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FOUR-POINT FUNCTIONS IN $N = 1$ SCFT*

Paulina Suchanek

M. Smoluchowski Institute of Physics, Jagiellonian University
Reymonta 4, 30-059 Kraków, Poland
suchanek@th.if.uj.edu.pl

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The main results concerning the 4-point super-conformal blocks in the $N = 1$ Neveu–Schwarz sector are resumed.

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1. Introduction

Two dimensional quantum field theories with conformal symmetry (CFT) constitute a rapidly developing branch of modern theoretical physics with a rich mathematical structure and a variety of physical applications. The beginning of an intense activity in that subject is connected with the BPZ work [1], where the basic objects in CFT were defined and the way of constructing examples of CFT models was presented. The latter are minimal models *i.e.* completely solvable conformal theories that can be identified with two-dimensional statistical systems at their critical points. The applications of CFT however go beyond the description of statistical systems. It is also commonly used language of string theory where the string scattering amplitudes can be expressed in terms of correlation functions of CFT. An additional motivation for advanced studies on conformal theories comes from the AdS CFT correspondence that has been rapidly developing field of research for last ten years.

In two dimensions conformal symmetry imposes strong conditions on correlation functions. Thanks to the conformal Ward identities and the assumption of associativity of the operator algebra it is possible to reduce any correlation function to 3-point coupling constants and conformal blocks,

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the latter being universal functions completely determined by the symmetry. Even in the case of 4-point conformal block it is extremely hard to calculate explicitly a general form of these objects. There are, however, recursive methods of an approximate, analytic determination of conformal blocks worked out by Zamolodchikov [2–4]. Using them one can numerically calculate any 4-point function if only structure constants of a given conformal theory are known.

The first recursive method is based on expansion of the block in terms of power series in projective invariant z. Coefficients of the expansion can be represented as a sum over poles in the central charge c and a regular in c term which is given by the limit of block for $c \to \infty$. Since the residues are proportional to block's coefficients the recursion relations for coefficients can be obtained. The second, more efficient method of determining block is so called elliptic recursion. In order to derive it one has to calculate the large intermediate weight asymptotic, which according to the Zamolodchikov's works is determined by the classical asymptotic of conformal blocks. The classical limit of block can be analyzed in terms of Liouville theory. Once the large Δ asymptotic of block is calculated it is possible to define elliptic block that can be expanded as a series in elliptic nome. The coefficients of this expansion can be represented as a sum over poles in Δ and a regular term, which does not depend on external weights and central charge. Thus it is possible to determine it from explicit analytic formulae of block derived in a certain model. As in the case of c dependence, because the residues in Δ are proportional to the elliptic block's coefficients, the closed recursive relation for the coefficients can be obtained.

As mentioned above, the Liouville theory is essential for deriving the classical limit of conformal block. An exact analytical expressions for the Liouville two and three-point functions were proposed by Otto and Dorn [5] and by Zamolodchikov and Zamolodchikov [6]. The recursive methods of calculating the conformal blocks were used in [6] for numerical checks of the bootstrap equation. The bootstrap program was completed by Ponsot and Teschner [7, 8] and Teschner [9, 10] and it resulted in a consistent operator formulation of the Liouville theory.

The problem of defining and determining 4-point blocks is even more interesting and difficult in the case of supersymmetric conformal field theories (SCFT). Since the algebra in SCFT is more general, there are two types of 3-point structure constants. Therefore, the structure of 3-point block is more complicated and there are more types of 4-point blocks than in the bosonic case. The definition and some properties of superconformal blocks were not worked out until [11, 12], where the recursive method of determination the block's coefficients of the expansion in z was derived as well. In order to obtain the elliptic recursion one has to analyze the classical limit of block in supersymmetric Liouville theory and calculate the explicit analytic formulae of superconformal block for special case of $c=\frac{3}{2}$ SCFT. This program was completed in [13, 14].

In the rest of the paper a brief outline of our results concerning superconformal blocks in Neveu–Schwarz sector of $N=1$ SCFT is presented.

2. Correlation functions in SCFT

The space of states in NS sector consists of superconformal NS Verma modules $\mathcal{V}_\Delta$, *i.e.* the $\frac{1}{2}\mathbb{Z}$-graded representations of the algebra:

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{12}m\left(m^2-1\right)\delta_{m+n}\,, \\ [L_m, S_k] &= \frac{m-2k}{2}S_{m+k}\,, \\ \{S_k, S_l\} &= 2L_{k+l} + \frac{c}{3}\left(k^2-\frac{1}{4}\right)\delta_{k+l}\,, \end{aligned}$$

where indices m, n are integers and k, l half-integers. The superconformal NS module of the highest weight Δ is a sum over free vector spaces $\mathcal{V}_\Delta^f$ generated by all vectors of the form

$$\nu_{\Delta,KM} \;\equiv\; S_{-k_i}\ldots S_{-k_1}L_{-m_j}\ldots L_{-m_1}\nu_\Delta\,, \tag{1}$$

where k and m are arbitrary ordered indices such that $k_1+\ldots+k_i+m_1+\ldots+m_j=f$. For a given level f each $\mathcal{V}_\Delta^f$ is an eigenspace of L_0 with the eigenvalue $\Delta+f$. Any state $\xi_\Delta\in\mathcal{V}_\Delta^f$ as an eigenstate of parity operator $(-1)^F=(-1)^{2(L_0-\Delta)}$ has defined parity.

Due to the superconformal Ward identities any correlation function of NS fields is completely determined up to two independent structure constants:

$$\begin{aligned} C_{321} &= \Big\langle \phi_3(\infty,\infty)\phi_2(1,1)\phi_1(0,0)\Big\rangle\,, \\ \widetilde{C}_{321} &= \Big\langle \phi_3(\infty,\infty)\widetilde{\phi}_2(1,1)\phi_1(0,0)\Big\rangle\,, \end{aligned} \tag{2}$$

where ϕ_i is superprimary field and $\widetilde{\phi}_i=\left\{S_{-1/2},\left[\bar{S}_{-1/2},\phi_i\right]\right\}$.

The 3-point correlation function can be written in terms of above constants and 3-point blocks:

$$\langle\xi_3,\bar{\xi}_3\,|\varphi_{\Delta_2,\bar{\Delta}_2}(\xi_2,\bar{\xi}_2|z,\bar{z})|\xi_1,\bar{\xi}_1\rangle = z^{\Delta_3(\xi_3)-\Delta_2(\xi_2)-\Delta_1(\xi_1)}\bar{z}^{\bar{\Delta}_3(\bar{\xi}_3)-\bar{\Delta}_2(\bar{\xi}_2)-\bar{\Delta}_1(\bar{\xi}_1)}$$
$$\times\rho_{\infty}^{\Delta_3}{}_{1}^{\Delta_2}{}_{0}^{\Delta_1}(\xi_3,\xi_2,\xi_1)\rho_{\infty}^{\bar{\Delta}_3}{}_{1}^{\bar{\Delta}_2}{}_{0}^{\bar{\Delta}_1}(\bar{\xi}_3,\bar{\xi}_2,\bar{\xi}_1)\times\left\{\begin{array}{l} C_{321} \\ \widetilde{C}_{321} \end{array}\right. .$$

The upper line corresponds to the case when the total parity of all states ξ_i is positive, while the lower line corresponds to the case of a negative total parity.

The 3-point block is the 3-form defined on NS Verma modules:

$$\rho^{\Delta_3\ \Delta_2\ \Delta_1}_{\infty\ \ z\ \ \ 0}(\xi_3,\xi_2,\xi_1) : \mathcal{V}_{\Delta_3}\times\mathcal{V}_{\Delta_2}\times\mathcal{V}_{\Delta_1} \ \mapsto\ \mathbb{C}\,,$$

normalized by the condition

$$\rho^{\Delta_3\ \Delta_2\ \Delta_1}_{\infty\ \ z\ \ \ 0}(\nu_3,\nu_2,\nu_1) = \rho^{\Delta_3\ \Delta_2\ \Delta_1}_{\infty\ \ z\ \ \ 0}(\nu_3,S_{-\frac{1}{2}}\nu_2,\nu_1) = 1\,,$$

where ν_3,ν_2,ν_1 are super-primary states.

The general 4-point function can be expressed through the correlation functions of primary fields, which in order can be written in terms of structure constants (2) and superconformal 4-point blocks. For the sake of clarity we shall analyze only the case of correlation function of superprimary fields[1], in the diagonal case $\Delta_i = \bar{\Delta}_i$:

$$\begin{aligned}\langle \Delta_4|\,\phi_3(1,1)\phi_2(z,\bar{z})\,|\Delta_1\rangle \ =\ \sum_p \Bigg(& C_{43p}C_{p21}\left|\mathcal{F}^{1}_{c,\Delta_p}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right](z)\right|^2 \\ & -\tilde{C}_{43p}\tilde{C}_{p21}\left|\mathcal{F}^{1/2}_{c,\Delta_p}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right](z)\right|^2\Bigg)\,. \end{aligned}\qquad (3)$$

The existence of two independent structure constants implies appearance of two types of blocks corresponding to factorization of 4-point function appropriately on even or odd states. This is non trivial modification in comparison with the bosonic case. The even block has the form:

$$\mathcal{F}^{1}_{c,\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right](z) \ =\ z^{\Delta-\Delta_2-\Delta_1}\left(1+\sum_{m\in\mathbb{N}} z^m F^{m}_{c,\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right]\right),$$

and the odd one:

$$\mathcal{F}^{1/2}_{c,\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right](z) \ =\ z^{\Delta-\Delta_2-\Delta_1}\sum_{k\in\mathbb{N}-\frac{1}{2}} z^k F^{k}_{c,\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right].$$

The coefficients are defined by the 3-point blocks:

$$\begin{aligned}F^{f}_{c,\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right] \ =\ \sum_{|K|+|M|=|L|+|N|=f} & \rho^{\Delta_4\ \Delta_3\ \Delta}_{\infty\ \ 1\ \ \ 0}(\nu_4,\nu_3,\nu_{\Delta,KM})\left[B^{f}_{c,\Delta}\right]^{KM,LN} \\ & \times\rho^{\Delta\ \Delta_2\ \Delta_1}_{\infty\ \ 1\ \ \ 0}(\nu_{\Delta,LN},\nu_2,\nu_1),\end{aligned}$$

[1] The other cases are discussed exactly in Ref. [11].

where $\left[B^f_{c,\Delta}\right]^{KM,LN}$ is the inverse matrix to the Gram matrix of superconformal Verma module on level f with respect to the basis (1).

The Gram matrix $\left[B^f_{c,\Delta}\right]_{KM,LN}$ is singular only if the supermodule $\mathcal{V}_\Delta$ contains singular vectors χ of degree $k \le f$, *i.e.* highest weight vectors satisfying condition $L_0\chi = (\Delta + k)\chi$. The determinant of this matrix is given by the Kac theorem

$$\det B^f_{c,\Delta} = K_f \prod_{1\leqslant rs\leqslant 2f} (\Delta - \Delta_{rs})^{P_{\rm NS}(f-\frac{rs}{2})}\,, \tag{4}$$

where K_f depends only on the level f, the sum $r+s$ must be even and

$$\begin{aligned}\Delta_{rs}(c) &= -\frac{rs-1}{4} + \frac{r^2-1}{8}\beta^2 + \frac{s^2-1}{8}\frac{1}{\beta^2}\,,\\ \beta &= \frac{1}{2\sqrt{2}}\left(\sqrt{1-\hat{c}} + \sqrt{9-\hat{c}}\right), \qquad \hat{c} = \frac{2}{3}c\,.\end{aligned}$$

The multiplicity of each zero in a degenerate weight Δ_{rs} is given by $P_{\rm NS}(f) = \dim \mathcal{V}^f_\Delta$.

3. Recursion relations for 4-point blocks

Despite the fact that blocks' coefficients are completely determined by the symmetry, an explicit calculation of them from definition is extremely troublesome. In order to derive the recursive methods of determining the blocks one has to work out their properties, which can be deduced from the structure of the Gram matrix and conditions that the Ward identities impose on the 3-point blocks.

The 4-point blocks are functions of external weights Δ_i, intermediate weight Δ and the central charge c. Their coefficients depend on external weights entirely through the 3-point blocks which are polynomials in all weights. As functions of the intermediate weight Δ and the central charge c the 4-point blocks' coefficients are rational functions. Moreover, from the multiplicity of zero $\Delta = \Delta_{rs}$ of the Kac determinant (4) and properties of the Gram matrix, it follows that blocks' coefficients can be expressed as a sum over simple poles in Δ and a regular term.

$$F^f_{c,\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right] = {\rm h}^f_{c,\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right] + \sum_{\substack{1\leq rs\leq 2f\\ r+s\in 2\mathbb{N}}} \frac{\mathcal{R}^f_{c,rs}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right]}{\Delta - \Delta_{rs}(c)}\,, \tag{5}$$

The same is true for the central charge dependence with simple poles in $c = c_{rs}$.

The structure of the residues is crucial for the recurrence relations for the blocks' coefficients. As it was shown in [11] from the factorization property of 3-point blocks follows that the residue of the coefficient $F^f_{c,\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right]$ at Δ_{rs} is proportional to the coefficient from the lower level and so called fusion polynomials:

$$\mathcal{R}^m_{c,rs}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right] = A_{rs}(c) P^{rs}_c\left[\begin{smallmatrix}\Delta_3\\ \Delta_4\end{smallmatrix}\right] P^{rs}_c\left[\begin{smallmatrix}\Delta_2\\ \Delta_1\end{smallmatrix}\right] F^{m-\frac{rs}{2}}_{c,\Delta_{rs}+\frac{rs}{2}}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right], \text{ for } m \in \mathbb{N}\cup\{0\} \tag{6}$$

and

$$\mathcal{R}^k_{c,rs}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right] = -A_{rs}(c) P^{rs}_c\left[\begin{smallmatrix}*\Delta_2\\ \Delta_1\end{smallmatrix}\right] P^{rs}_c\left[\begin{smallmatrix}*\Delta_2\\ \Delta_1\end{smallmatrix}\right] F^{k-\frac{rs}{2}}_{c,\Delta_{rs}+\frac{rs}{2}}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right] \text{ for } k \in \mathbb{N}-\tfrac{1}{2}\,. \tag{7}$$

The fusion polynomials are defined in terms of weights

$$\Delta_i = -\frac{1}{8}\left(\beta - \frac{1}{\beta}\right)^2 + \frac{\alpha_i^2}{8}$$

and
$p = r-1-2k, q = s-1-2l$ $(0 \le k \le r-1,\ 0 \le l \le s-1)$:

$$P^{rs}_c\left[\begin{smallmatrix}\Delta_2\\ \Delta_1\end{smallmatrix}\right] = \prod_{p=1-r}^{r-1}\prod_{q=1-s}^{s-1}\left(\frac{\alpha_2-\alpha_1+p\beta-q\beta^{-1}}{2\sqrt{2}}\right)\left(\frac{\alpha_2+\alpha_1+p\beta-q\beta^{-1}}{2\sqrt{2}}\right),$$

where p, q are such that $k+l \in 2\mathbb{N}\cup 0$, and

$$P^{rs}_c\left[\begin{smallmatrix}*\Delta_2\\ \Delta_1\end{smallmatrix}\right] = \prod_{p=1-r}^{r-1}\prod_{q=1-s}^{s-1}\left(\frac{\alpha_2-\alpha_1+p\beta-q\beta^{-1}}{2\sqrt{2}}\right)\left(\frac{\alpha_2+\alpha_1+p\beta-q\beta^{-1}}{2\sqrt{2}}\right)$$

with $k+l \in 2\mathbb{N}-1$.

The coefficient of proportionality $A_{rs}(c)$ can be deduced from the properties of Gram matrix and is given by the limit:

$$A_{rs}(c) = \lim_{\Delta\to\Delta_{rs}}\left(\frac{\langle\chi^\Delta_{rs}|\chi^\Delta_{rs}\rangle}{\Delta-\Delta_{rs}(c)}\right)^{-1}.$$

The exact form of this coefficient was "guessed" on the basis of higher equations of motion in $N=1$ supersymmetric Liouville theory by Belavin and Zamolodchikov [15]:

$$A_{rs}(c) \;=\; \frac{1}{2}(-1)^{rs-1}\prod_{m=1-r}^{r}\prod_{n=1-s}^{s}\left(\frac{1}{\sqrt{2}}\left(r\beta-\frac{s}{\beta}\right)\right)^{-1}, \qquad m+n\in 2\mathbb{Z}\,,\;\;(m,n)\neq(0,0),(r,s)\,.$$

The last missing information necessary for calculating the blocks recursively is the exact form of the regular terms in (5), which can be determined from the behavior of the 4-point blocks for large Δ or c, respectively. In the case of c-dependence, nonsingular term is simply a limit of the block for $c\to\infty$. This follows from the fact that the blocks coefficients depend on c only through inverse Gram matrix and thus they are given by non positive power of c.

As discussed in [13] the large Δ behavior of blocks is much more complicated. However, the recursive methods of approximate determination of general 4-point blocks are more efficient in that case.

The essential observation for deriving the large Δ asymptotic of the block made by Zamolodchikov [2–4] is the following: to write down the block's asymptotic it is necessary to study the classical limit of the block. The first two terms of the expansion of classical block in terms of large classical intermediate weight δ fully determine the dependence on external weights and central charge of the first two terms in the $\frac{1}{\Delta}$ expansion of conformal quantum block. It was also indicated that the classical limit of the block can be investigated by analyzing Liouville theory.

Analogical reasoning works for the superconformal quantum blocks. Studying the supersymmetric Liouville theory for $b\to 0\,,\; b^2\Delta_i\to\delta_i$ one can found out that there is one universal classical block $f_\delta\left[\begin{smallmatrix}\delta_3 & \delta_2\\ \delta_4 & \delta_1\end{smallmatrix}\right](z)$ for every type of quantum superconformal block:

$$\mathcal{F}^{1}_{\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right](z)\sim e^{(1/2b^2)f_\delta\left[\begin{smallmatrix}\delta_3 & \delta_2\\ \delta_4 & \delta_1\end{smallmatrix}\right](z)},\quad \mathcal{F}^{1/2}_{\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right](z)\sim b^2 e^{(1/2b^2)f_\delta\left[\begin{smallmatrix}\delta_3 & \delta_2\\ \delta_4 & \delta_1\end{smallmatrix}\right](z)}\,.$$

The contribution from fermions is noticeable just in the power of b in coefficient proportional to the exponent, what makes the difference in behavior of even or odd block.

From the $1/\delta$ expansion of the classical block follows the form of asymptotics:

$$\mathcal{F}^{1,1/2}_{\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right](z) \;=\; (16q)^{\Delta-\frac{c-3/2}{24}}\, z^{\frac{c-3/2}{24}-\Delta_1-\Delta_2}\,(1-z)^{\frac{c-3/2}{24}-\Delta_2-\Delta_3} \times \theta_3^{\frac{c-3/2}{2}-4(\Delta_1+\Delta_2+\Delta_3+\Delta_4)}\,\mathcal{H}^{1,1/2}_{\Delta}\left[\begin{smallmatrix}\Delta_3 & \Delta_2\\ \Delta_4 & \Delta_1\end{smallmatrix}\right](q)\,,$$

where $q = e^{i\pi\tau}$ is the elliptic nome, $\tau(z) = i\frac{K(1-z)}{K(z)}$ and $K(z)$ is the complete elliptic integral of the first kind. The elliptic blocks can be written as series in nome:

$$\mathcal{H}^{1,1/2}_{\Delta}\begin{bmatrix}\Delta_3 & \Delta_2 \\ \Delta_4 & \Delta_1\end{bmatrix}(q) \;=\; \sum_f (16q)^f H^f_{c,\Delta}\begin{bmatrix}\Delta_3 & \Delta_2 \\ \Delta_4 & \Delta_1\end{bmatrix},$$

where f is non negative for even block and half-integer for odd block. The elliptic blocks have the same structure as the superconformal ones and therefore one can derive the recursion relations for their coefficients. Moreover, the expansion in elliptic nome has better properties of convergence in comparison with z-expansion.

The elliptic recurrence representation takes the form:

$$\begin{aligned} H^f_{c,\Delta}\begin{bmatrix}\Delta_3 & \Delta_2 \\ \Delta_4 & \Delta_1\end{bmatrix} \;=\; & g_f + \sum_{\substack{r,s>0 \\ r,s\in 2\mathbb{N}}} \frac{A_{rs}(c)\, P^{rs}_c\begin{bmatrix}\Delta_3 \\ \Delta_4\end{bmatrix} P^{rs}_c\begin{bmatrix}\Delta_2 \\ \Delta_1\end{bmatrix}}{\Delta - \Delta_{rs}} H^{f-\frac{rs}{2}}_{\Delta_{rs}+\frac{rs}{2}}\begin{bmatrix}\Delta_3 & \Delta_2 \\ \Delta_4 & \Delta_1\end{bmatrix} \\ & - \sum_{\substack{r,s>0 \\ r,s\in 2\mathbb{N}+1}} \frac{A_{rs}(c)\, P^{rs}_c\begin{bmatrix}*\Delta_2 \\ \Delta_1\end{bmatrix} P^{rs}_c\begin{bmatrix}*\Delta_2 \\ \Delta_1\end{bmatrix}}{\Delta - \Delta_{rs}} H^{f-\frac{rs}{2}}_{\Delta_{rs}+\frac{rs}{2}}\begin{bmatrix}\Delta_3 & \Delta_2 \\ \Delta_4 & \Delta_1\end{bmatrix}, \end{aligned} \tag{8}$$

where nonsingular terms

$$g^1(q) = \sum_{m\in\mathbb{N}} (16q)^m g_m\,, \qquad\qquad g^{1/2}(q) = \sum_{k\in\mathbb{N}-\frac{1}{2}} (16q)^k g_k\,,$$

are independent of external weights and central charge. Thus they can be determined from some explicit formulae of elliptic blocks calculated for given external weights and c. The examples of such blocks for equal external weights $\Delta_0 = \frac{1}{8}$, $c = \frac{3}{2}$ and arbitrary intermediate weight were derived in [14]:

$$\mathcal{H}^1_{\Delta}\begin{bmatrix}\Delta_0 & \Delta_0 \\ \Delta_0 & \Delta_0\end{bmatrix}(q) \;=\; \theta_3(q^2)\,, \qquad \mathcal{H}^{1/2}_{\Delta}\begin{bmatrix}\Delta_0 & \Delta_0 \\ \Delta_0 & \Delta_0\end{bmatrix}(q) \;=\; \frac{1}{\Delta}\,\theta_2(q^2)\,.$$

Thus one can easily read off the regular terms $g^1(q) = \theta_3(q^2) = \sum_{n=-\infty}^{\infty} q^{2n^2}$, $g^{1/2}(q) = 0$, which for the type of blocks discussed here are the same as conjectured in [16].

4. Conclusions

We presented a short sum up of the results concerning the $N = 1$ Neveu–Schwarz superconformal blocks that appear in factorization of the correlation function of four superprimary fields. One should emphasize that there are tree more types of NS superconformal blocks, all defined in [11] where the properties of each block are analyzed and recursion relations for blocks' coefficients of expansion in z are derived. Analogical to presented here arguments lead to the efficient elliptic recurrence representation of all NS superconformal blocks [13,14]. The results allowed for verification of the bootstrap in NS sector of supersymmetric Liouville field theory what was done by Belavin, Belavin, Neveu and Zamolodchikov [16, 17].

The natural continuation of the studies on superconformal blocks is the problem of blocks in Ramond sector of $N = 1$ SCFT. The analysis is more difficult that in NS sector because the Ward identities have much more complicated form in that case. The main problem is that correlation functions of one fermionic current, two Ramond fields and one NS field are double valued. For this reason the structure and properties of 3-point superconformal Ramond blocks are complicated and the definition of 4-point block is not straightforward. Nevertheless this problem found recently its solution in work [18], where the Ramond blocks are defined and the elliptic recurrence for them is derived. These results can be used for the check of the consistency of $N = 1$ super Liouville theory [19] or its $c \to \frac{3}{2}$ limit [20] in both Ramond and NS sectors.

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