]. However, *a priori* another situation is possible for the highly relativistic spinning particle. Naturally, this situation must be investigated carefully.

Note that the very condition (3) was derived in some papers by different methods [10–12], and we agree with the conclusion that this condition "... arises in a natural fashion in the course of the derivation", [11], p. 112. That is, the condition (3) is necessary, though often, with high accuracy, it can be substituted by the condition (5).

We stress: the existence of the superfluous (oscillatory) solutions of equations (1), (2) under condition (3) is not a reason to ignore this condition. The point of importance is that just among all solutions of equations (1)–(3) the single solution describing the motion of the particle's proper center of mass can be found. Obviously, it is necessary to know how this solution can be identified among others.

In the focus of this paper are just the initial Mathisson equations (1)–(3). Our purpose is to investigate non-oscillatory solutions describing highly relativistic motions of the spinning particle in a Schwarzschild field.

Note that the information on all possible types of motions of the spinning test particles in the gravitational fields is important for astrophysics, for more fine investigations of the gravitational collapse and other astrophysical phenomena.

This paper is organized as follows. The known integrals of the strict Mathisson equations (1)–(3) in a Schwarzschild field, the energy and angular momentum, are used in Section 2 for reducing the order of differentiation in these equations. The possible procedure of finding the values of the energy and momentum parameters, which correspond just to the motions of the proper center of mass, is discussed in Section 3. This procedure is realized in Section 4 for the motions close to highly relativistic equatorial circular orbits in a Schwarzschild field. In Section 5 the illustrations of computer calculation are presented. We conclude in Section 6.

2. Mathisson equations for equatorial motions in a Schwarzschild field

Let us consider equations (1)–(3) for Schwarzschild's metric in the standard coordinates $x^1 = r,\ x^2 = \theta,\ x^3 = \varphi,\ x^4 = t$ for the equatorial motions of a spinning particle with spin orthogonal to the motion plane $\theta = \pi/2$. Then the non-zero components of the metric tensor $g_{\mu\nu}$ are:

$$g_{11} = -\left(1 - \frac{2M}{r}\right)^{-1}, \qquad g_{22} = g_{33} = -r^2\,, \qquad g_{44} = 1 - \frac{2M}{r}\,, \qquad (8)$$

where M is the Schwarzschild mass. Due to the symmetry of Schwarzschild's metric equations (1), (2) have the integrals of the energy E and the angular momentum L which for the equatorial motions can be written as [13]

$$E = mu_4 + g_{44}u_\mu \frac{DS^{4\mu}}{ds} + \frac{1}{2}g_{44,1}S^{14}\,,$$
$$L = -mu_3 - g_{33}u_\mu \frac{DS^{3\mu}}{ds} - \frac{1}{2}g_{33,1}S^{13}\,. \qquad (9)$$

For the equatorial motions with spin orthogonal to the motion plane $\theta = \pi/2$ equations (2) can be solved separately from (1). Indeed, taking into account relations (3) and (8) it is not difficult to obtain from (2) all non-zero components $S^{\mu\nu}$:

$$S^{13} = -S^{31} = -\frac{u_4 S_0}{r}\,, \qquad S^{14} = -S^{41} = \frac{u_3 S_0}{r}\,, \qquad S^{34} = -S^{43} = -\frac{u_1 S_0}{r}\,, \qquad (10)$$

where S_0 is the known constant of spin [10]

$$S_0^2 = \frac{1}{2}S_{\mu\nu}S^{\mu\nu}\,. \qquad (11)$$

(We stress that expressions (10) satisfy all equations of set (2).)

Now we shall consider equation (1). It is known that these equations under condition (3) contain the second order derivatives u^μ with respect to s. However, in our case of Schwarzschild's metric, due to the integral E and L, it is possible to reduce the order of differentiation by the standard procedure of the differential equations theory. Using (8)–(10) and the relation $u_\mu u^\mu = 1$ after direct calculations (which are quite simple but rather lengthly) we get from equation (1) the two non-trivial equations for $r(s)$ and $\varphi(s)$:

$$\ddot r = \frac{\dot r^2}{r} + 2r\left(1 - \frac{3M}{r}\right)\dot\varphi^2 - \frac{rE}{S_0}\dot\varphi + \frac{1}{r}\left(1 - \frac{3M}{r}\right) + \frac{L}{rS_0}\left[\dot r^2 + \left(1 - \frac{2M}{r}\right)(1 + r^2\dot\varphi^2)\right]^{1/2}, \qquad (12)$$

$$\ddot\varphi = -\frac{\dot r\dot\varphi}{r} + r\left(1 - \frac{3M}{r}\right)\frac{\dot\varphi}{\dot r} + \frac{m + L\dot\varphi}{rS_0\dot r}\left[\dot r^2 + \left(1 - \frac{2M}{r}\right)(1 + r^2\dot\varphi^2)\right]^{1/2}, \qquad (13)$$

where a dot denote the usual derivatives with respect to s. (Two other equations of set (1) are satisfied identically.) So, equations (12), (13) do not contain the third coordinate derivatives. However, in these equations the quantities E and L are present as the parameters which are not determined by the initial values of r, φ, $\dot r \equiv u^1$, $\dot\varphi \equiv u^3$ only. (According to (9), for the determination of E and L the second coordinate derivatives must be given as well.) That is, equations (12), (13), as well as the initial Mathisson equations (1), describe the motions both of the proper center of mass and the non-proper centers.

In other words, equations (12), (13) contain the non-oscillatory and oscillatory solutions. The question of importance is: which values E and L correspond just to the proper center of mass at the arbitrary initial values of r, φ, $\dot{r}$, $\dot{\varphi}$ for a spinning test particle? It is easy to answer this question when the motion of such a particle is close to the geodesic motion: then approximately $E = mu_4$ and $L = -mu_3$, *i.e.*, we write the relations for the geodesic motion. However, we have not any proof that the worldline of a spinning particle is close to the corresponding geodesic worldline for all physically admitted initial values of r, φ, $\dot{r}$, $\dot{\varphi}$. On the contrary, our results of investigations of the gravitational spin–orbit interaction in a Schwarzschild field [14–16] show that just highly relativistic motions of a spinning particle must be studied carefully. Therefore, in the next section we shall consider the procedure of choosing the values E and L for the proper center of mass for highly relativistic motions.

For further calculations it is convenient to write equations (12), (13) in terms of the non-dimensional quantities

$$\tau \equiv \frac{s}{M}\,, \qquad Y \equiv \frac{dr}{ds}\,, \qquad Z \equiv M\frac{d\varphi}{ds}\,, \qquad \rho \equiv \frac{r}{M}\,, \qquad \varepsilon \equiv \frac{S_0}{mM}\,. \tag{14}$$

(In the following we shall put $\varepsilon > 0$, without any loss in generality.) Then according to equations (12), (13) we have the set of the first-order differential equations

$$\begin{aligned}
\frac{dY}{d\tau} &= \frac{Y^2}{\rho} + \rho\left(1 - \frac{3}{\rho}\right)\left(2Z^2 + \frac{1}{\rho^2}\right) - \mu Z\rho \\
&\quad + \frac{\nu}{\rho}\left[Y^2 + \left(1 - \frac{2}{\rho}\right)(1 + Z^2\rho^2)\right]^{1/2}, \qquad (15)\\
\frac{dZ}{d\tau} &= -\frac{YZ}{\rho} + \rho\frac{Z^2 + 1/\rho^2}{Y}\left(Z - \frac{3Z}{\rho} - \mu\right) \\
&\quad + \frac{1}{\rho Y}\left(\frac{1}{\varepsilon} + \nu Z\right)\left[Y^2 + \left(1 - \frac{2}{\rho}\right)(1 + Z^2\rho^2)\right]^{1/2}, \qquad (16)\\
\frac{d\rho}{d\tau} &= Y\,, \qquad (17)\\
\frac{d\varphi}{d\tau} &= Z\,, \qquad (18)
\end{aligned}$$

where

$$\mu \equiv \frac{ME}{S_0}, \qquad \nu \equiv \frac{L}{S_0}\,. \tag{19}$$

For correct studying the physical conclusions of equations (15)–(18), it is necessary to take into account the Wald condition for a test spinning particle [17], therefore, we put

$$\varepsilon \equiv \frac{S_0}{mM} \ll 1\,. \tag{20}$$

Because in the following we shall compare some solutions of equations (15)–(18) with solutions of the geodesic equations in Schwarzschild's metric, it is useful to write here the last equations for the equatorial motions in the coordinates r, φ

$$\ddot{r} \;=\; \dot{\varphi}^2 r\left(1-\frac{3M}{r}\right)-\frac{M}{r^2}\,, \tag{21}$$

$$\ddot{\varphi} \;=\; -\frac{2}{r}\dot{r}\dot{\varphi}\,. \tag{22}$$

Using notation (14) we rewrite equations (21), (21) as the set of the first-order differential equations

$$\frac{dY}{d\tau} \;=\; Z^2\rho\left(1-\frac{3}{\rho}\right)-\frac{1}{\rho^2}\,, \tag{23}$$

$$\frac{dZ}{d\tau} \;=\; -2\frac{YZ}{\rho}\,, \tag{24}$$

plus two equations which coincide with (17), (18). Equations (15), (16) and (23), (24) correspondingly are essentially different. As well as equations (12), (13), the first two equations of set (15)–(18) contain the constants of the energy and angular momentum of a spinning particle. According to the discussion above, different values of the parameters μ and ν in (15), (16) at the fixed values of Y, Z, ρ correspond to different centers of mass, namely to the single proper center of mass and to the set of non-proper centers.

3. Possible way of finding the parameters μ and ν for the proper center of mass

The right-hand sides of equations (15), (16) are too complicated for investigations in general case. However, *a priori* we cannot exclude the possibility of showing the essential difference between the motions of the proper and non-proper centers of mass during the short time interval after the beginning of the corresponding motions, namely, when the displacement of the values Y, Z, ρ, φ from their initial values $Y_0, Z_0, \rho_0, \varphi_0$ are considered in the linear approximations in the quantities

$$\xi_1 \equiv \frac{Y-Y_0}{Y_0}\,, \qquad \xi_2 \equiv \frac{Z-Z_0}{Z_0}\,, \qquad \xi_3 \equiv \frac{\rho-\rho_0}{\rho_0}\,. \tag{25}$$

(In (25) we do not write the displacement $\varphi - \varphi_0$ because the right-hand sides of equations (15)–(18) do not depend on φ and equation (18) is trivial, *i.e.*, according to (18) the value φ is determined by simple integration of Z.) Let us check this possibility, *i.e.*, consider equations (15)–(17) in the linear in ξ approximation. Then by direct calculations we obtain

$$\begin{aligned}\frac{d\xi_1}{d\tau} &= (a_{10} + a_{11}\nu + a_{12}\mu)\,\xi_1 + (a_{20} + a_{21}\nu + a_{22}\mu)\,\xi_2 \\ &\quad + (a_{30} + a_{31}\nu + a_{32}\mu)\,\xi_3 + a_{00} + a_{01}\nu + a_{02}\mu\,, \end{aligned} \tag{26}$$

$$\begin{aligned}\frac{d\xi_2}{d\tau} &= \left(b_{10} + b_{11}\nu + b_{12}\mu + b_{13}\frac{1}{\varepsilon}\right)\xi_1 + \left(b_{20} + b_{21}\nu + b_{22}\mu + b_{23}\frac{1}{\varepsilon}\right)\xi_2 \\ &\quad + \left(b_{30} + b_{31}\nu + b_{32}\mu + b_{33}\frac{1}{\varepsilon}\right)\xi_1 + b_{00} + b_{01}\nu + b_{02}\mu + b_{03}\frac{1}{\varepsilon}\,, \end{aligned} \tag{27}$$

$$\frac{d\xi_3}{d\tau} = c_{10}\xi_1 + c_{00}\,, \tag{28}$$

where the coefficients a, b, c with the corresponding indexes are expressed through Y_0, Z_0, ρ_0. We shall use the expressions of these coefficients in the approximation

$$Z_0^2\rho_0^2 \gg 1\,, \qquad Y_0^2 \ll Z_0^2\rho_0^2\,. \tag{29}$$

According to notation (14), relations (29) mean that the tangential component of the particle's initial velocity is highly relativistic, and, in addition, that the tangential component is much greater than the radial component. First, it corresponds with our aim to investigate just highly relativistic motions. Second, because the deviation of a spinning particle is caused by the gravitational spin–orbit interaction, most clearly this deviation can be shown in the case when the tangential velocity is dominant (in particular, for the circular or closer to the circular orbits [16]).

According to the theory of differential equations, the general solution of linear equations (26)–(28) is determined by the combination of $e^{\lambda_i \tau}$ ($i = 1, 2, 3$), where λ_i are the solutions of the third-order algebraic equation

$$\lambda^3 + C_2\lambda^2 + C_1\lambda + C_0 = 0\,. \tag{30}$$

Here the coefficients C_j ($j = 0, 1, 2$) can be expressed through a, b, c and depend both on $\rho_0, Y_0, Z_0, \varepsilon$ and on the parameters μ, ν. Our task is to find such concrete values μ, ν which at the fixed $\rho_0, Y_0, Z_0, \varepsilon$ determine just the motion of the proper center of mass. We begin by analyzing the expressions C_j for the simple case when the worldline of a spinning particle is close to the corresponding geodesic worldline.

3.1. Expressions C_j for quasi-geodesic motions

If the parameter ε in (20) is sufficiently small, for any fixed values ρ_0, Y_0, Z_0 the motion of the proper center of mass is close to the geodesic motion. Then we can write approximately $E = mu_4$, $L = -mu_3$ (see expressions (9)), and according to (14), (19) we have

$$\mu = \frac{1}{\varepsilon}|Z_0|\rho_0 \left(1 - \frac{2}{\rho_0}\right)^{1/2}, \qquad \nu = \frac{1}{\varepsilon}\rho_0^2 Z_0 . \tag{31}$$

Let us consider expressions C_j at μ, ν from (31). It is easy to show that all these expressions have a common feature: all greatest terms with the large value $1/\varepsilon$ are canceled. It means that the corresponding largest terms with $1/\varepsilon$ are absent in the expressions λ_j as well. So, just the values μ, ν for the proper center of mass give the minimum values of λ_j. We stress that the similar situation takes place for motions of the proper center of mass in the Minkowski spacetime: for the proper center $\lambda_j = 0$ (it corresponds to the straightforward motions), and for the non-proper centers λ_j are proportional to M/S_0 (the oscillatory motions).

3.2. Expressions C_j for the highly relativistic circular motion with $r = 3M$

In [16] we have considered the case of the circular motion of the proper center of mass in a Schwarzschild field with $r = 3M$. By the notation (14) in this case we write

$$\rho = 3, \qquad Y = 0, \qquad Z = -\frac{3^{-3/4}}{\sqrt{\varepsilon}} . \tag{32}$$

For values (19), from (9) we obtain

$$\mu = 3^{-1/4}\varepsilon^{-3/2}, \qquad \nu = -3^{5/4}\varepsilon^{-3/2} . \tag{33}$$

Let us estimate the values C_j. Taking into account (33) it is easy to check that, as well as in the previous case, in all expressions C_j the largest terms with $1/\varepsilon$ are canceled.

It is naturally to suppose that the similar feature takes place not only in the two partial cases above. Therefore, below we shell check do the criterion of excluding the largest terms with $1/\varepsilon$ in the expressions C_j can be used for finding the values μ and ν which pick out just the motion of the proper center of mass.

4. Expressions μ and ν for the motions of the proper center of mass at $2 < \rho_0 < 3$ under condition (29)

As we pointed out in [16] equations (1)–(3) admit in a Schwarzschild field the solutions describing the circular equatorial orbits in the region $2M < r < 3M$. By notation (14) for these orbits we have

$$2 < \rho < 3\,, \qquad Y = 0\,, \qquad Z = -\frac{1}{\rho}\left(1 - \frac{2}{\rho}\right)^{1/4}\left|1 - \frac{3}{\rho}\right|^{-1/2}\varepsilon^{-1/2}. \quad (34)$$

The expression Z from (34) is valid beyond the small neighborhood of the value $\rho = 3$. (This neighborhood was considered in [16].) At expressions (34), it follows from (9) that

$$\mu \sim \varepsilon^{-1/2}\,, \qquad \nu \sim \varepsilon^{-1/2}\,. \quad (35)$$

It is interesting to consider the non-circular orbits which deviate from (34) due to $Y_0 \neq 0$ under condition (29). Let us consider the expressions C_j in the case

$$2 < \rho_0 < 3\,, \qquad 1 \ll Y_0^2 \ll Z_0^2\rho_0^2\,, \qquad Z_0 = k\varepsilon^{-1/2}\,, \quad (36)$$

where according to (34), (35)

$$k = -\frac{1}{\rho_0}\left(1 - \frac{2}{\rho_0}\right)^{1/4}\left|1 - \frac{3}{\rho_0}\right|^{-1/2}\,, \quad (37)$$

$$\mu = k_1\varepsilon^{-1/2}\,, \qquad \nu = k_2\varepsilon^{-1/2}\,, \quad (38)$$

and k_1, k_2 are some coefficients which do not depend on ε. Our task is to find such values k_1, k_2 which ensure excluding the largest terms with $1/\varepsilon$ in the expressions C_j.

Using μ, ν from (37) we obtain the conditions under which the largest terms with $1/\varepsilon$ in the expressions C_j are canceled:

$$3k^2(\rho_0 - 3) - 2\rho_0 kk_1 - 2k\left(1 - \frac{2}{\rho_0}\right)^{1/2} k_2 - \left(1 - \frac{2}{\rho_0}\right)^{1/2} = 0\,, \quad (39)$$

$$-k^2(\rho_0 - 3) + \rho_0 kk_1 + kk_2\left(1 - \frac{2}{\rho_0}\right)^{1/2} + \left(1 - \frac{2}{\rho_0}\right)^{1/2} = 0\,, \quad (40)$$

$$k^2\rho_0 - \rho_0 kk_1 - kk_2\frac{1}{\rho_0}\left(1 - \frac{2}{\rho_0}\right)^{-1/2} - \frac{1}{\rho_0}\left(1 - \frac{2}{\rho_0}\right)^{-1/2} = 0\,. \quad (41)$$

It is easy to check that at the value k from (37) among three linear in k_1, k_2 algebraic equations (39)–(41) there are only two independent equations. Then the solution of (39)–(41) is:

$$k_1 = -\frac{1}{\sqrt{\rho_0}}\left(1-\frac{2}{\rho_0}\right)^{1/4}\left|1-\frac{3}{\rho_0}\right|^{-3/2}\left(1-\frac{3}{\rho_0}+\frac{3}{\rho_0^2}\right), \quad (42)$$

$$k_2 = \sqrt{\rho_0}\left(1-\frac{2}{\rho_0}\right)^{-1/4}\left|1-\frac{3}{\rho_0}\right|^{-3/2}\left(1-\frac{9}{\rho_0}+\frac{15}{\rho_0^2}\right). \quad (43)$$

Relations (42), (43) can be used in computer integration of equations (15)–(18) under conditions (36).

5. Examples of computer integration of equations (15)–(18)

Graphs of $\rho(\tau)$ and $\rho(\varphi)$ according to equations (15)–(18) under relations (42), (43) are shown by the thick lines in Fig. 1 and Fig. 2 correspondingly. For comparison, the thin lines in the same figures show graphs of $\rho(\tau)$ and $\rho(\varphi)$ according to geodesic equations (23)–(24). If relations (42), (43) are violated, computer integration shows the oscillatory solutions. Similar situation takes place for orbits with $\rho_0 = 3$. An example of the oscillatory solution is presented in Fig. 3.

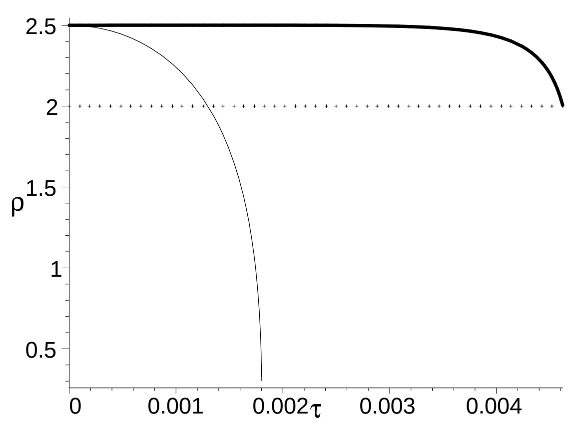

Fig. 1. Graph of $\rho(\tau)$ according to equations (15)–(18) under relations (42), (43) at $\varepsilon = 10^{-6}$, $\rho_0 = 2.5$, $Y_0 = 0.3$ and Z_0 determined by (36), (37) (thick line). Graph of $\rho(\tau)$ according to geodesic equations (23)–(24) at the same ρ_0, Y_0, Z_0 (thin line). The horizontal line $\rho = 2$ corresponds to the horizon surface.

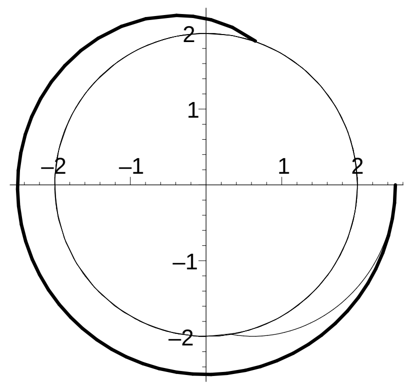

Fig. 2. Graph of $\rho(\varphi)$ in the polar coordinates according to equations (15)–(18) (thick line) and (23)–(24) (thin line). The values ρ_0, Y_0, Z_0 are the same as in Fig. 1. The circle $\rho = 2$ corresponds to the horizon surface.

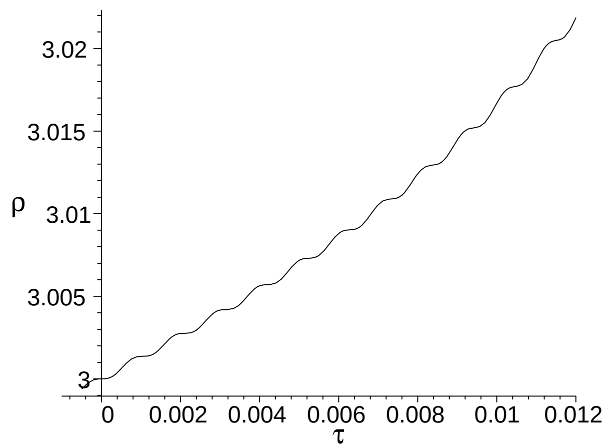

Fig. 3. An example of the oscillatory solution for $\rho(\tau)$ according to equations (15)–(18) at $\varepsilon = 10^{-4}$, $\rho_0 = 3$, $Y_0 = 2.5 \times 10^{-3}$ and Z_0 determined by (31) when relations (33) are slightly violated.

6. Summary

The computer integration of equations (15)–(18) at relations (42), (43) shows that these relations are suitable for choosing solutions, which describe the motions of the proper center of mass. As a result, according to Fig. 1 and Fig. 2, we conclude that (under those conditions indicated in the captions at these figures) the force of the interaction of the particle's spin with the gravitational field acts as the repulsive one. Due to this force the spinning particle falls on the horizon surface during longer time as compared to the corresponding particle without spin (Fig. 1). Moreover, according to Fig. 2, considerable space separation of the corresponding spinning and non-spinning particles takes place within a short time, *i.e.*, within the time of the particle's fall on the Schwarzschild horizon.

There is also a possibility of generalizing the above onto case of a Kerr field.

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