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BPS STATES IN PERTURBATIVE $\mathcal{N}=4$ SYM*

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The partition function of 1/16 BPS states in $\mathcal{N}=4$ SYM is found using the one loop dilatation operator. The result matches precisely the AdS/CFT prediction, *i.e.* it coincides with the partition function of the gas of supergravitons in $\mathrm{AdS}_5\times\mathrm{S}^5$.

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1. Introduction

AdS/CFT correspondence [1] is one of the most exciting and intensively studied subjects in modern theoretical physics (see *e.g.* [2] for the reviews). According to the conjecture a certain ($\mathcal{N}=4$ supersymmetric) Yang–Mills theory in 4 dimensional Minksowski space is equivalent to one of the superstring theories (type IIB) in 10 dimensional, curved background — $\mathrm{AdS}_5\times\mathrm{S}^5$. The conjecture is an example of the strong–weak coupling duality *i.e.* it translates the difficult (nonperturbative) problems on the gauge theory side into doable (perturbative) ones on the string theory side and *vice versa.* Consequently it is a hard problem to fully verify the conjecture if one is left entirely with perturbative expansion tools. However, there is a distinguished sector of field configurations, the BPS operators, that do not receive quantum corrections and hence play an important role in studying AdS/CFT correspondence nonperturbatively. These states, by definition, preserve some fractions of supersymmetry. Since the symmetries on the gauge theory side are exactly the symmetries of the superstring in $\mathrm{AdS}_5\times\mathrm{S}^5$, there is a one-to-one correspondence between BPS states in $\mathcal{N}=4$ SYM

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and superstring theory, respectively. Of particular interest are the 1/16 BPS configurations which preserve the least amount of supersymmetries. At low energies and strong coupling the partition function of gauge theory 1/16 BPS states should correspond to the 1/16 BPS supergravity configurations, while at high energies to the 1/16 BPS black holes.

Surprisingly, it was found [3] that the partition function of 1/16 BPS states in free $\mathcal{N}=4$ SYM overcounts the states as compared to the supegravity prediction (both in high and low energy regimes). However, it was also pointed out that once the gauge theory interaction is turned on, there should be no disagreement. In this paper we focus on [4] where it was argued that this is precisely the case, by using the complete one-loop result [5] for the dilatation operator.

2. Preliminaries

The $\mathcal{N}=4$ SYM on-shell field are the gauge field A^a_μ, $\mu=0,\ldots,3$, $a=1,\ldots,N^2-1$, the real scalar field ϕ^a_{ij}, $i=1,\ldots,4$ (in the antisymmetric representation of SU(4)), and the complex fermionic field (Weyls fermions) $\psi^a_{\alpha i}$, $\alpha=1,2$ — all in the adjoint representation of SU(N). The Lagrangian is given by $\mathrm{Tr}\,\mathcal{L}$ where

$$\mathcal{L}=-\frac{1}{4}F^2_{\mu\nu}-\frac{1}{2}D_\mu\phi_{ij}D^\mu\phi^{ij}-g^2[\phi_{ij},\phi_{kl}][\phi^{ij},\phi^{kl}]-\frac{1}{2}\bar{\psi}_i D\psi^i+ig\bar{\psi}^i[\phi_{ij},\psi^j]$$

and can be obtained *e.g.* after reducing the $\mathcal{N}=1$ SYM in 10 spacetime dimensions to 4 spacetime dimensions. The theory has the global SU(4) (= SO(6)) symmetry and the superconformal symmetry. Accordingly, any state can be labeled by the appropriate quantum numbers D, (j_1,j_2) and (R_1,R_2,R_3) corresponding to the dilatation operator, Lorentz spins and SU(4) Cartan generators, respectively.

Any gauge invariant operator in $\mathcal{N}=4$ SYM can be presented as a linear combination of products of single-trace operators. The latter are obtained by taking a trace over an arbitrary product of ϕ_{ij}, $\psi_{\alpha i}$, $\bar{\psi}^i_{\dot\alpha}$, $\mathcal{F}_{\alpha\beta}$, $\bar{\mathcal{F}}_{\dot\alpha\dot\beta}$ (selfdual, anti-selfdual components of field-strength in spinor notation) and $D_{\alpha\dot\beta}$ (the covariant derivative in spinor notation).

2.1. BPS operators

Among the generators of the superconformal symmetry (which consists of the Poincaré generators P_μ, $M_{\mu\nu}$, the special conformal transformation K_μ, the dilatation operator H, the supersymmetry charges $Q^{\alpha i}$, $\bar{Q}^{\dot\alpha}_i$ and the special conformal supercharges $S_{\alpha i}$, $\bar{S}^i_{\dot\alpha}$) the $Q^{\alpha i}$'s and the $S_{\alpha i}$'s play an important role in the discussion of BPS operators. Let us consider a BPS operator O preserving the least amount of supersymmetries (the 1/16 BPS operators) *i.e.* $[Q/\bar{Q},O]=[S/\bar{S},O]=0$ for some $\alpha,\dot{\alpha},i$. It follows from superconformal algebra that

$$\{S_{\alpha i},Q^{\beta j}\}=\delta_i^j(J_1)_\alpha^\beta+\delta_\alpha^\beta R_i^j+\frac{1}{2}\delta_i^j\delta_\alpha^\beta H\,, \tag{1}$$

where $(J_1)_\alpha^\beta$ are SU(2) generators in spinor notation and R_i^j are the SU(4) generators. Therefore, a consistency condition $[\{Q,S\},O]=\{Q,[S,O]\}+\{S,[Q,O]\}=0$ yields a new constraint for BPS operators.

2.2. The oscillator picture

Due to equations of motion and Bianchi identities, gauge invariant operators are in general algebraically dependent. To avoid this it is helpful to introduce a bosonic $a_\alpha^\dagger$, a_α, $b_{\dot{\alpha}}^\dagger$, $b_{\dot{\alpha}}$ and fermionic $c_i^\dagger$, c_i creation/annihilation operators which provide the Fock space representation $\mathcal{H}$ of the full symmetry of $\mathcal{N}=4$ SYM [6]. Furthermore, one can find a dictionary between states in $\mathcal{H}$ and gauge invariant operators according to [5]

$$\begin{aligned}
D_{\alpha\dot{\beta}}\dots D_{\gamma\dot{\delta}}\mathcal{F}_{\pi\sigma} &\longleftrightarrow a_\alpha^\dagger b_{\dot{\beta}}^\dagger\dots a_\gamma^\dagger b_{\dot{\delta}}^\dagger a_\pi^\dagger a_\sigma^\dagger|0\rangle\,,\\
D_{\alpha\dot{\beta}}\dots D_{\gamma\dot{\delta}}\psi_{\pi i} &\longleftrightarrow a_\alpha^\dagger b_{\dot{\beta}}^\dagger\dots a_\gamma^\dagger b_{\dot{\delta}}^\dagger a_\pi^\dagger c_i^\dagger|0\rangle\,,\\
D_{\alpha\dot{\beta}}\dots D_{\gamma\dot{\delta}}\phi_{ij} &\longleftrightarrow a_\alpha^\dagger b_{\dot{\beta}}^\dagger\dots a_\gamma^\dagger b_{\dot{\delta}}^\dagger c_i^\dagger c_j^\dagger|0\rangle\,,\\
D_{\alpha\dot{\beta}}\dots D_{\gamma\dot{\delta}}\bar{\psi}_{\dot{\pi}i} &\longleftrightarrow a_\alpha^\dagger b_{\dot{\beta}}^\dagger\dots a_\gamma^\dagger b_{\dot{\delta}}^\dagger b_{\dot{\pi}}^\dagger\epsilon^{ijkl}c_j^\dagger c_k^\dagger c_l^\dagger|0\rangle\,,\\
D^k\bar{\mathcal{F}}\sim D_{\alpha\dot{\beta}}\dots D_{\gamma\dot{\delta}}\bar{\mathcal{F}}_{\dot{\pi}\dot{\sigma}} &\longleftrightarrow a_\alpha^\dagger b_{\dot{\beta}}^\dagger\dots a_\gamma^\dagger b_{\dot{\delta}}^\dagger b_{\dot{\pi}}^\dagger b_{\dot{\sigma}}^\dagger\epsilon^{ijkl}c_i^\dagger c_j^\dagger c_k^\dagger c_l^\dagger|0\rangle\,,
\end{aligned} \tag{2}$$

where $|0\rangle$ is the Fock vacuum (corresponding to field = 1).

In the planar limit, the correlation functions involving multiple trace operators factorizes into a product of correlation functions corresponding to single trace operators. Therefore, it is enough to consider operators of the form $\mathrm{Tr}(\chi_1\dots\chi_L)$ where χ_s is any, out of 5, operators listed in (2).

In the Fock space representation it is convenient to introduce the site index s: $a_\alpha^\dagger$, $b_{\dot{\alpha}}^\dagger$, $c_i^\dagger\to a_{\alpha s}^\dagger$, $b_{\dot{\alpha}s}^\dagger$, $c_{is}^\dagger$, $\mathcal{H}\to\mathcal{H}_s$ which corresponds to the position of the operator inside the trace, $s=1,\dots,L$. Now, a generic gauge invariant single trace operator, with L sites, is simply an element of $\otimes_{s=1}^L\mathcal{H}_s$. We note, however, that there is a constraint (the central charge constraint) on bosonic and fermionic occupation numbers namely at each site s

$$C=n_{a_1}+n_{a_2}-n_{b_1}-n_{b_2}+n_{c_1}+n_{c_2}+n_{c_3}+n_{c_4}-2=0\,,$$

(where n_x is the number of quanta corresponding to the operator x) which can be seen from (2).

2.3. The one loop dilatation operator

In conformal filed theories, a two point correlation function $\langle \bar{O}(x)O(y)\rangle$ of an arbitrary operator O, must be proportional to $|x-y|^{-2D}$. It is convenient to split the dimension D of the operator O as $D = D_0 + \gamma$ where D_0 is the bare dimension and γ is the anomalous part.

An important result in planar $\mathcal{N} = 4$ SYM is the exact calculation of γ at one loop in perturbation theory [5, 7] with

$$\gamma = \lambda \,\mathrm{spectrum}(H_{1-\mathrm{loop}})\,, \qquad H_{1-\mathrm{loop}} = \frac{1}{8\pi^2}\sum_{s=1}^{L} H_{s,s+1}\,, \qquad \lambda = g^2 N\,,$$

where $H_{1-\mathrm{loop}}$ is the Hamiltonian whose Fock space representation is obtained in the following way. The action of $H_{s,s+1}$ introduces an interaction between *only* the neighboring sites (where the last site and the first site are assumed to be neighbors) and is the same for all s. For this reason it is enough to consider an arbitrary pair of such sites $|v\rangle = |w_1\rangle \ldots |w_s\rangle|w_{s+1}\rangle \ldots |w_L\rangle$. In order to calculate $H_{s,s+1}|v\rangle$ one first considers all oscillator hoppings form $|w_s\rangle$ to $|w_{s+1}\rangle$ (and form $|w_{s+1}\rangle$ to $|w_s\rangle$) so that the resulting state has the same central charge. Each state obtained in this way is multiplied by

$$c_{n,n_{12},n_{21}} = (-1)^{1+n_{12}n_{21}} \frac{\Gamma\left(\frac{1}{2}n_{12} + \frac{1}{2}n_{21}\right)\Gamma\left(1 + \frac{1}{2}n - \frac{1}{2}n_{12} - \frac{1}{2}n_{21}\right)}{\Gamma\left(1+\frac{1}{2}n\right)}\,,$$

where n_{12} and n_{21} are the numbers of oscillators hopping from $|w_s\rangle$ to $|w_{s+1}\rangle$ and form $|w_{s+1}\rangle$ to $|w_s\rangle$, respectively, n is the number of all quanta in $|w_s\rangle|w_{s+1}\rangle$.

3. BPS states at zero coupling

The operator $\{S_{\alpha i}, Q^{\alpha i}\}$ (no sum) has an important property namely (semi)positive-definiteness. This can be seen by noting that $(Q^{\alpha i})^\dagger = S_{\alpha i}$ (which in turn follows from the condition $P_\mu = K_\mu^\dagger$ that one has to impose in quantization of conformal field theories). Let us introduce $\varDelta := 2\{S, Q\}$ where $S = S_{-1/2,1}$ and $Q = Q^{-1/2,1}$. It is clear that $\varDelta|s\rangle = 0$ if $|s\rangle$ is the 1/16 BPS state however, since $Q = S^\dagger$ the converse is also true namely (following the standard argument) if $\varDelta|s\rangle = 0$ then $\langle s|\varDelta|s\rangle = \|Q|s\rangle\|^2 + \|S|s\rangle\|^2 = 0$ hence $Q|s\rangle = S|s\rangle = 0$.

Therefore, according to (1), the condition for 1/16 BPS states becomes

$$\varDelta = H - 2(J_1)^1_1 + (R_3)^1_1 = H - 2J_1 - \frac{1}{2}(3R_1 + 2R_2 + R_3) = 0\,,$$

where $J_1 = (J_1)^1_1$ and R_i are SU(4) Cartan generators (see Appendix A in [3] for more details). The partition function of 1/16 BPS states can, therefore, be written as function of 5 variables x, z, y, v, w corresponding to H, J_1, J_2, R_2, R_3, respectively

$$Z_{\frac{1}{16}\mathrm{BPS}} = \sum_{\Delta=0} x^{2H} z^{2J_1} y^{2J_2} u^{R_2} v^{R_3} \,. \tag{3}$$

There exists an elegant way to evaluate $Z_{1/16\mathrm{BPS}}$ in the planar, free $\mathcal{N} = 4$ SYM using only the letter partition function $z(x) = z_{\mathrm{B}}(x) + z_{\mathrm{F}}(x)$, $x = (x, z, y, u, v)$ (where we made a split into fermionic and bosonic part) of the theory [3,8,9]. The single trace partition function is given, according to Polya theorem, by

$$Z_{\mathrm{s.t.}} = -\sum_{n=1}^{\infty} \frac{\phi(n)}{n} \log\left(1 - z_{\mathrm{B}}(x^n) - (-1)^{n+1} z_{\mathrm{F}}(x^n)\right) \tag{4}$$

while the multiple trace partition function is

$$Z_{\mathrm{m.t.}} (= Z_{\frac{1}{16}\mathrm{BPS}}) = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} \left\{ Z^{\mathrm{B}}_{\mathrm{s.t.}}(x^n) + (-1)^{n+1} Z^{\mathrm{F}}_{\mathrm{s.t.}}(x^n) \right\}\right) . \tag{5}$$

We note that these formulas are applicable only in the $N = \infty$ case (moreover, formula (4) cannot be used in the interacting theory) however, in the case of free gauge theories there exists a possibility to obtain $Z_{\mathrm{m.t.}}$ directly from $z(x)$ for finite N, namely

$$Z^{(N)}_{\mathrm{m.t.}} = \int\limits_{U \in \mathrm{SU}(N)} DU \exp\left\{ \sum_{n=1}^{\infty} \left(z_{\mathrm{B}}(x^n) + (-1)^{n+1} z_{\mathrm{F}}(x^n)\right) \frac{\mathrm{Tr}\, U^n \,\mathrm{Tr}\, U^{-n}}{n} \right\} . \tag{6}$$

One can show [8,9] that in the large N limit, $Z^{(N)}_{\mathrm{m.t.}}$ becomes $Z_{\mathrm{m.t.}}$ if the parameters x are small. However, there exists a critical value of the parameter above which $\ln Z^{(N)}_{\mathrm{m.t.}} \sim N^2$ resembling the confinement/deconfinement phase transition [8,9].

According to AdS/CFT conjecture $Z_{\mathrm{m.t.}}$ should correspond to the partition function over the whole (including multi-particle states) Fock space in supergravity while $Z^{(N)}_{\mathrm{m.t.}}$ at large energies should correspond to the partition function of 1/16 BPS black holes (at least for the special values of parameters x considered in [3]). In [3] the partition functions $Z_{\mathrm{m.t.}}$ and $Z^{(N)}_{\mathrm{m.t.}}$ were calculated using (4)–(6) and found to be in disagreement with the supergravity prediction.

3.1. Back to the oscilator picture

In the next section we argue that the apparent disagreement can be cured (at least partly) once we consider 1/16 BPS operators at one-loop. It is useful for later purposes to rewrite the Lorentz spins and Cartan generators in terms of creation/annihilation operators discussed earlier (see Appendix A in [3] for more details)

$$\begin{aligned} R_1 &= n_{c_2} - n_{c_2}\,, \qquad R_2 = n_{c_3} - n_{c_2}\,, \qquad R_3 = n_{c_4} - n_{c_3}\,, \\ J_1 &= \frac{1}{2}(n_{a_2} - n_{a_1})\,, \qquad J_2 = \frac{1}{2}(n_{b_2} - n_{b_1})\,. \end{aligned} \tag{7}$$

The dilatation operator H_0 in free theory also has a similar representation

$$H_0 = n_{a_1} + n_{a_2} + \frac{1}{2}(n_{c_1} + n_{c_2} + n_{c_3} + n_{c_4})$$

hence the 1/16 BPS condition $\Delta = 0$ in free $\mathcal{N} = 4$ SYM becomes

$$\Delta_{\lambda=0} = 2n_{a_1} + 2n_{c_1} = 0\,.$$

Therefore, the 1/16 BPS states do not have any $a_1^\dagger$ and $c_1^\dagger$ operators in the Fock space representation. The partition function (3) can now be rewritten as

$$\begin{aligned} Z_{\text{m.t.}}(a_2, b_1, b_2, c_2, c_3, c_4) &= \sum_{\Delta=0, C=0} a_1^{n_{a_1}} a_2^{n_{a_2}} b_1^{n_{b_1}} b_2^{n_{b_2}} c_1^{n_{c_1}} c_2^{n_{c_2}} c_3^{n_{c_3}} c_4^{n_{c_4}} \\ &= \sum_{C=0} a_2^{n_{a_2}} b_1^{n_{b_1}} b_2^{n_{b_2}} c_2^{n_{c_2}} c_3^{n_{c_3}} c_4^{n_{c_4}}\,. \end{aligned} \tag{8}$$

It can be evaluated using (4), (5) and the letter partition function

$$z = \frac{a_2^2 + c_2c_3 + c_2c_4 + c_3c_4}{(1-b_1a_2)(1-b_2a_2)} + \frac{a_2(c_2+c_3+c_4) + (b_1+b_2-a_2b_1b_2)c_2c_3c_4}{(1-b_1a_2)(1-b_2a_2)}\,, \tag{9}$$

which follows directly from the counting over the Fock space states (taking into consideration the central charge constraint $C = 0$).

The functions (3) and (8) represent the same quantity and differ only by the change of variables. Using (7) one finds that the transformation from $(a_2, b_1, b_2, c_2, c_3, c_4)$ to (x, z, y, v, w) is

$$a_2 = x^2 z\,, \quad b_1 = \frac{1}{y}\,, \quad b_2 = y\,, \quad c_2 = \frac{x}{v}\,, \quad c_3 = \frac{xv}{w}\,, \quad c_4 = xw\,. \tag{10}$$

4. The one loop result

The dilatation operator at one loop takes the form $H = H_0 + \lambda H_{1-\text{loop}}$ where $H_{1-\text{loop}}$ is described in Section 2.3. Accordingly, the 1/16 BPS condition is

$$\Delta_{\lambda=0} + \lambda H_{1-\text{loop}} = 0\,. \tag{11}$$

Since (11) is supposed to be true for an arbitrary small λ one concludes that the condition is actually $\Delta_{\lambda=0} = 0$ *and* $H_{1-\text{loop}} = 0$. Therefore, the 1/16 BPS partition function becomes

$$Z_{\text{m.t.}}(a_2, b_1, b_2, c_2, c_3, c_4) = \sum_{H_{1-\text{loop}}=0} a_2^{n_{a_2}} b_1^{n_{b_1}} b_2^{n_{b_2}} c_2^{n_{c_2}} c_3^{n_{c_3}} c_4^{n_{c_4}}\,.$$

The letter partition function z is of little use at one loop, however, one can still use $Z_{\text{s.t.}}$ and (5) to obtain $Z_{\text{m.t.}}$. The single trace partition function is obtained from

$$Z_{\text{s.t.}} = z + \sum_{L=2}^{\infty} Z_{\text{s.t.}}^{(L)}, \tag{12}$$

where $Z_{\text{s.t.}}^{(L)}$ is the partition function of 1/16 BPS states corresponding to L sites. In order to determine $Z_{\text{s.t.}}^{(L)}$ one can use the one-loop dilatation operator described in Section 2.2. The condition $H_{1-\text{loop}} = 0$ is satisfied only for states which are eigenstates corresponding to 0 eigenvalue of $H_{1-\text{loop}}$. In general, a number $D_{n_{a_2},n_{b_1},n_{b_2},n_{c_2},n_{c_3},n_{c_4},L}$ of such states in the sector with n_{a_2}, n_{b_1}, n_{b_2}, n_{c_2}, n_{c_3}, n_{c_4} number of quanta and L sites respectively, could be obtained by looking at the spectrum of $H_{1-\text{loop}}$ and then finding its kernel. With use of the computer code implementation of $H_{1-\text{loop}}$, we determined the numbers $D_{n_{a_2},n_{b_1},n_{b_2},n_{c_2},n_{c_3},n_{c_4},L}$ exactly for several thousands of cases. The generating function we are looking for

$$Z_{\text{s.t.}}^{(L)} = \sum_{\substack{n_{a_2},n_{b_1},n_{b_2}=0,\ldots,\infty \\ n_{c_2},n_{c_3},n_{c_4}=0,\ldots,L}} D_{n_{a_2},n_{b_1},n_{b_2},n_{c_2},n_{c_3},n_{c_4},L}\, a_2^{n_{a_2}} b_1^{n_{b_1}} b_2^{n_{b_2}} c_2^{n_{c_2}} c_3^{n_{c_3}} c_4^{n_{c_4}}\,,$$

(the sum over fermionic variables runs from 0 to L due to the Pauli exclusion principle) is, therefore, found up to some powers in the Taylor expansion. It is not clear that such data can determine the whole function $Z_{\text{s.t.}}^{(L)}$, nevertheless our analysis shows that the Taylor expansion of $Z_{\text{s.t.}}^{(L)}$ coincides with the expansion of certain rational function. After some guess work we found that (see [4] for more details)

$$Z_{\text{s.t.}}^{(L)} = \frac{P}{(1-a_2 b_1)(1-a_2 b_2)}\,, \tag{13}$$

$$
\begin{aligned}
P \;=\; & \sigma_{L,L,0} + a_2\sigma_{L,L-1,0} + a_2^2\sigma_{L-1,L-1,0} \\
& + (b_1+b_2)\left(\sigma_{L,L,1} + a_2\sigma_{L,L-1,1} + a_2^2\sigma_{L-1,L-1,1}\right) \\
& + b_1b_2\left(\sigma_{L,L,2} + a_2\sigma_{L,L-1,2} + a_2^2\sigma_{L-1,L-1,2}\right) \,, \qquad (14)
\end{aligned}
$$

where $\sigma_{n_1,n_2,n_3} = \sigma_{n_1,n_2,n_3}(c_2,c_3,c_4)$ is the Schur polynomial.

Remarkably enough, substituting (9), (13) to (12) and then changing the variables *via* (10) one recovers precisely the supergraviton partition function obtained in [3]! The construction of $Z_{\rm m.t}$ from $Z_{\rm s.t.}$ is mathematically the same in gauge theory side and in supergravity side, therefore, we obtain a complete agreement with AdS/CFT correspondence.

5. Outlook

In this paper we discussed the BPS sector of $\mathcal{N}=4$ SYM and its relation to supergravity. According to the AdS/CFT conjecture there should be a one-to-one correspondence between the BPS states on both sides, regardless of the energy regimes. It turns out that the corresponding generating functions do not match if in the gauge theory calculation one does not consider the interaction [3]. However, as argued in [4], when the interaction is turned on one finds a complete agreement in low energy regimes. The result is obtained using the oscillator representation of the complete one loop dilatation operator [5]. We note that a crucial step in [4] is Eq. (13) which in turn was guessed supported by the computer analysis. Therefore, in view of future perspectives, one would like to find a rigorous derivation of (13) or of what follows.

Even more important is the question about the high energy regimes. This is an outstanding problem which has not been solved so far. At high energies, the partition function of 1/16 BPS configurations on the supergravity side is presumably dominated by the 1/16 BPS black hole solutions and the corresponding free energy $F = \ln Z_{\rm blackholes}$ scales like N^2. Unfortunately it is difficult, if at all possible, to obtain this scaling using the results from the *planar* gauge theory, therefore, it seems that the non-planar analysis is required [4].

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