EINSTEIN–SMOLUCHOWSKI EQUATION AND TIME-DEPENDENT MODULATION OF GALACTIC COSMIC RAYS*

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We develop three dimensional (3D) hybrid model of galactic cosmic ray (GCR) propagation in the heliosphere based on the Parker's transport equation. The hybrid model consists of two parts-stationary for high rigidities of GCR particles and non-stationary for relatively low rigidities. It is supposed that scattering of GCR particles in the irregularities (turbulence) of the interplanetary magnetic field (IMF) can be considered as a Brownian motion, and the Einstein–Smoluchowski relation $\langle x^2\rangle = bKt_{\rm sc}$ is valid; $\langle x^2\rangle$ is the mean-square diffusion distance of the GCR particles, K is diffusion coefficient and $t_{\rm sc}$ scattering time; $b = 2, 4$ and 6 for one, two and three dimensional space, respectively. We show that a construction of the hybrid model is possible owing to the dependence of diffusion coefficient on the rigidity of GCR particles. We applied the hybrid model to describe the Forbush effect of the GCR intensity. For the assumed Forbush effect the hybrid model consists of the stationary part for rigidities > 21 GV and of the non-stationary part for rigidities < 21 GV. This model needs $\sim 30\%$ less time for numerical solution than the non-stationary model.

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1. Introduction

Solar activity is generally characterized by number of sunspots on the surface of the Sun, changing from year to year. The increase and drop of sunspot number is called a solar cycle. Reasonably well established

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average cycle is the 11-year periodicity; it lasts from one minimum (period when few sunspots appear or they are not visible at all) to another minimum epochs. At the maximum epochs (a period with a peak in the sunspot number), the polarity of the Sun's global magnetic field reverses, so that the North magnetic pole becomes the South and vice versa. As far every 22 years the magnetic polarity of the Sun is returning to its earlier state, there exists the 22-year solar magnetic cycle. The exceptional phenomenon for the activity of the Sun is solar wind — an extension of the outer atmosphere of the Sun (the corona) into interplanetary space [1-2]. Owing to the rotation of the Sun there are observed the 27-day periodicities of different parameters of solar activity and solar wind. Also, there are observed the outstanding solar flares and the intensive solar coronal mass ejecta (CME) causing the powerful disturbances in the interplanetary space. They appear in time by chance, sporadically, without any regularity, increasing its frequency in the maximum epochs of solar activity. Consequently, the interplanetary space is filled with the electro-magnetic fields consisting of the regular and turbulent components developing dynamically with different time and spatial ranges. This space around the Sun, where the solar wind dominates, is called the heliosphere.

Galactic cosmic ray (GCR) protons and electrons, as well anomalous cosmic rays propagating through the heliosphere are exposed to the influence (modulation) of the diverged solar wind, and the combination of the turbulent (to be stochastic with the zero mean) and regular (averaged) interplanetary magnetic field. The modulation of the GCR intensity is generally characterized with different long and short scale quasi periodic variations (22-years, 11-years and 27-days) and short time (a few days) irregular changes. A short time decrease and recovery of the GCR intensity during 8–10 days is called the Forbush effect [3-4]. In general, there are observed two types of Forbush effects: (1) sporadic Forbush effects characterized by the asymmetric time profile — the GCR intensity rapidly decreases during one-two days and then it recovers gradually in 5–7 days, and (2) recurrent Forbush effects with the (approximately) symmetric decrease and recovery time-profiles of the GCR intensity and a duration of 8–12 days. Sporadic Forbush effects of the GCR intensity are related with the shock waves and magnetic clouds in the interplanetary space created after the outstanding solar flares and the intensive solar CME [5-8]. The recurrent Forbush effects of the GCR intensity are associated with the co-rotating interaction regions (CIRs) in the interplanetary space [9-12] and with the active heliolongitudes existing for the several solar rotations.

In this paper we consider two mathematical models of GCR transport to describe the Forbush effect: (1) the non-stationary 3D model, and (2) 3D model, stationary for high rigidities and non-stationary for relatively low rigidities of GCR particles. Further in this paper, the last model will be called as a "hybrid" model of Forbush effect. Generally, the model of the Forbush effect is very complicated for the three dimensional case; it is true, that besides the complexity of the construction of the compatible numerical model, there is a problem of large computational time requiring its solution. We use both — non-stationary and hybrid models to describe the recurrent Forbush effect of the GCR intensity, and show that the hybrid model gives a possibility to reduce significantly a computer time for numerical solution. We demonstrate that it is possible owing to the dependence of the diffusion coefficient on the rigidity of the GCR particles. Therefore, our aim is to construct the hybrid 3D model of the Forbush effect, and then by comparing the expected temporal changes of the GCR intensity and rigidity spectrum of the Forbush effect to show its compatibility with the non-stationary model.

2. Motivation

According to the quasi linear theory (QLT) [13-16], the nonlinear guiding center theory (NLGCT) [17] and weakly nonlinear theory (WNLT) [18] a diffusion coefficient K depends on the rigidity R of the GCR particles as, $K = AR^{\alpha}$ for the $R > 1\,\mathrm{GV}$ [19-20]. The parameter α is determined by the state of the interplanetary magnetic field (IMF) turbulence, particularly $\alpha = 2 - \nu$, where ν is the exponent of the power spectral density (PSD) of the IMF turbulence (PSD $\propto f^{-\nu}$, and f is the frequency). The dependence of diffusion coefficient of the GCR particles on the parameter α is assumed for the IMF turbulence with a Gaussian distribution.

A propagation of the GCR particles through the regular and stochastic (turbulent) IMF is described by the transport equation [21–28]:

$$\frac{\partial N}{\partial t} = \nabla_i(K_{ij}\nabla_j N) - \nabla_i(U_i N) + \frac{1}{3}\frac{\partial}{\partial R}(NR)(\nabla_i U_i)\,. \tag{1}$$

where N and R are density and rigidity of cosmic ray particles, respectively; U_i is the solar wind velocity, t time; K_{ij} is the anisotropic diffusion tensor of GCR, which can be taken, *e.g.* for the three dimensional IMF [29–30]. We set the dimensionless density $f = N/N_0$, time $\tau^* = t/t_0$, distance $\rho = r/L$, and rigidity $R^* = R/1\mathrm{GV}$, where N and N_0 are density in the interplanetary space and in the local interstellar medium (LISM), respectively; $N_0 = 4\pi I_0$, where the intensity I_0 in the LISM [31-32] has the form:

$$I_0 = \frac{21.1T^{-2.8}}{1 + 5.85T^{-1.22} + 1.18T^{-2.54}}\,,$$

T is kinetic energy in GeV ($T = \sqrt{R^2 + 0.938^2} - 0.938$), t_0 is the characteristic time of GCR modulation corresponding to the change of the electromagnetic conditions in the modulation region for the certain class of GCR variation; r is the radial distance and L is the size of the modulation region. For the case of Forbush effect we assumed that t_0 is equal to the Sun's complete rotation period (27-days $\sim 2.3328 \times 10^6$ s) and the size of modulation region L equals 30 astronomical unites (AU). The Eq. (1) for the dimensionless variables f, τ^* and ρ in the spherical coordinate system (ρ, θ, φ) can be written:

$$A_{12}\frac{\partial f}{\partial \tau^*} = Mf + A_{11}\frac{\partial f}{\partial R}\,, \tag{2}$$

where

$$\begin{aligned} M \;=\; & A_1\frac{\partial^2}{\partial\rho^2} + A_2\frac{\partial^2}{\partial\theta^2} + A_3\frac{\partial^2}{\partial\varphi^2} + A_4\frac{\partial^2}{\partial\rho\partial\theta} + A_5\frac{\partial^2}{\partial\theta\partial\varphi} + A_6\frac{\partial^2}{\partial\rho\partial\varphi} \\ & + A_7\frac{\partial}{\partial\rho} + A_8\frac{\partial}{\partial\theta} + A_9\frac{\partial}{\partial\varphi} + A_{10}\,. \end{aligned} \tag{3}$$

The coefficients A_1, A_2, ..., A_{12} are functions of the spherical coordinates (ρ, θ, φ), rigidity R of GCR particles and time τ^*.

Equation (1) describes a normal diffusion of the GCR particles, for which is accepted that a mean squared displacement $\langle x^2\rangle$ of the GCR particles is proportional to the scattering time $t_{\rm sc}$, as $\langle x^2\rangle \propto t_{\rm sc}$. A statement of the normal diffusion ($\langle x^2\rangle \propto t_{\rm sc}$) is based on the postulation that the motion of the GCR particles in the irregularities (turbulence) of the IMF can be considered as a Brownian motion [27, 33].

In 1905 and 1906 Einstein and Smoluchowski independently published the explanation of the Brownian motion phenomena based on the kinetic theory of matter [34-36]. They found that the irregular motions of the grains are caused by subsequent collisions with medium particles and the following relation (the Einstein–Smoluchowski relation) takes place, $\langle x^2\rangle = bKt_{\rm sc}$; $\langle x^2\rangle$ is the mean squared displacement of a diffusing particle, K diffusion coefficient and $t_{\rm sc}$ a scattering time; $b = 2, 4$ and 6 for one, two and three dimensional space, respectively. Late, in 1908, the same relation was obtained by Langevin [37] based on the solution of the stochastic differential equation, well-known as the first generalized Langevin-equation for the stochastic dynamical system [37]. Late Wiener, [38–39] provided a complete mathematical description of the Brownian motion as a stochastic process (Wiener process). The conception of normal diffusion of GCR particles subtended in the Eq. (1) is based on the assumption that the motion of GCR particles in the irregularities (turbulence) of the IMF can be considered as a Brownian motion and the Einstein–Smoluchowski relation is valid

[34–36, 27] particularly, the magnetic irregularities (turbulence) of the IMF in the solar wind are the centers of the scattering of the GCR particles with the energies of 10^9–10^{12} eV. In case of GCR modulation by the solar wind, the mean displacement $L = \sqrt{\langle x^2 \rangle}$ of GCR particles in the stochastic IMF is accepted (in Eq. (1)) as a size (radius) of the spherical modulation region of GCR.

Due to dynamical extension of the solar corona in the interplanetary space the electro-magnetic conditions continuously change with the rate of the solar wind velocity U in the heliosphere. So, the average convection time $t_{\rm c}$ of the changes of the electro-magnetic conditions on the distance L equals $t_{\rm c} = L/U$, while the average scattering time $t_{\rm sc}$ on the same distance L equals $t_{\rm sc} = L^2/(bK)$, according to the Einstein–Smoluchowski relation. We assume that the length L is equal to the radius of the modulation region. We set a modulation parameter τ, as a ratio of the scattering time $t_{\rm sc}$ of GCR particles and the time $t_{\rm c}$ of the change of the electro-magnetic conditions in the modulation region, $\tau = t_{\rm sc}/t_{\rm c} = (UL)/(bK)$, where $t_{\rm sc} = L^2/(bK)$ and $t_{\rm c} = L/U$ [*e.g.*, 40]. The modulation parameter $\tau = (UL)/(bK)$ determines a character of diffusion of GCR particles in the interplanetary space. Diffusion coefficient K depends on the rigidity R of GCR particles as, $K = \beta R^\alpha$ for $R > 1\,$GV, according to [14, 16–18]; the coefficient β includes any spatial changes of diffusion coefficient and normalization multiplier. For $\alpha = 2 - \nu$, the modulation parameter $\tau = (UL)/(\beta b) R^{\nu - 2}$. It seems that τ depends single-valued on the rigidity R of the GCR particle for the other equal parameters U, L, β, b and ν. Thus, the modulation parameter τ determines a character of diffusion *versus* the rigidity R of the GCR particles. Particularly, when $\tau < 1$ a modulation of GCR can be considered as a strongly stationary process ($\partial f/\partial \tau^* = 0$ in Eq. (2)); when $\tau > 1$, a modulation of GCR is non-stationary ($\partial f/\partial \tau^* \neq 0$ in Eq. (2)), while when $\tau \approx 1$, there is a special quasi-stationary case. In this case we have to estimate the contribution of the term $\partial f/\partial \tau^*$ in Eq. (2) in order to find a transition rigidity R separating stationary and non-stationary regimes of GCR modulation.

3. Theoretical modeling

Generally the full transport equation (2) is very complicated, even, after some simplification *e.g.*:

1. when $\partial f/\partial R = 0$, Eq. (2) is a parabolic time-dependent three dimensional (3D) equation (Cauchy problem)

$$\frac{\partial f}{\partial \tau^*} = \frac{Mf}{A_{12}}\,, \tag{4}$$

2. when $\partial f/\partial\tau^* = 0$, Eq. (2) is a stationary, but it remains as a 3D Cauchy problem with respect to the rigidity R

$$\frac{\partial f}{\partial R} = -\frac{Mf}{A_{11}}\,. \tag{5}$$

We rewrite the equation (2) as:

$$\frac{\partial f}{\partial R} = \left(A_{12}\frac{\partial f}{\partial\tau^*} - Mf\right)\Big/A_{11}\,. \tag{6}$$

The proposing scenario for modeling of the recurrent Forbush effect concerns with the assumption that the temporal changes of the exponent γ of the rigidity spectrum $((\delta D(R))/(D(R)) \propto R^{-\gamma})$ [41-45] of the recurrent Forbush effect is related with the temporal changes of the exponent ν of the PSD of the IMF turbulence. It was shown [41-45] that the temporal changes of the rigidity spectrum of the Forbush effect of the GCR intensity found by neutron monitors experimental data can be provided from the theoretical modeling only if the changes of the IMF turbulence are considered. So, we assume that the Forbush effect of the GCR intensity is caused by the changes of the diffusion coefficient due to the varying of the IMF turbulence. However, we do not exclude important roles of the other parameters of solar wind, which are not included in the present model. We consider two models of the recurrent Forbush effect of the GCR intensity: (1) the non-stationary model, and (2) the hybrid model. In the both models of the Forbush effect a change of diffusion coefficient K takes place in the disturbed vicinity of the interplanetary space restricted in the heliolongitudes $\varphi \in (80^\circ, 280^\circ)$, heliolatitudes $\theta \in (60^\circ, 120^\circ)$ and distance $r < 15\,\mathrm{AU}$.

3.1. Non-stationary model

For the non-stationary model (Eq. (6)) the parallel $K_\parallel$, perpendicular $K_\perp$ and drift diffusion coefficient K_d of cosmic ray particles change as:

$$\begin{aligned}
K_\parallel &= K_0 K(r) K(R,\nu(\varphi,\tau^*))\,,\\
K_0 &= 4.5\times 10^{21}\mathrm{cm}^2/\mathrm{s}\,,\\
K(r) &= 1 + 0.5\left(\frac{r}{1\,\mathrm{AU}}\right)\,,\\
K(R,\nu(\varphi,\tau^*)) &= R^{2-\nu(\varphi,\tau^*)}\,,\\
\nu(\varphi,\tau^*) &= 0.8 - 0.2\,(\cos(2\pi\tau^*) - 0.35) - 0.1(\cos\varphi - 0.2)\,,\\
K_\perp &= \frac{K_\parallel}{1+\omega^2\tau'^2}\,,
\end{aligned}$$

$$
\begin{aligned}
K_{\mathrm{d}} &= K_{\|} \frac{\omega\tau^{'}}{1+\omega^2\tau^{'2}}\,,\\
\omega\tau^{'} &= 300 B\lambda R^{-1}\,,
\end{aligned}
\tag{7}
$$

where the magnitude B of the IMF equals 5.6×10^{-5} Gauss at the Earth orbit and changes according Parker's spiral field model and the mean free path $\lambda \approx 10^{12}$ cm for 10 GV of GCR particle. The solar wind velocity $U = 400$ km/s throughout the modulation region.

The Eq. (6) was transformed to the algebraic system of equation using the implicit finite difference scheme (a grid is $91 \times 91 \times 72 \times 25 \times 81$; 91 steps in distance, 91 in heliolatitudes, 72 in heliolongitudes, 81 in time and 25 in rigidities) and then solved by the Gauss–Seidel iteration method.

The boundary conditions for non-stationary model have a form:

$$
\begin{aligned}
f|_{r=30\,\mathrm{au}} &= 1\,,\\
\frac{\partial f}{\partial r}|_{r=0} &= 0\,,\\
\frac{\partial f}{\partial \theta}|_{\theta=0^\circ} &= \frac{\partial f}{\partial \theta}|_{\theta=180^\circ} = 0\,,\\
f|_{\varphi=\varphi_1} &= f|_{\varphi=\varphi_{L+1}}\,,\\
f|_{\varphi=\varphi_{-1}} &= f|_{\varphi=\varphi_{L-1}}\,.
\end{aligned}
$$

The initial condition with respect to the rigidity R (25 steps up to 1 GV) is $f|_{R=150\,\mathrm{GV}} = 1$ but with respect to time τ^* (for each fixed rigidity R) it is $f(\rho,\theta,\varphi,R,\tau^*)|_{\tau^*=0} = f(\rho,\theta,\varphi,R)$.

3.2. Hybrid model

To construct the hybrid model we have to find the transition rigidity R from the stationary to non-stationary regime. For this purpose we examine a behavior of the modulation parameter τ *versus* the rigidity R of the GCR particle, $\tau = (UL)/(\beta b)\,R^{\nu-2}$. In Fig. 1 there are presented the changes of the modulation parameter τ *versus* the rigidity R of the GCR particle for the determined values of the exponent ν of the PSD of the IMF turbulence, (*e.g.* $\nu = 0$, 0.8, 1.5).

As was mentioned above, when $\tau < 1$ a modulation of GCR can be considered as a strongly stationary process ($\partial f/(\partial \tau^*) = 0$ in Eq. (6)), while when $\tau > 1$, a modulation of GCR is non-stationary ($\partial f/(\partial \tau^*) \neq 0$ in Eq. (6)); $\tau \approx 1$, is a special quasi-stationary case. Fig. 1 shows that a modulation of GCR can be considered either stationary (steady state) or non-stationary (time-dependent) *versus* the rigidity R of GCR particles. As it can be seen from Fig.1 the stationary approximation is valid for the higher

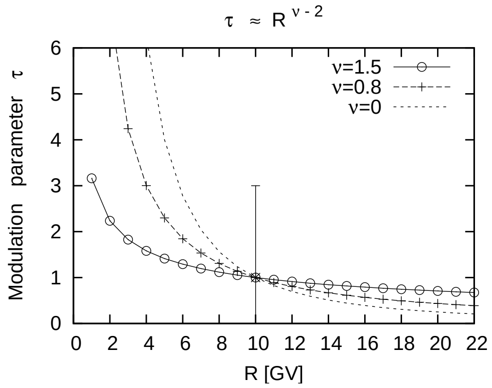

Fig. 1. The changes of the modulation parameter τ *versus* the rigidity R of the GCR particle for the various values of the exponent ν of the PSD of the IMF turbulence ($\nu = 0, 0.8, 1.5$).

rigidities (*e.g.* $R > 21$ GV, $\tau < 1$) of the GCR particles, while for the lower rigidities the non-stationary approximation should be applied (*e.g.* $R < 21$ GV, $\tau > 1$). We show below that this approaching gives a possibility to optimize the time of the numerical solution of the transport equation.

In the hybrid model all parameters are changing as in non-stationary model (Eq. (7)) except the exponent ν of the PSD of the IMF turbulence. The exponent ν changes as:

- $\nu(\varphi) = 0.8 - 0.31\,(\cos\varphi - 0.2)$
  for the rigidities > 21 GV (stationary stage) *versus* the heliolongitudes φ

- $\nu(\varphi, \tau^*) = 0.8 - 0.2(\cos(2\pi\tau^*) - 0.35) - 0.1(\cos\varphi - 0.2)$
  for the rigidities < 21 GV (non-stationary regime) *versus* the heliolongitudes φ and time τ^*.

For the hybrid model the boundary conditions and initial condition with respect rigidity R are the same as for the non-stationary model. Below the rigidity 21 GV there is non-stationary regime, so the initial condition with respect time τ^* is added.

3.3. Comparison of non-stationary and hybrid models

The amplitudes of the Forbush effect calculated using the solutions of the Eq. (6) for the non-stationary model (time of numerical solution equals $\sim$ 45 hours) and for the hybrid model (time of numerical solution equals $\sim$ 30 hours) are presented in Figs. 2(a) and 2(b) for the rigidities of $R = 3$, 5, 10, 21 GV of GCR particles.

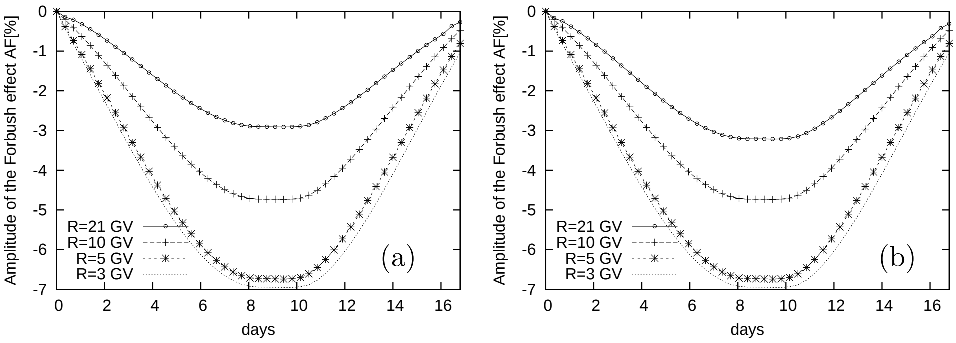

Fig. 2. Changes of the expected amplitudes of the Forbush effect of the GCR intensity for different rigidities *versus* the time for (a) the non-stationary model, (b) the hybrid model.

Figs. 2(a) and 2(b) show that the amplitudes of the Forbush effect are basically the same for the non-stationary and hybrid models, however the time of calculations is 1.5 times greater for the non-stationary model. The difference of the calculation times is more recognizable solving the complicated models when the linear system is 5–10 times larger, than presented in this paper. We can judge about the compatibilities of the proposed models not only by the comparing of the amplitudes of Forbush effects, but by match up the expected rigidity spectra of the Forbush effect for the both models, as well.

In Fig. 3 there are presented the temporal changes of the exponent γ of the power law rigidity spectrum $\delta D(R)/D(R)$ of GCR intensity found as, $\delta D(R)/D(R) = (1/f)\, df/dR \propto R^{-\gamma}$ for the non-stationary model (circles) and for the hybrid model (crosses). One day at the Earth orbit (86400 s)

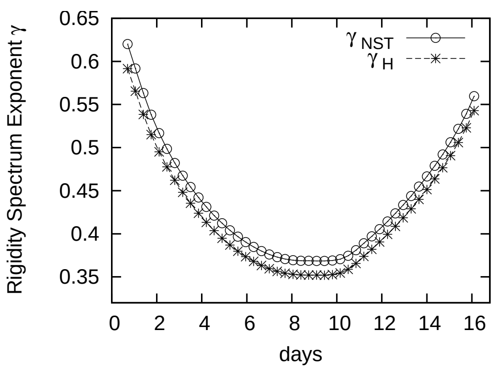

Fig. 3. Temporal changes of the expected rigidity spectrum exponent γ for non-stationary ($\gamma_{\rm NST}$) and hybrid ($\gamma_{\rm H}$) models.

corresponds to the 13.3° of the heliolongitudes. We see from Fig. 3 that the temporal changes of the rigidity spectrum exponent γ for non-stationary ($\gamma_{\rm NST}$) and hybrid model ($\gamma_{\rm H}$) are basically the same; correlation coefficient between them equals 0.99 ± 0.05.

4. Conclusions

1. Einstein–Smoluchowski formula gives a possibility to compose a hybrid model of the Forbush effect of the GCR intensity consisting of the stationary regime for the rigidities $> 21\,$GV, and the non-stationary regime for rigidities $< 21\,$GV. The transition rigidity limit (21 GV) of the GCR particles for the presented hybrid model is changeable depending on the character of the dependence of diffusion coefficient on the rigidity of GCR particles.
2. The same temporal changes of the expected amplitudes and rigidity spectra for both models of the Forbush effect demonstrate an explicit compatibility of the hybrid model with the non-stationary model of recurrent Forbush effect of GCR.
3. Numerical solution of the hybrid model needs $\sim 30\%$ less time than the numerical solution of the non-stationary model. Thus, to study the Forbush effect of the GCR intensity the non-stationary model can be replaced by the economy hybrid model. The proposed hybrid model can be successfully applied to other non-stationary processes in cosmic ray physics.

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