Vol. 39 (2008) *ACTA PHYSICA POLONICA B* No 11

ON THE MASS SPECTRUM OF 2^1S_0 MESON STATE*

Xue-Chao Feng

Department of Technology and Physics, Zhengzhou University of Light Industry
450002 Zhengzhou, China

(Received May 26, 2008; revised version received August 4, 2008)

In the framework of meson–meson mass mixing matrix and Regge trajectory, in the presence of the $\eta(1295)$, $\eta(1475)$ and $\eta_c(2S)$ being the 2^1S_0 meson state, we predict the mass spectrum of the 2^1S_0 meson state, and quarkonium content. The decays of isoscalar state are also presented. The results are in good agreement with the values predicted by other theoretical approaches. Our results should be tested in the experiment in the future.

PACS numbers: 14.40.–n, 11.40.–q

1. Introduction

Quantum Chromodynamics (QCD) is accepted as a successful theory of strong interactions in particle physics. However, the understanding of strong interactions is far from completeness and it is difficult to interpret the experimental data from first principles. To be able to interpret the nature of new resonances, it is necessary to build models comparing observed states with theoretical predictions. The constituent quark model offers the most complete description of hadron properties and is probably the most successful phenomenological model of hadron structure. In the last decade, the constituent quark model has also been used to study the low-lying hadron spectrum. In the constituent quark model, quarks with a given set of quantum numbers J^{PC} appear as meson states. In the present work, we firstly review the assignment of 2^1S_0 meson state in the $q\bar{q}$ quark model (see Table I).

In the new edition of Particle Data Group (PDG) [1], the states $\eta(1295)$ and $\eta(1475)$ have been well established as the isoscalar member in the 2^1S_0 meson state. The $K(1460)$ is assigned as isodoublet member. However, this assignment is still based on very weak experimental signals. Till now, the

* fxchao@zzuli.edu.cn

TABLE I

Assignment of the 2^1S_0 meson state in PDG [1] ($n = u, d$), s, c denote the constituent quark.

$^{2s+1}l_j$	$n\bar{n}$	$n\bar{n}(s\bar{s})$	$n\bar{s}$	$c\bar{c}$	$c\bar{n}$	$c\bar{s}$
2^1S_0	$\pi(1300)$	$\eta(1295)\eta(1475)$	$K(1460)$	$\eta_c(2S)$	D	D_s

$K(1460)$ has been reported by two experiments $K^-p \to K^-2\pi p$ [2] (with mass $\sim 1460\,\text{MeV}$ and width $\sim 260\,\text{MeV}$) and $K^\pm p \to K^+2\pi p$ [3] (with mass $\sim 1400\,\text{MeV}$ and width $\sim 250\,\text{MeV}$). For the isovector state, the candidate is $\pi(1300)$, but the mass has large error, $1300 \pm 100\,\text{MeV}$.

In the last few years, heavy-light mesons have been observed in the experiments, *i.e.* $D^*_{s0}(2317)$ [4], $D_{s1}(2460)$ [5], $Ds(2632)$ [6], $D_{sJ}(2860)$ [7] and $D_{sJ}(2715)$ [8], the spectrum of charmed and charmed strange mesons become a very active field. The assignment and identification of these states attract more attention [9]–[13]. For the 2^1S_0 meson state, the situation is even worse, the members of charmed and charmed strange mesons have not been observed in the experiment.

In this work, relaying on the fact that the isoscalar states $\eta(1295)$ and $\eta(1475)$ can mix to form the physical states in the $q\bar{q}$ quark model and employing the linear Regge trajectory, we establish new mass relations which relate the mass spectrum of meson state and constituent quark masses. Inserting the corresponding constituent quark masses, we reexamine the mass spectrum of the 2^1S_0 meson state. The results could be useful for comparing with the experiment data in a new experiment.

2. Meson–meson mass mixing matrix of isoscalar states and Regge phenomenology

In the quark model, the two isoscalar states with the same J^{PC} will mix to form the physical isoscalar states. We can establish the mass-squared matrix in the $s\bar{s}$ and $N = (u\bar{u} + d\bar{d})/\sqrt{2}$ basis [14],

$$M^2 = \begin{pmatrix} M^2_{\pi(1300)} + 2A_{nn} & \sqrt{2}A_{ns} \\ \sqrt{2}A_{ns} & 2M^2_{K(1460)} - M^2_{\pi(1300)} + A_{ss} \end{pmatrix}, \qquad (1)$$

where $M_{\pi(1300)}$ and $M_{K(1460)}$ are the masses of isovector and isodoublet states of the 2^1S_0 meson nonet, respectively; A_{nn}, A_{ns} and A_{ss} are mixing parameters which describe the $q\bar{q} \leftrightarrow q'\bar{q}'$ transition amplitudes. In order to reduce the number of parameters, we adopt the similar expression of the transition amplitudes in the $q\bar{q} \leftrightarrow q'\bar{q'}$ process which is widely used in Refs. [15,16],

$$\begin{aligned} A_{nn} &= \frac{\Lambda}{m_n m_n}\,, \\ A_{ss} &= \frac{\Lambda}{m_s m_s}\,, \\ A_{ns} &= \frac{\Lambda}{m_n m_s}\,, \end{aligned} \tag{2}$$

where Λ is a phenomenological parameter. From the isospin symmetry, we have $m_u = m_{\bar{u}} = m_d = m_{\bar{u}}$, $m_s = m_{\bar{s}}$ (m_n, m_s denote the mass of light quark u, d and s).

In the 2^1S_0 meson nonet, we assume the physical states $\eta(1295)$ and $\eta(1475)$ are the eigenstates of mass-squared matrix and the masses square of $M^2_{\eta(1295)}$ and $M^2_{\eta(1475)}$ are the eigenvalues, respectively. The physical states $\eta(1295)$ and $\eta(1475)$ can be related to the $s\bar{s}$ and $N=(u\bar{u}+d\bar{d})/\sqrt{2}$ by

$$\begin{pmatrix} |\eta(1295)\rangle \\ |\eta(1475)\rangle \end{pmatrix} = U \begin{pmatrix} |N\rangle \\ |S\rangle \end{pmatrix} \tag{3}$$

and the unitary matrix U can be described as

$$UM^2U^\dagger = \begin{pmatrix} M^2_{\eta(1295)} & 0 \\ 0 & M^2_{\eta(1475)} \end{pmatrix}. \tag{4}$$

From the relations (1)–(4) we obtain

$$2\frac{\Lambda}{m_n^2} + 2M^2_{K(1460)} + \frac{\Lambda}{m_s^2} = M^2_{\eta(1295)} + M^2_{\eta(1475)}\,, \tag{5}$$

$$\begin{aligned} &\left(M^2_{\pi(1300)} + 2\frac{\Lambda}{m_n^2}\right)\left(2M^2_{K(1460)} - M^2_{\pi(1300)} + \frac{\Lambda}{m_s^2}\right) - 2\frac{\Lambda}{m_s^2 m_n^2} \\ &= M^2_{\eta(1295)} * M^2_{\eta(1475)}\,. \end{aligned} \tag{6}$$

In the relations (5) and (6), the masses of $K(1460)$ and $\pi(1300)$ are related to the constituent quark masses and phenomenological parameter Λ.

Regge theory is cornered with the particle spectrum, the forces between particles, and the high energy behavior of scattering amplitudes. Since Regge trajectories offer an effective way for the assignment and classification of meson states, it also becomes an active field with many new particles and resonances that were observed in the experiment in the last decade. In Ref. [17], the authors investigated the masses of different meson multiplets, and suggested that the quasi-linear Regge trajectories could describ the meson mass spectrum. Khruschov [18], using the phenomenology formulae derived from the Regge trajectories, predicts the masses of excited meson states. Anisovich *et al.* [19] show that meson states can fit quasi-linear Regge trajectories with good accuracy.

Assuming that the hadron with a set of given quantum number belongs to a quasi-linear trajectory, we have the following relation [17]

$$J = \alpha_{i\bar{i}'}(0) + \alpha'_{i\bar{i}'} M^2_{i\bar{i}'}\,, \tag{7}$$

where $i\bar{i}'$ refers to the quark (antiquark) flavor, J and $M_{i\bar{i}'}$ are, respectively, the spin and mass of the $i\bar{i}'$ meson. The parameters $\alpha'_{i\bar{i}'}$ and $\alpha_{i\bar{i}'}(0)$ are the slope and intercept of the trajectory, respectively. The intercepts and slopes can be parameterized as follows [17,20]

$$\alpha_{i\bar{i}}(0) + \alpha_{j\bar{j}}(0) \;=\; 2\alpha_{i\bar{j}}(0)\,, \tag{8}$$

$$\frac{1}{\alpha'_{i\bar{i}'}} + \frac{1}{\alpha'_{j\bar{j}'}} \;=\; \frac{2}{\alpha'_{i\bar{j}'}}\,. \tag{9}$$

The relation (8) is satisfied in two-dimensional QCD [21], the dual-analytic model [22], and the quark bremsstrahlung model [23]. The relation (9) is derived from the topological and the $q\bar{q}$-string picture of hadrons [24]. According to the data available for meson states, Burakovsky constructed a slope formula (10) for all quark flavors [25]

$$\frac{\pi}{4}\,\alpha'_{ji} + \frac{\pi}{4}\sqrt{\alpha'}\,\frac{m_i + m_j}{2}\,\alpha'_{ji} = \alpha'\,, \tag{10}$$

where m_i,m_j are the corresponding constituent quark masses, and $\alpha' = 0.88\,\mathrm{GeV}^{-2}$ is the standard Regge slope in the light quark sector.

From equations (7)–(10), we obtained the following relations

$$\frac{m^2_{\pi(1300)}\alpha'}{2 + 2\sqrt{\alpha'}m_n} + \frac{m^2_{\eta(1475)}\alpha'}{2 + 2\sqrt{\alpha'}m_s} = \frac{2m^2_{K(1460)}\alpha'}{2 + \sqrt{\alpha'}(m_n + m_s)}\,, \tag{11}$$

$$\frac{m^2_{\pi(1300)}\alpha'}{2 + 2\sqrt{\alpha'}m_n} + \frac{m^2_{\eta_c(2S)}\alpha'}{2 + 2\sqrt{\alpha'}m_c} = \frac{2m^2_{D}\alpha'}{2 + \sqrt{\alpha'}(m_n + m_c)}\,, \tag{12}$$

$$\frac{m^2_{\eta(1475)}\alpha'}{2 + 2\sqrt{\alpha'}m_s} + \frac{m^2_{\eta_c(2S)}\alpha'}{2 + 2\sqrt{\alpha'}m_c} = \frac{2m^2_{D_s}\alpha'}{2 + \sqrt{\alpha'}(m_s + m_c)}\,. \tag{13}$$

3. The results and conclusions

Using the relations (5), (6), (11), (12) and (13), the masses of $K(1460)$, $\pi(1300), D$ and D_s of the 2^1S_0 meson state are related to constituent quark masses. In this work, we use the average values of the constituent quark masses as input parameters (see Table II). Moreover, we choose the following well-established resonances taken from PDG [1] as inputs: $\eta(1295), \eta(1475)$, $\eta_c(2S)$ PDG [1]. Our results are presented in Table III.

TABLE II

Constituent quark masses (in MeV) in different phenomenological models.

Mass	Ref.[20]	Ref.[26]	Ref.[27]	Ref.[28]	Ref.[29]	Average
$m_n (n = u, d)$	290	360	337.5	311	310	321
m_s	460	540	486	487	483	491.2
m_c	1650	1710	1550	1592	—	1625.5

TABLE III

Predicted masses (in MeV) of $K(1460)$, $\pi(1300), D$ and D_s of the 2^1S_0 meson state.

Mass	Our results	Ref.[1]	Ref.[30]	Ref.[31]	Ref.[32]	Ref.[33]	Ref.[34]
$M_{\pi(1300)}$	1261.76	1300±100	—	1290	—	—	—
$M_{K(1460)}$	1369.17	$\sim$ 1460	1391.91	1400	—	—	—
M_D	2498.6	—	—	2450	2579	2580	2500
M_{D_s}	2589.85	—	—	2560	2670	2670	2610

If $M_{\pi(1300)} = 1261.76\,\text{MeV}$, $M_{K(1460)} = 1369.17$ MeV, $M_{\eta(1295)}$, $M_{\eta(1475)}$ are assigned as the member of the 2^1S_0 meson nonet, we obtain the quarkonium content of the isoscalar state. Based on the previous assumption that physical states $\eta(1295)$ and $\eta(1475)$ are the eigenvectors of mass-squared matrix, the unitary matrix U can be described as

$$U = \begin{pmatrix} X_{\eta(1295)} & Y_{\eta(1295)} \\ X_{\eta(1475)} & Y_{\eta(1475)} \end{pmatrix} =$$

$$\begin{pmatrix} \dfrac{\sqrt{2}\Lambda}{\sqrt{2\Lambda^2 + a\left(M^2_{\eta(1295)} - M^2_{\pi(1300)} - 2\frac{\Lambda}{m_n^2}\right)^2}} & \dfrac{a\left(M^2_{\eta(1295)} + M^2_{\pi(1300)} - 2\frac{\Lambda}{m_n^2}\right)^2}{\sqrt{2\Lambda^2 + a\left(M^2_{\eta(1295)} - M^2_{\pi(1300)} - 2\frac{\Lambda}{m_n^2}\right)^2}} \\ \dfrac{\sqrt{2}\Lambda}{\sqrt{2\Lambda^2 + a\left(M^2_{\eta(1475)} - M^2_{\pi(1300)} - 2\frac{\Lambda}{m_n^2}\right)^2}} & \dfrac{a\left(M^2_{\eta(1295)} + M^2_{\pi(1300)} - 2\frac{\Lambda}{m_n^2}\right)^2}{\sqrt{2\Lambda^2 + a\left(M^2_{\eta(1295)} - M^2_{\pi(1300)} - 2\frac{\Lambda}{m_n^2}\right)^2}} \end{pmatrix}, \quad (14)$$

where $a = (m_n m_s)^2$.

Inserting the masses of isoscalar states, the quarkonium content of $\eta(1295)$ and $\eta(1475)$ are determined to be

$$|\eta(1295)\rangle = -0.9968|N\rangle - 0.0787|S\rangle\,, \tag{15}$$

$$|\eta(1475)\rangle = 0.0787|N\rangle - 0.9968|S\rangle\,. \tag{16}$$

In order to check our predictions, we discuss the decays of the isoscalar states $\eta(1295)$ and $\eta(1475)$. According to the Refs. [15], [35]–[37] we have the following relations

$$\frac{\Gamma(\eta(1295)\to\gamma\gamma)}{\Gamma(\pi(1300)\to\gamma\gamma)} = \frac{1}{9}\left(\frac{M_{\eta(1295)}}{M_{\pi(1300)}}\right)^3 (5X_{\eta(1295)}+\sqrt{2}Y_{\eta(1295)})^2\,, \tag{17}$$

$$\frac{\Gamma(\eta(1475)\to\gamma\gamma)}{\Gamma(\pi(1300)\to\gamma\gamma)} = \frac{1}{9}\left(\frac{M_{\eta(1475)}}{M_{\pi(1300)}}\right)^3 (5X_{\eta(1475)}+\sqrt{2}Y_{\eta(1475)})^2\,, \tag{18}$$

$$\frac{\Gamma(\eta(1295)\to\omega\gamma)}{\Gamma(\eta(1295)\to\rho\gamma)} = \frac{1}{9}\left(\frac{M^2_{\eta(1295)}-M^2_\omega}{M^2_{\eta(1295)}-M^2_\rho}\right)^3 \times\left(\frac{0.999X_{\eta(1295)}+0.06Y_{\eta(1295)}}{X_{\eta(1295)}}\right)^2\,, \tag{19}$$

$$\frac{\Gamma(\eta(1475)\to\omega\gamma)}{\Gamma(\eta(1475)\to\rho\gamma)} = \frac{1}{9}\left(\frac{M^2_{\eta(1475)}-M^2_\omega}{M^2_{\eta(1475)}-M^2_\rho}\right)^3 \times\left(\frac{0.999X_{\eta(1475)}+0.06Y_{\eta(1475)}}{X_{\eta(1475)}}\right)^2\,, \tag{20}$$

$$\frac{\Gamma(\eta(1295)\to\phi\gamma)}{\Gamma(\eta(1295)\to\rho\gamma)} = \frac{1}{9}\left(\frac{M^2_{\eta(1295)}-M^2_\phi}{M^2_{\eta(1295)}-M^2_\rho}\right)^3 \times\left(\frac{0.03X_{\eta(1295)}+1.99Y_{\eta(1295)}}{X_{\eta(1295)}}\right)^2\,, \tag{21}$$

$$\frac{\Gamma(\eta(1475)\to\phi\gamma)}{\Gamma(\eta(1475)\to\rho\gamma)} = \frac{1}{9}\left(\frac{M^2_{\eta(1475)}-M^2_\phi}{M^2_{\eta(1475)}-M^2_\rho}\right)^3 \times\left(\frac{0.03X_{\eta(1475)}+1.99Y_{\eta(1475)}}{X_{\eta(1475)}}\right)^2\,. \tag{22}$$

TABLE IV

Predicted results of relations (15)–(20).

Relations	Our work	Relations	Our work
(15)	3.111	(18)	0.016
(16)	0.184	(19)	3.64×10^{-4}
(17)	0.521	(20)	26.384

In summary, on the basis of the meson–meson mass mixing and Regge model, we investigated masses of $K(1460)$, $\pi(1300)$, D and D_s as 2^1S_0 meson states and gave more precise mass range of this state. Moreover, the quarkonium content of $\eta(1295)$ and $\eta(1475)$ was explained, which imply that $\eta(1295)$ is nearly a pure $u\bar{u}+d\bar{d}$ state and $\eta(1475)$ is nearly a pure $s\bar{s}$ state. Our results should be useful for the assignment and identification of the members of the 2^1S_0 meson states in new experiments.

REFERENCES

[1] W.M. Yao *et al., J. Phys. G* **33**, 1 (2006).

[2] C. Daum, *Nucl. Phys.* **B187**, 1 (1981).

[3] G.W. Brandenburg, *Phys. Rev. Lett.* **36**, 1239 (1976).

[4] [BaRar Collaboration] B. Aubert *et al., Phys. Rev. Lett.* **90**, 242001 (2003).

[5] [CLEO Collaboration] D. Besson *et al., Phys. Rev.* **D68**, 032002 (2003).

[6] [SELEX Collaboration] A. Evdokimov *et al., Phys. Rev. Lett.* **93**, 242001 (2003).

[7] [BaBar Collaboration] B. Aubert *et al., Phys. Rev. Lett.* **97**, 222001 (2006).

[8] [Belle Collaboration] K. Abe *et al.*, `arXiv:hep-ex/0608031`.

[9] E.V. Beveren, G. Rupp, *Phys. Rev. Lett.* **97**, 202001 (2006).

[10] P. Colangelo, F.D. Fazio, S. Nicotri, *Phys. Lett.* **B642**, 48 (2006).

[11] F.E. Close, C.E. Tomas, O. Lakhina, E.S. Swanson, *Phys. Lett.* **647**, 159 (2007).

[12] Y.R. Liu, Y.B. Dai, C. Liu, S.L. Zhu, *Phys. Rev.* **D70**, 094009 (2004).

[13] W.A. Bardeen, E.J. Eichten, C.T. Hill, *Phys. Rev.* **D68**, 054024 (2003).

[14] M.M. Brisudova, L. Burakovsky, T. Goldman, *Phys. Rev.* **D58**, 114015 (1998).

[15] S. Godfrey, N. Isgur, *Phys. Rev.* **D32**, 189 (1985).

[16] I. Cohen, H.J. Lipkin, *Nucl. Phys.* **B151**, 16 (1979); N. Isgur, *Phys. Rev.* **D21**, 779 (1980).

[17] D.M. Li, B. Ma, Y.X. Li, Q.K. Yao, H. Yu, *Eur. Phys. J.* **C37**, 323 (2004).

[18] V.V. Khruschov, `arXiv:hep-ph/0504077`.

[19] A.V. Anisovich, V.V. Anisovich, A.V. Sarantsev, *Phys. Rev.* **D62**, 051502 (2000).

[20] L. Burakosky, T. Goldman, *Phys. Lett.* **B434**, 251 (1998).

[21] R.C. Brower, J. Ellis, M.G. Schmidt, J.H. Weis, *Nucl. Phys.* **B128**, 75 (1977).

[22] N.A. Kobylinsky, E.S. Martynov, A.B. Prognimak, *Ukr. Phys. Zh.* **24**, 969 (1979).

[23] V.V. Dixit, L.A. Balazs, *Phys. Rev.* **D20**, 816 (1979).

[24] A.B. Kaidalov, *Z. Phys.* **C12**, 63 (1982).

[25] L. Burakovsky, T. Goldman, *Phys. Rev. Lett.* **82**, 457 (1999).

[26] M. Karliner, H.J. Lipkin, `arXiv: hep-ph/0307243`.

[27] M.D. Scadron, R. Delbourgo, G. Rupp, *J. Phys. G* **32**, 735 (2006).

[28] V.B. Jovanovic, *Phys. Rev.* **D76**, 105011 (2007).

[29] M. Lavelle, D. McMullan, *Phys. Rep.* **279**, 1 (1997).

[30] D.M. Li, B. Ma, Q.K. Yao, J.L. Feng, X.C. Feng, H. Yu, *Mod. Phys. Lett.* **A18**, 2775 (2003).

[31] V.V. Khruschev, S.V. Semenov, *Part. Nucl. Lett.* **5**, 5 (2002).

[32] D. Ebert, V.O. Galkin, R.N. Faustov, *Phys. Rev.* **D57**, 5663 (1998).

[33] S. Godfrey, *Phys. Rev.* **D43**, 1679 (1991).

[34] J. Zeng, J.W. Van Orden, W. Roberts, *Phys. Rev.* **D52**, 5229 (1995).

[35] W.S. Carvalho, A.B.C. Antunes, A.S. de Castro, *Euro. Phys. J.* **C7**, 95 (1999).

[36] I. Kitamura, N. Morisita, T. Teshima, *Int. J. Mod. Phys.* **A9**, 5489 (1994).

[37] A. Bramon, R. Escribanon, M.D. Scadron, *Phys. Lett.* **B403**, 339 (1997).