ON SOME PROPERTIES OF THE FINE STRUCTURE CONSTANT

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The effects of the inhomogeneity of the mass distribution in the early universe and of the cosmological constant on the variation of the fine structure constant have been investigated. It has been suggested that the variation of the fine structure constant may be attributed to the intrinsic scale dependence of the fundamental constants of nature. The effect of the vacuum polarisation on the variation of fine structure constant has also been investigated and some interesting observations are made.

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The possibility of variation of physical quantities has been a long standing problem and widely discussed by a number of authors [1] including Dirac himself [2]. The idea that the charge of the electron or the fine structure constant might vary in cosmological time is proposed by Teller [3]. With the progress of the observational cosmology the experimental verification of the suggestions become plausible. Recent observations of the distant quasars have suggested that the fine structure constant varies with cosmological time scale and the variation is $\approx d\alpha/\alpha \approx 10^{-5}$ [4] over the time period since the emission of the quasar light. Kuhne [5] has pointed out that the time varying fine structure constant is compatible with cosmologies which demand large value of the cosmological constant. He has suggested that under the present convention of Planck's units to be the fundamental, any time variation of fine structure constant implies the variation of the unit of electric charge. In the context of discussing the renormalisation, Veltman [6] has also pointed out that electric charge is a free parameter and is not known from any basic principle. However, recently Bank *et al.* [7] have argued that the present observational value of the $d\alpha/\alpha$ cannot be explained by any field theory as in the field theory the fields are not stable under renormalisation and require

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massive fine tuning which is difficult in present day physics. In the context of discussing the contribution of monopoles near the Planck scale, Laperashvili *et al.* [8] have discussed the RG equations for electric and magnetic fine structure constant and possible consequences on the unification of scales. Uzan [9] has made an excellent review on the variation of the fine structure constant and indicated that the variation of α may induce new cosmological constant problem as a varying α cannot naturally be explained in a field theoretical approach.

In the present work we have investigated the effect of the mass fractal dimension on the possible variation of the fine structure constant in the context of the early universe between matter and radiation dominated era. We have also investigated possible variation of α from the quantum vacuum effect and discussed the consequences of the variation of the electric charge as the relative distance becomes smaller than the Compton scale of the electron.

The FRW metric for homogeneous universe and corresponding Einstein equation runs as [10]:

$$\left[\frac{dR(t)/dt}{R}\right]^2 = \frac{4\pi G\rho}{3} - \frac{K}{R(t)^2} + \frac{\Lambda}{3}\,, \tag{1}$$

where the symbols have their usual meanings. In our previous work [11] we have introduced a small inhomogeneity in the mass distribution of the early universe for large value of R where the energy density varies through $\rho(R) \sim R^{-d-3}$ where d is the mass fractal dimension and lies between $0 < d < 1$ for matter ($d = 0$) and radiation dominated era ($d = 1$), respectively. We have obtained [11]

$$R(t) \sim t^{\beta}\,, \tag{2}$$

where $\beta = 2/(d + 3)$ and

$$H = \beta t^{-1}\,, \tag{3}$$

with the conventional nomenclature of present time t as in Weinberg [12] where $t_0 < H_0$, H is the Hubble factor. In the context of the effective field theory and M theory, the change of the fine structure constant is obtained by coupling the dynamical scalar field ϕ to the photon kinetic term in the low energy effective action. Moreover, to study the cosmology of the field ϕ, it has been assumed that ϕ is governed by a Lagrangian $L = (\delta\phi)^2 - V(\phi)$, where the potential energy is given by generic form $\mu^4 f(\phi/M)$ [7]. ϕ and M are microphysical parameters. Considering the motion to be friction dominated to incorporate the slow time variation of α over the matter dominated era, the expression for $d\alpha^{-1}$ can be written as [7]:

$$d\alpha^{-1} = \frac{4\pi\varepsilon\mu^4}{3M^2}\int dt\,\frac{f'(\phi/M)}{H}\,. \tag{4}$$

With ϕ/M approximately constant during this era, we may recast the above equation as

$$d\alpha^{-1} = K \int dt/H \,. \tag{5}$$

Now using the time dependence of Hubble parameter during the matter dominated era as obtained in (3), we get

$$d\alpha^{-1} = \beta K \ln t \,, \tag{6}$$

where β is $2/3$ for the matter dominated era and $1/2$ for radiation dominated era. Teller [3] has suggested that the cosmological time variation of α behaves like $\alpha^{-1} = \ln t$. In the context of the present investigation we get the similar type of result but with a dependence on the mass fractal dimension d. Again, in the generic quantum field theory varying of an arbitrary dimensionless coupling such as α will lead to a variation in the vacuum energy V controlled by the cutoff scale λ [7] as:

$$dV = C\delta\alpha\lambda^4 \,. \tag{7}$$

In the context of discussing the cosmological constant problem in the fractal universe [12] we have shown that the energy conservation equation demands that the cosmological constant Λ should scales as in the same manner as the density distribution scales in the early universe. Again, as stated earlier, we have assumed $\rho \sim R^{-d-3}$ to incorporate a tiny inhomogeneity in the matter density where d is small parameter in the range of $0 \leq d \leq 1$ [10]. Hence we may write $\Lambda \sim R^{-d-3}$. It may be mentioned here that Weinberg [13] in his review on the cosmological constant indicated that Λ represents the vacuum density. With the concept of cosmological constant Λ representing the vacuum energy density and the cut off scale, we come across $\Lambda \sim V \sim R^{-d-3}$ (where R is usual radius parameter) which suggests that Λ itself would behave as a scale parameter in the evolution of the universe. With the above considerations we may rewrite the expression (7)

$$\frac{d\alpha}{dR} \sim (d+3)R^{3d-1} \,. \tag{8}$$

Using Eq. (2) we may recast the above expression as

$$\frac{d\alpha}{dt} \sim t^{3d\beta-1} \,. \tag{9}$$

Thus we observe that the time variation of the fine structure constant shows a power law behaviour if the scaling behaviour of the Λ is incorporated and it shows dependence on the mass fractal dimension *i.e.* on the inhomogeneity of the universe in the cosmological time. It is interesting to observe that α becomes constant in time during the matter dominated era ($d = 0$), whereas during the radiation dominated era ($d = 1$) it follows a power law behaviour such as $d\alpha/dt \sim t^{1/2}$. Hence if we assume that the cosmological constant scales as in the same manner the energy distribution scales and if Λ represents the vacuum energy density, the time variation of α obeys a power law behaviour at least during the era of the early universe. However, it may be mentioned here that according to Dyson *et al.* [14] the power law behaviour of $d\alpha/\alpha$ is ruled out in the recent geological past.

The radiative correction to Coulomb's law may be described as resulting from the polarisation of the vacuum around a point charge. It has been assumed that the radiative corrections are generated from the interaction between photon field and e^+e^- field. The interaction of the photon with the vacuum may be taken into account by associating the internal closed loops in the self energy photon diagrams with electrons. Considering radiative correction to the energy level of the non-relativistic electrons and treating the Coulomb field as a weak perturbation, we come across [15]

$$\Delta E_n = \frac{-4\alpha^4 m}{15\pi n^3}\,, \tag{10}$$

where α is fine structure constant. In terms of the Bohr radius a_0, we may rewrite the above expression as

$$\frac{-4\alpha^5 m}{15\pi} = \frac{-4a_0^{-5}}{15\pi m^4}\,. \tag{11}$$

From the expression (11) we may find that

$$\frac{d\alpha}{\alpha} = \frac{-a_0^{-6}da_0}{m^5\alpha^5} \tag{12}$$

with the input of the conventional values of the parameters we have obtained $d\alpha/\alpha \approx 0.282\times 10^{-5}da_0$. The Compton wavelength $\lambda_{\rm C}$ for electron is 3.86×10^2 fm. It is observed that the variation of Bohr radius $da_0 \approx 0.0005\lambda_{\rm C}$ yields $d\alpha/\alpha = 10^{-5}$. From the expression (12) it is evident that the variation of the α can only be attributed to the variation of a_0, all other quantities being supposed to be constant. It is well known that when the relative distance becomes smaller than the Compton scale of the electron the nature between the two nearby charges changes due to the pair creation and annihilation for the time interval $\Delta t \approx h/2m_e c^2$ *i.e.*, distance smaller than $\lambda_{\rm C}/2$ [16]. It has been pointed out by Khune [5] that in principle the time variation of electric charge e implies the variation of α. We have found that the variation of $(d\alpha/\alpha)$ implies the variation of Bohr radius in the context of the atomic spectra. If the distortion of the electron cloud is of the order of $\sim 0.0005\lambda_{\rm C}$, $d\alpha/\alpha$ is obtained as $\sim 10^{-5}$ suggested by the recent observation [4]. Chengalur *et al.* [17] have estimated $\Delta\alpha = (-1.88 \pm 0.06) \times 10^{-5}$ using 18 cm OH lines, whereas Srianand *et al.* [18] have extracted the value as $(-0.06 \pm 0.06) \times 10^{-5}$ within the red shift range of $0.4 \leq 2 \leq 2.3$. Uzan [9] has made an excellent review on the variation of the fine structure constant in the context of the atomic spectra.

Khune [5] has argued that time varying fine structure constant demands large value of the cosmological constant. In the context of discussing the cosmological constant problem Nottale [16] has pointed that the large value of Λ at the Planck's scale is due to the intrinsic scale dependence of Λ and variation of α in the cosmological scale is consistent with the observation made by Khune [5]. In quantum electrodynamics also the charge becomes a scale dependant quantity from the electron scale to W/Z scale. In the present work we have observed that the time variation of α follows a power law behaviour if we assume that the cosmological constant scales as in the same way as the vacuum energy density scales in the early universe [12]. Moreover, we have observed that the inhomogeneity of the density distribution has also contribution to the time varying α as we move from radiation dominated era to matter dominated era of the early universe. We have also observed that the variation of α may be simulated through a small distortion in the Bohr radius in the atomic scale. Thus it may not be far from reality to suggest that starting from the cosmological scale to microphysics the variation of α represents the scale relativity which is supposed to be the fundamental properties of nature. However, it may be pointed out here that the study of the variation of the fine structure constant needs the study of a domain which includes cosmology, astrophysics, high energy physics. Much more theoretical and observational efforts are needed to understand the origin of the variations of the fundamental constants of nature.

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