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ON TWO-POINT CORRELATION FUNCTIONS IN AdS/QCD*

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In this talk we study the hard wall AdS/QCD model, the simplest model of bottom-up approach to AdS/QCD. We reveal the relations between fields in the model and operators in QCD, fix parameters of the model and calculate several quantities of interest. We underline the problems of the model and propose the way to solve them.

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1. Introduction

In the last few years a great attention was paid to the so-called phenomenological AdS/QCD theories. The essence of these models is to use the AdS/CFT correspondence [2] to describe QCD in large $N_{\rm c}$ limit via its 5-dimensional dual theory. The exact structure of this 5D theory, describing all specific features of QCD is not clear, but some simple models have been proposed [3–8], which already give promising results.

In this talk we study the simplest of these settings, the so-called hard wall AdS/QCD model (see for example [3,5], first proposed in [9]). Our goal is to find proper relation between fields in the theory and QCD currents, fix the free parameters of the model and study results, that this model gives without any tuning. After that we propose some ways to tune up the model in order to reproduce results of QCD. We will calculate vector, axial vector and pseudoscalar two-point functions of QCD, that will allow us to find values of meson masses and f_π.

This talk is mainly based on the paper by Krikun [1], so all details and references can be found there.

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2. Description of the model

We consider the simplest holographic model of low energy QCD, proposed in [3,5,7] (see also [14–16]), the so-called "Hard wall AdS/QCD model". In what follows we will work with conventions and notations used in [3].

In the AdS/CFT prescription, the fields in 5-dimensional space are dual to operators in 4D, and the global flavor symmetry of the 4D field theory corresponds to the gauge symmetry in its 5D dual. So we will study 4D QCD with $\mathrm{SU}(2)_{\mathrm{L}}\times\ \mathrm{SU}(2)_{\mathrm{R}}$ global symmetry via the gauge theory in AdS with $\mathrm{SU}(2)_{\mathrm{L}}\times\ \mathrm{SU}(2)_{\mathrm{R}}$ gauge group. In this model only the fields dual to QCD operators with the lowest dimensions are considered.

We have the $\mathrm{SU}(2)_{\mathrm{L}}\times\ \mathrm{SU}(2)_{\mathrm{R}}$ gauge field theory in AdS_5 space with the metric:

$$ds^2 = \frac{R^2}{z^2}(-dz^2 + dx^\mu dx_\mu)\,, \tag{1}$$

where R is the AdS curvature radius, cut at z coordinate: $0 < z \leq z_m$.

Later, we will denote 5-dimensional indices with $A, B \ldots \in \{0,1,2,3,z\}$, and 4D indices with $\mu, \nu, \ldots \in \{0,1,2,3\}$.

The theory includes left- and right-handed gauge vector fields $\mathrm{SU}_{\mathrm{L}}(2)\times \mathrm{SU}_{\mathrm{R}}(2)$ (A_{L} and A_{R}, respectively) and bifundamental scalar $X_{\alpha\beta}$. According to AdS/CFT 5D fields correspond to operators in QCD:

$$\begin{aligned} A^a_{\mathrm{L}\mu} &\leftrightarrow \bar{q}_{\mathrm{L}}\gamma^\mu t^a q_{\mathrm{L}}\,, \\ A^a_{\mathrm{R}\mu} &\leftrightarrow \bar{q}_{\mathrm{R}}\gamma^\mu t^a q_{\mathrm{R}}\,, \\ \left(\frac{2}{z}\right) X^{\alpha\beta} &\leftrightarrow \bar{q}^\alpha_{\mathrm{R}} q^\beta_{\mathrm{L}}\,, \end{aligned} \tag{2}$$

with the boundary conditions imposed at $z = z_m$:

$$\partial_z V(z_m) = 0\,, \qquad \partial_z A(z_m) = 0\,.$$

The action is:

$$S = \int d^5x\sqrt{g}\,\mathrm{Tr}\left\{\Lambda^2\left(|DX|^2 + \frac{3}{R^2}|X|^2\right) - \frac{1}{4g_5^2}\left(F_{\mathrm{L}}^2 + F_{\mathrm{R}}^2\right)\right\}\,, \tag{3}$$

where

$$\begin{aligned} D_B X &= \partial_B X - \imath A_{\mathrm{L}B} X + \imath X A_{\mathrm{R}B}\,, \\ A_{\mathrm{L(R)}} &= A^a_{\mathrm{L(R)}} t^a\,, \\ F_{BD} &= \partial_B A_D - \partial_D A_B - \imath[A_B, A_D]\,, \end{aligned}$$

and we introduce the normalization constant Λ of field X.

From the equation of motion for X we can get classical solution:

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3 \,.$$

According to AdS/CFT [17], we argue, that M corresponds to quark mass matrix, *i.e.* the source of operator $\bar{q}_{\rm R}^{\alpha} q_{\rm L}^{\beta}$ and Σ to condensates, *i.e.* vacuum expectation value of $\bar{q}_{\rm R}^{\alpha} q_{\rm L}^{\beta}$. We can make M to be quark masses exactly by appropriate definition of normalization Λ, but the relation between Σ and condensates is to be ascertained. In further discussion we set $\Sigma = \sigma\mathbf{1}$, $M = m\mathbf{1}$, assuming the equality of quark masses. Hence

$$X_0(z) = \frac{1}{2}v(z)\mathbf{1} \,, \qquad v(z) = mz + \sigma z^3 \,. \tag{4}$$

We will decompose X in modulus and phase:

$$X = X_0 e^{\imath 2\pi^a (t^a)} = \mathbf{1}\frac{v(z)}{2} e^{\imath 2\pi^a t^a} \,.$$

It is convenient to introduce vector and axial vector fields:

$$\begin{aligned} V &= (A_{\rm L} + A_{\rm R})/2 \,, \\ A &= (A_{\rm L} - A_{\rm R})/2 \,. \end{aligned}$$

We set $A_z = V_z = 0$, use transverse gauge for V_μ and decompose A_μ on longitudinal and transverse parts:

$$\partial_\mu V_\mu = 0 \,, \qquad A_\mu = A_{\perp\mu} + \partial_\mu \phi \,. \tag{5}$$

One can relate the pseudoscalar current $\bar{q}\gamma_5 q$ to axial vector current $\bar{q}\gamma_5\gamma_\mu q$ via relation:

$$\partial_\mu \left(\bar{q}\gamma_5\gamma_\mu q\right) = 2m \left(\bar{q}\gamma_5 q\right) \,.$$

So we can write out the following table of correspondence:

$$\begin{aligned} V_\mu &\leftrightarrow \bar{q}\gamma^\mu q = J_V \,, \\ A_\mu &\leftrightarrow \bar{q}\gamma_5\gamma^\mu q = J_A \,, \\ \frac{Q^2}{2m}\phi &\leftrightarrow \bar{q}\gamma_5 q = J_\pi \,. \end{aligned} \tag{6}$$

With this table we can calculate two-point functions of QCD via our 5D theory, using the AdS/CFT recipe. For example:

$$\begin{aligned} \langle J_V(q_1) J_V(q_2) \rangle &= \frac{\delta}{\delta V_0(q_1)} \frac{\delta}{\delta V_0(q_2)} S(V_{\rm classic})|_{V_0=0} \,, \\ V_0(q) &= V_{\rm classic}(q,z)|_{z=0} \,. \end{aligned} \tag{7}$$

In order to perform this calculation one needs to find classical solutions, which are solutions to equations of motion, obtained by variation of $S(V_\mu, A_\mu, \partial_\mu\phi, A_z)$:

$$\begin{aligned}
\left[\partial_z\left(\frac{1}{z}\partial_z V^a_\mu\right) + \frac{q^2}{z}V^a_\mu\right]_\perp &= 0\,,\\
\left[\partial_z\left(\frac{1}{z}\partial_z A^a_\mu\right) + \frac{q^2}{z}A^a_\mu - \frac{R^2 g_5^2 \Lambda^2 v^2}{z^3}A^a_\mu\right]_\perp &= 0\,,\\
\partial_z\left(\frac{1}{z}\partial_z \phi^a\right) + \frac{R^2 g_5^2 \Lambda^2 v^2}{z^3}(\pi^a - \phi^a) &= 0\,,\\
-q^2\partial_z\phi^a + \frac{R^2 g_5^2 \Lambda^2 v^2}{z^2}\partial_z \pi^a &= 0\,.
\end{aligned} \tag{8}$$

One can see that X does not interact with vector field in quadratic action, so the equation for v is exactly solvable. This is in contrast to other equations, which can be solved perturbatively in the limits of large or small momenta.

3. Parameter fixing

Let us calculate some two-point functions. We start with vector current, associated with the field V in the model. The solution for V is:

$$V(Q,z) = -V_0(Q)\frac{1}{I_0(Qz_m)}Qz[K_0(Qz_m)I_1(Qz) - I_0(Qz_m)K_1(Qz)]\,. \tag{9}$$

We substitute it to the variation of metric with respect to boundary value V_0 , which can be presented in the form

$$\delta S_V = -\int d^4x \frac{R}{g_5^2}\left[\delta V_\mu \frac{\partial_z V_\mu}{z}\right]_{z=\epsilon}$$

and the result for current correlator is:

$$\langle J^a_{V\mu}(q) J^b_{V\nu}(q)\rangle = \delta^{ab}(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V(q^2)\,,$$

where

$$\Pi_V(Q^2) = -\frac{R}{g_5^2}\frac{\Big(K_0(Qz_m) - I_0(Qz_m)[\ln(Q\epsilon/2) + \gamma]\Big)}{I_0(Qz_m)}\,. \tag{10}$$

The poles of Euclidean correlator correspond to masses of bound states. Consequently the first pole corresponds to the ρ-meson mass (the meson, associated with vector current in QCD), so we can fix the value of z_m:

$$I_0(\imath M_\rho z_m) = 0 \qquad \Longrightarrow \qquad z_m = \frac{2.4}{M_\rho} = \frac{1}{323}\mathrm{MeV}^{-1}\,. \tag{11}$$

The masses of higher states we will obtain automatically: each one will correspond to the zero of Bessel function. The problem here is that this spectrum does not demonstrate the Regge behavior. In order to solve it, one has to change form of IR boundary. If at z_m metric has a factor of $\exp(-z^2)$ instead of a simple cut, the Regge behavior will take place [15,16].

In the large Q^2 limit we have

$$\Pi_V(Q^2) = -\frac{R}{2g_5^2} \ln Q^2\epsilon^2 \,.$$

This result has the same form as in QCD and can be compared with the QCD sum rules leading term [18]:

$$\Pi_V(Q^2) = -\frac{N_{\rm c}}{24\pi^2} \ln Q^2\epsilon^2 \,.$$

This fixes the 5D coupling constant g_5

$$\frac{g_5^2}{R} = \frac{12\pi^2}{N_{\rm c}} \,. \tag{12}$$

To compute correlator of pseudoscalar current J_π we find solutions for coupled ϕ and π near the boundary

$$\begin{aligned} \phi(z) &= \phi_0(q) Q z K_1(Qz) \,, \\ \pi(z) &= -\phi_0(q) \frac{Q^2}{g_5^2 R^2 \Lambda^2 m^2} Q z K_1(Qz) \,. \end{aligned} \tag{13}$$

The variation of action with respect to $\phi_0(q)$ gives

$$\delta S_\pi = \int d^4x \;\; \frac{R}{g_5^2} \left[\delta\partial_\mu\phi \frac{\partial_z\partial_\mu\phi}{z}\right]_{z=\epsilon} - \Lambda^2 R^3 \left[\delta\pi \frac{v^2}{z^3}\partial_z\pi\right]_{z=\epsilon} \,.$$

And we get for the correlator:

$$\langle J_\pi(q), J_\pi(q)\rangle = 2\frac{R}{g_5^2}\frac{1}{g_5^2 R^2 \Lambda^2} Q^2 \ln\left(Q^2\epsilon^2\right) \,.$$

One can find that this result also has the same form as in QCD and comparison with the sum rules leading term [18]

$$\langle J_\pi(q), J_\pi(q)\rangle_{QCD} = \frac{N_{\rm c}}{16\pi^2} Q^2 \ln\left(Q^2\epsilon^2\right)$$

gives the value of Λ

$$\Lambda^2 = \frac{8}{3}\frac{1}{g_5^2 R^2} = \frac{2N_{\rm c}}{9\pi^2}\frac{1}{R^3} \,. \tag{14}$$

We can also calculate the value of the chiral condensate to fix the value of σ. As was mentioned above, it should be proportional to σ but we need to find the coefficient. In QCD the chiral condensate is defined as:

$$\langle \bar{q}q \rangle = \frac{\delta \varepsilon_{\mathrm{QCD}}}{\delta m_q} |_{m_q=0} \,.$$

On the AdS side this corresponds to:

$$\langle \bar{q}q \rangle = \left. \frac{\delta\ S(X_0)}{\delta m} \right|_{m=0} = 3R^3 \Lambda^2 \sigma = \frac{2N_{\mathrm{c}}}{3\pi^2}\sigma \,. \tag{15}$$

We see, that $\langle \bar{q}q \rangle$ is proportional to σ and, because chiral condensate is linear in N_{c}, σ turns out to be $O(N_{\mathrm{c}}^0)$. For $N_{\mathrm{c}} = 3$ $\sigma = (462\mathrm{MeV})^3$.

4. Results

After we have fixed all parameters in the model, we can write out the action in terms of QCD values:

$$S = \frac{N_{\mathrm{c}}}{12\pi^2} \int d^5x \left\{ -\frac{1}{4z} \left(F_A^2 + F_V^2 \right) + \frac{4}{3z^3} v(z)^2 (\partial\pi - A)^2 + \frac{4}{z^5} v(z)^2 \right\} . \tag{16}$$

We can calculate the axial two-point function. It is a little more difficult than in vector case, because the equation of motion cannot be solved exactly. We solve it in the limit of large Q^2 perturbatively and corrections of the order of $1/Q^n$ in the classical solution give next to leading terms of OPE. We can write out the result with corrections of the order of σ^2/Q^6 and $(m\sigma)/Q^4$:

$$\Pi_A(Q^2) = -\frac{N_{\mathrm{c}}}{24\pi^2} \left[\ln Q^2 + \frac{128}{15} \frac{\sigma^2}{Q^6} - \frac{64}{9} \frac{\sigma m}{Q^4} \right] . \tag{17}$$

One can see, that coefficients here do not coincide with sum rules calculation. We could tune this result by adding a perturbation in the metric [13]:

$$ds^2 = \omega(z) \left(-dz^2 + dx^\mu dx_\mu \right) , \qquad \omega(z) = \frac{R^2}{z^2} + A\sigma^2 z^4 + B\sigma m z^2 \,,$$

which would cause additional corrections to the classical solution to appear, and this would change coefficients as desired.

The interesting object is "left-right" correlator $\Pi_{\mathrm{LR}} = \Pi_A - \Pi_V$

$$\Pi_{\mathrm{LR}} = -\frac{N_{\mathrm{c}}}{9\pi^2} \left[\frac{16}{5} \frac{\sigma^2}{Q^6} - \frac{8}{3} \frac{\sigma m_q}{Q^4} \right] . \tag{18}$$

Note here that it has not powers of R, namely it has the order λ'^0 because in AdS/CFT:

$$\frac{R^4}{4\pi\alpha'^2} = \lambda' = N_\mathrm{c} g_\mathrm{YM}^2 \,. \tag{19}$$

If we denote in this formula coefficients as f and ρ, we find that at $\lambda' \to \infty$ our calculation predicts:

$$f(\lambda') \sim \rho(\lambda') \sim \lambda'^0 \,,$$

while at weak coupling regime (sum rules):

$$\rho(\lambda') \sim \lambda'^0 \,, \qquad f(\lambda') = -4\pi\alpha_\mathrm{s} \sim \lambda' \,.$$

The different behavior of the results is an evidence that two approaches, AdS/QCD and sum rules, are applicable at different regimes. AdS/QCD works at strong coupling, whereas sum rules correspond to weak coupling regime.

One more value that we can find using AdS/QCD is f_π. We use the relation

$$\Pi_A(Q)|_{Q\to 0} = \frac{f_\pi^2}{Q^2} \,.$$

We can solve the equation of motion of A in the limit $Q^2 \to 0$ to obtain the two-point function at small Q^2. We find:

$$\begin{aligned} f_\pi^2 &= -\frac{R}{g_5}\frac{\partial_z a(z)}{z}|_{z=0,Q=0} \\ &\approx \frac{R}{g_5} 2.16\sigma^{2/3} = \frac{N_\mathrm{c}}{12\pi^2} 2.16 \left(\frac{3\pi^2}{2}\frac{\langle\bar{q}q\rangle}{N_\mathrm{c}}\right)^{2/3} \sim 40\mathrm{MeV} \,. \end{aligned} \tag{20}$$

This value obviously does not coincide with expected 140 MeV. We see that f_π is related to the parameters of classical solution of X. One can introduce additional potential for X in the 5D bulk. Consequently the classical solution will change and the value of f_π can be tuned.

5. Conclusion

The model under consideration has several free parameters but still has some predictive power. It gives qualitatively satisfactory results but numbers differ. Study of such simple model gives an insight to common features of AdS/QCD and proposes modifications needed to obtain more realistic results.

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