NEGATIVE RADIATION PRESSURE IN THE CASE OF TWO INTERACTING FIELDS*

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The new mechanism of an interesting phenomenon of the negative radiation pressure is presented. Force exerted by radiation on the kink in a simple toy model is calculated using perturbation scheme. The results are compared with numerical calculations. The interaction of vortices and radiation is discussed and possible explanation of the negative radiation pressure is examined.

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1. Introduction

Among many other features, some topological defects reveal a very interesting phenomenon which we have called the negative radiation pressure (NRP). When exposed to some kind of radiation most topological defects are pushed by the radiation in a process which is very similar to the radiation pressure known from electrodynamics. However, certain defects in certain theories behave in a completely different way. They accelerate towards the source of radiation. The importance of the negative radiation pressure is not yet well understood, however we believe that it may be very important in many processes including stability of systems of topological defects. The explanation of the phenomenon is rather simple. During the scattering process incoming wave are transformed into waves which carry more momentum than the initial ones. The surplus of momentum is passed on the defect.

Currently we know three mechanisms responsible for the appearance of the NRP. The first mechanism was discovered in the very well known and studied model of ϕ^4 scalar field [1, 2]. The theory exhibits a static kink

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solution. Linear analysis of small excitations around this kink solution shows that the kink is transparent to the waves (this is a very rare feature of so called reflectionlessness). However, higher order corrections show, that some parts of the waves are transformed into waves with twice the frequency of the initial wave. They carry away more momentum. The surplus of the momentum must be compensated with kink's motion. The kink experiences a force which pulls it towards the source of radiation. This phenomenon is purely nonlinear and the force is proportional to the forth power of amplitude of the wave. The phenomenon was proved to be robust with respect to small perturbation of the field-theory potential.

The second mechanism requires two interacting fields with different masses. During scattering of more massive field over the defect some energy is transformed into waves of less massive field. These waves carry more momentum and, in favourable conditions, again the surplus of momentum can be observed and the motion of kink must compensate it. This can happen even in the linear approximation and the force is proportional to square of amplitude of incoming wave. This situation does not require full transparency of a defect and seems to be more common. It was numerically seen in case of vortices in Goldstone and Abelian Higgs models.

The third process does not involve static objects but rather oscillating in time. Our preliminary numerical results show that the negative radiation pressure exists also for oscillons. In $1+1d$ theories oscillons can be treated as a bound state of two kinks [3]. They are similar to breathers in integrable model described by sine-Gordon equation. In contrary to the breathers they radiate and finally decay to vacuum, but their lifetime is extremely long. When a wave interacts with a localized oscillon, waves with frequencies equal to sum and difference of an oscillon and wave frequencies are created. Sometimes this can also lead to creation of surplus of momentum and NRP.

In the present paper we will focus on the second mechanism which in our opinion should be the most frequent.

The paper is organized as follows. First we introduce a simple toy model on which we show how the mechanism of NRP works. Because of its simplicity it is quite easy to obtain some perturbative analytic results which can be compared with numerical solutions of the full partial differential equation. In the fourth section we discuss our preliminary results in case of vortices. We point out the similarities to the toy model and present the numerical results.

2. A toy model

As we stated in the introduction there are at least three different mechanisms leading to the negative radiation pressure. One of them, which we believe is the most common, is the one involving two scalar fields with different masses: $m_1 > m_2$. An intuitive picture can be following. Suppose that the object (*i.e.* topological defect) is hit with a wave with the larger mass. During scattering process some part of the energy is transferred to the field with smaller mass. Reflection always pushes the object but sometimes more radiation goes through the defect and reflected part is negligible.

The incoming wave caries energy and momentum, flow of which can be expressed as:

$$\dot{\mathcal{E}}_1 = \frac{1}{2}A^2\omega\sqrt{\omega^2 - m_1^2}\,, \qquad \dot{\mathcal{P}}_1 = \frac{1}{2}A^2(\omega^2 - m_1^2)\,. \tag{1}$$

For simplicity let us assume that the whole energy goes to the field with smaller mass, m_2. After scattering the flows are following:

$$\dot{\mathcal{E}}_2 = \frac{1}{2}B^2\omega\sqrt{\omega^2 - m_2^2}\,, \qquad \dot{\mathcal{P}}_2 = \frac{1}{2}B^2(\omega^2 - m_2^2)\,. \tag{2}$$

Because of energy conservation law $\dot{\mathcal{E}}_1 = \dot{\mathcal{E}}_2$ which gives the square of the amplitude of the scattered wave

$$B^2 = \sqrt{\frac{\omega^2 - m_1^2}{\omega^2 - m_2^2}}A^2\,. \tag{3}$$

Note that if the kink is stationary at the beginning ($v = 0$) we do not need to take its kinetic energy to account $\dot{E} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\dot{v} = 0$. Now we can compare the momentum flow:

$$\dot{\mathcal{P}}_2 = \frac{1}{2}\sqrt{\left(\omega^2 - m_2^2\right)\left(\omega^2 - m_1^2\right)}A^2\,. \tag{4}$$

If $m_1 > m_2$ then $\dot{\mathcal{P}}_2 > \dot{\mathcal{P}}_1$ and the surplus of momentum will be seen as a force exerted on the defect. In this case the force will be pushing the defect towards the direction from where the radiation came. This is a case of the negative radiation pressure. If the defect would be hit with a wave with smaller mass (in case when $m_1 < m_2$) it would be simply pushed by radiation and ordinary positive radiation pressure could be observed.

Now let us discuss a full field-theoretic toy model example where the above situation appears. This is a model of two interacting scalar fields in 1+1 dimensions [4]. Without interaction one of them describes the standard ϕ^4 theory and the second is governed by Klein–Gordon equation. We add the interaction term which couples the two fields. Lagrangian can be written as:

$$\mathcal{L} = \tfrac{1}{2}\left(\phi_t^2 - \phi_x^2\right) + \tfrac{1}{2}\left(\psi_t^2 - \psi_x^2\right) - \tfrac{1}{2}\bigg((\phi^2 - 1) + \kappa\psi\bigg)^2 - \tfrac{1}{2}\left(m^2 - \kappa^2\right)\psi^2\,, \tag{5}$$

or equivalently

$$\mathcal{L} = \tfrac{1}{2}\left(\phi_t^2 - \phi_x^2 - (\phi^2-1)^2\right) + \tfrac{1}{2}\left(\psi_t^2 - \psi_x^2 - m^2\psi^2\right) - \kappa(\phi^2-1)\psi\,, \quad (6)$$

where m is the mass of ψ field and κ is the coupling constant. For small values of $\kappa \ll m$, vacuum consists of merely two disconnected points ($\phi = \pm 1$, $\psi = 0$) revealing $\mathbb{Z}_2$ symmetry and domain walls (in general) or kinks (in one-dimensional models) are possible.

The equations of motion are:

$$\begin{cases} \phi_{tt} - \phi_{xx} + 2\phi(\phi^2-1) + 2\kappa\phi\psi = 0\,, \\ \psi_{tt} - \psi_{xx} + m^2\psi + \kappa(\phi^2-1) = 0\,. \end{cases} \quad (7)$$

The ϕ-part of static solution with non-zero topological charge is a kink which is very similar to the one in ϕ^4 theory. The ψ-part is a bell-shaped function similar to Gauss function (see Fig. 1). However, for $\kappa \neq 0$ no analytic solutions are known.

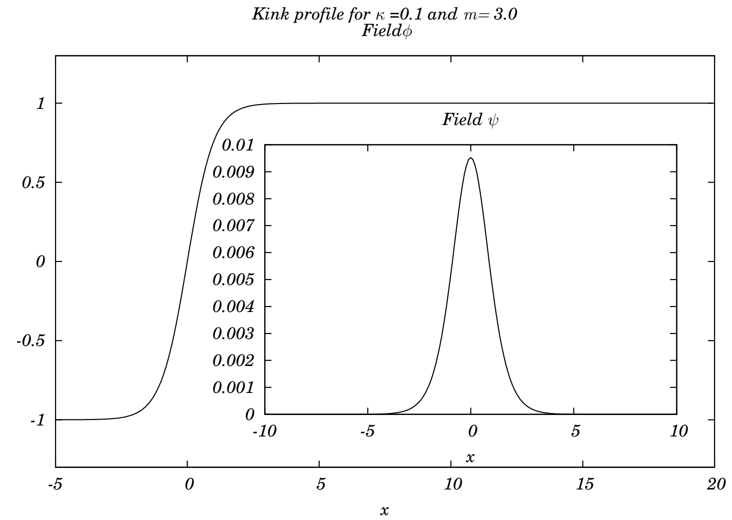

Fig. 1. Static solutions (numerical) for the kink for $\kappa = 0.1$ and $m = 3.0$.

Let us consider a process when the kink is hit with a ψ-field wave with frequency ω coming from $+\infty$. In order to calculate the leading contribution to the radiation pressure we do not need to know the full kink solution. All we need is the perturbative series in the coupling constant κ:
$\phi = \phi^{(0)} + \kappa\phi^{(1)} + \kappa^2\phi^{(2)} + \cdots$ and $\psi = \psi^{(0)} + \kappa\psi^{(1)} + \kappa^2\psi^{(2)} + \cdots$.

In the order $\mathcal{O}(\kappa^0)$ the equations take the form:

$$\phi^{(0)}_{tt} - \phi^{(0)}_{xx} + 2\phi^{(0)}\left({\phi^{(0)}}^2 - 1\right) = 0\,, \tag{8a}$$

$$\psi^{(0)}_{tt} - \psi^{(0)}_{xx} + m^2\psi^{(0)} = 0\,. \tag{8b}$$

We choose the special solution to the above equations which satisfy our conditions: the static kink plus a travelling wave:

$$\phi^{(0)} = \tanh x\,, \qquad \psi^{(0)} = A\cos(kx + \omega t)\,, \tag{9}$$

where A is an amplitude of the wave and $k = \sqrt{\omega^2 - m^2}$ is a wave number. The first order equations are:

$$\phi^{(1)}_{tt} - \phi^{(1)}_{xx} + 2\left(3{\phi^{(0)}}^2 - 1\right)\phi^{(1)} + 2\phi^{(0)}\psi^{(0)} = 0\,, \tag{10a}$$

$$\psi^{(1)}_{tt} - \psi^{(1)}_{xx} + m^2\psi^{(1)} + \left({\phi^{(0)}}^2 - 1\right) = 0\,. \tag{10b}$$

The equation (10b) has only static inhomogeneous part so it gives only correction to the static solution of the kink.

Since $\psi^{(0)} = \frac{1}{2}e^{\mathrm{i}\omega t + kx} + \text{c.c.}$ the solution to the equation (10a) can be sought in the following form:

$$\phi^{(1)} = \tfrac{1}{2}e^{\mathrm{i}\omega t}\xi_+(x) + \tfrac{1}{2}e^{-\mathrm{i}\omega t}\xi_-(x)\,, \tag{11}$$

where ξ_+ is a solution to

$$\left(\frac{d^2}{dx^2} + q^2 + \frac{6}{\cosh^2 x}\right)\xi_+ = 2Ae^{\mathrm{i}kx}\tanh x\,, \tag{12}$$

with $q^2 = \omega^2 - 4$ and $\xi_- = \xi_+^*$. Fortunately we already know the solution η_q to the homogeneous part of the above equation (see [2])[1]

$$\eta_q(x) = \frac{3\tanh^2 x - 1 - q^2 - 3\mathrm{i}q\tanh x}{\sqrt{(q^2+1)(q^2+4)}}e^{\mathrm{i}qx}\,, \tag{13}$$

and we can use the Green function technique to construct the solution:

$$\xi_+(x) = -\frac{\eta_{-q}}{W}\int\limits_{-\infty}^{x} dx'\,\eta_q\left(x'\right)f\left(x'\right) - \frac{\eta_q}{W}\int\limits_{x}^{\infty} dx'\,\eta_{-q}\left(x'\right)f\left(x'\right)\,, \tag{14}$$

[1] Note that there is no reflected part proportional to $e^{-\mathrm{i}qx}$.

where $W = -2\mathrm{i}q$ is Wronskian and $f(x) = Ae^{\mathrm{i}kx}\tanh x$ is the r.h.s. of the equation (12). The solution for large values of x can therefore be written as:

$$\xi_+(x \to +\infty) = R(q,k)\eta_{-q}(x) - \frac{e^{\mathrm{i}kx}}{k^2 - q^2}\,, \tag{15}$$

where

$$R(q,k) = \frac{\pi\left(3k^2 - q^2 - 4\right)}{2q\sqrt{(q^2+1)(q^2+4)}\sinh\left(\frac{q+k}{2}\pi\right)} \tag{16}$$

is a reflection coefficient. For $x \to -\infty$ similar procedure leads to the following solution

$$\xi_+(x \to -\infty) = T(q,k)\eta_{-q}(x) + \frac{e^{\mathrm{i}kx}}{k^2 - q^2}\,, \tag{17}$$

where $T(q,k) = R(-q,k)$ is a transition coefficient. One can see that for $q = k \Leftrightarrow m = 2$ the transition coefficient becomes infinite. Of course, this means that our perturbation scheme fails in the vicinity of this point.

Another important observation is that for $q^2 = 3k^2 - 4 \Leftrightarrow \omega^2 = 3m^2/2$ the numerator vanishes and the kink becomes transparent.

2.1. The force

Having the asymptotic form of solution representing travelling wave

$$\phi(x,t) = \begin{cases} \frac{1}{2}A\kappa e^{\mathrm{i}\omega t}\Big(R(q,k)\eta_{-q}(x) - \frac{e^{\mathrm{i}kx}}{k^2-q^2}\Big) + \text{c.c.} & \text{for } x- \to +\infty \\ \frac{1}{2}Ab\kappa e^{\mathrm{i}\omega t}\Big(T(q,k)\eta_{q}(x) + \frac{e^{\mathrm{i}kx}}{k^2-q^2}\Big) + \text{c.c.} & \text{for } x- \to -\infty \end{cases} \tag{18a}$$

$$\psi(x,t) = \begin{cases} A\cos(\omega t + kx) & \text{for } x \to +\infty \\ Ab\cos(\omega t + kx) & \text{for } x \to -\infty \end{cases} \tag{18b}$$

we can calculate the force which is exerted on the kink by this wave. In the second equation we changed the amplitude from A to Ab. Actually as we will shortly show $b = 1 + \mathcal{O}(\kappa^2)$ so this correction is not visible at this point of perturbation series but it is necessary to fulfil the energy conservation law. If the kink is initially at rest, the rate of energy flowing into a large box containing the kink is given by

$$\left\langle \dot{E} \right\rangle_T = \langle \phi_t\phi_x + \psi_t\psi_x \rangle_T \Big|_{-L}^{L} = \frac{1}{2}\omega A^2\left(-q\kappa^2R^2 + k - kb^2 - q\kappa^2T^2\right)\,, \tag{19}$$

and since energy must be conserved ($\langle \dot{E} \rangle = 0$) we obtain

$$b^2 = 1 - \kappa^2\frac{q}{k}\left(R^2 + T^2\right)\,. \tag{20}$$

This correction to the amplitude of ψ wave would be obtained after calculating next step in our perturbation series but since the energy must be conserved we obtained this result as a consistency condition. (Similar result was discussed in more details in [2].) The force exerted on the kink can be calculated using the momentum conservation law:

$$F = \left\langle \dot{P} \right\rangle_T = \frac{1}{2} A^2 \left(-k^2 - \kappa^2 R^2 q^2 + b^2 k^2 + \kappa^2 T^2 \right) , \tag{21}$$

or using (20):

$$F = \frac{1}{2} A^2 q \left(T^2 (q-k) - R^2 (q+k) \right) . \tag{22}$$

The above force is proportional to A^2 (contrary to A^4 in ϕ^4 model [2]). Moreover, it can be positive (kink accelerates towards the source of radiation — negative radiation pressure) or negative (the kink accelerates with the wave due to the positive radiation pressure). Note that when $q < k$ *i.e.* $m < 2$ the force is always negative (or at most zero) whatever the coefficients R and T are. When $m > 2$ the direction in which the kink accelerates can be determined only after substituting the values of R and T. The negative radiation pressure appears for all $q > k > 0$.

3. Numerical calculations

As we have stated before, even in this simple model we do not know the exact analytical form of static solutions. Therefore, they have to be obtained numerically. We have done this using collocation Chebyshev spectral method in the variable $s = \tanh x$. In this way we obtained the solutions $\phi_0(x)$ and $\psi_0(x)$ depicted in the Fig. 1.

The next step was to find the initial condition representing the travelling ψ wave. Obtaining a solution satisfying all assumptions of the travelling wave is rather cumbersome. Therefore, we have decided to use only the approximation:

$$\phi(x, t=0) = \phi_0(x) , \qquad \dot{\phi}(x, t=0) = 0 , \tag{23}$$

and

$$\psi(x, t=0) = \psi_0(x) + A \cos(kx) , \qquad \dot{\psi}(x, t=0) = -A\omega \sin(kx) . \tag{24}$$

Disadvantage of the above initial data is that at the very beginning the kink has a no-zero initial velocity. This is due to the fact that the above initial conditions are combination of many eigenstates of the linearized equation around the kink, including a translational mode. However, this type of conditions show how robust and generic our considerations are.

We solved our PDE equations (7) using five-point discretization of the second spatial derivative and integrating the obtained system of ODEs with 4-th order Runge–Kutta method. The example trajectory of the kink is presented in Fig. 2. The trajectory of the kink is very similar to parabola (as expected) and we can fit it with a function in the form $X(t) = \frac{1}{2}at^2+vt+c$ obtaining its initial velocity v and acceleration a.

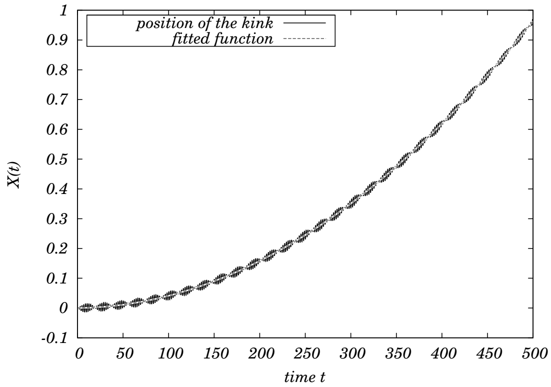

Fig. 2. Trajectory of the kink along with fitted parabola $X(t) = \frac{1}{2}at^2 + vt + c$ for $\kappa = 0.1$, $m = 2.5$, $A = 0.1$ and $\omega = 3.5$. The fitted values: $a = (7.57898 \pm 0.00069) \times 10^{-6}$, $v = (3.23 \pm 0.18) \times 10^{-5}$ and $c = (1.1 \pm 1.9) \times 10^{-4}$.

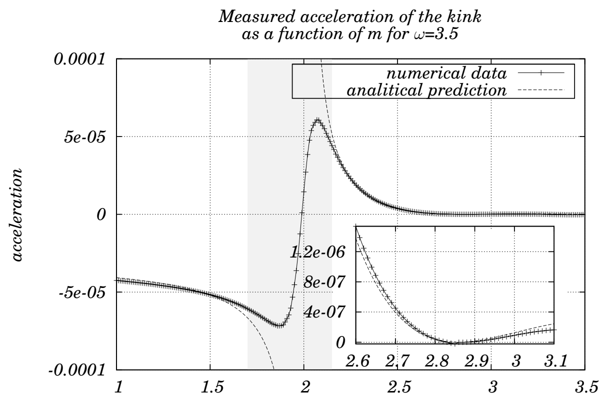

Fig. 3. Fitted acceleration compared with the analytical prediction as a function of m. $\omega = 3.5$, $\kappa = 0.1$, $A = 0.1$. The smaller figure shows a plot around $m = \sqrt{2/3}\,\omega = 2.86$ where the force vanishes.

In Fig. 3 we have depicted the measured acceleration as a function of mass parameter m together with theoretically predicted functions (the force from equation (22) divided by kink mass for $\kappa = 0$: $M_k = \frac{4}{3}$). This figure shows that analytical predictions are quite precise (outside the shaded region — vicinity of $m = 2$, when our perturbation scheme fails). Except this low frequency limit and the vicinity of $m = 2$ our theory coincides with numerical simulation with average error of a few percent which is satisfying having in mind all the assumptions and simplifications we used. We also confirmed that the leading term is proportional to $A^2\kappa^2$

4. Vortices

The toy model discussed in the previous sections showed that the negative radiation pressure appearing in the mechanism of two interacting field is quite easy to find. In contrary to the mechanism in ϕ^4 model, discussed in [2] where the transparency of kinks was needed, in our toy model it was sufficient that there were two interacting fields with different masses. Of course, not all theories of two fields reveal negative radiation pressure. Sometimes the reflection from the defect is simply too large. However, we can point out at least two theories which are of much physical interest were the NRP exists. Let us consider vortices in $2 + 1$ Goldstone's model of complex field

$$\ddot{\phi} - \Delta\phi + 2\phi(\phi\phi^* - 1) = 0\,. \tag{25}$$

As usual the vortex solution has the form

$$\phi_s(r, \theta) = f(r)e^{\mathrm{i}N\theta}\,, \tag{26}$$

where N is a winding number. Vortices with winding number larger than 1 are unstable so we will consider only the case $N = 1$.

Let us consider a small perturbation $\delta\phi$ of the field around the static vortex solution ϕ_s. An equation describing the small perturbation of the vortex has the form

$$\ddot{\delta\phi} - \Delta\delta\phi + 2\left(2f^2 - 1\right)\delta\phi + 2f^2e^{2\mathrm{i}N\theta}\delta\phi^* = 0\,. \tag{27}$$

One can seek the solution in the form:

$$\delta\phi = \sum_{l=-\infty}^{\infty} e^{\mathrm{i}(N+l)\theta}\left(e^{\mathrm{i}\omega t}s_l^+ + e^{-\mathrm{i}\omega t}s_l^-\right)\,. \tag{28}$$

It is more convenient to introduce the following variables:

$$a_l = \tfrac{1}{2}\left(s_l^+ + s_l^{-*}\right)\,, \tag{29a}$$

$$g_l = \tfrac{1}{2}\left(s_l^+ - s_l^{-*}\right)\,. \tag{29b}$$

The equations for new functions are

$$\begin{bmatrix} \hat{\boldsymbol{D}}_a & \frac{2Nl}{r^2} \\ \frac{2Nl}{r^2} & \hat{\boldsymbol{D}}_g \end{bmatrix} \begin{bmatrix} a_l \\ g_l \end{bmatrix} = \omega^2 \begin{bmatrix} a_l \\ g_l \end{bmatrix}, \tag{30}$$

where

$$\hat{\boldsymbol{D}}_a = -\partial_{rr} - \frac{1}{r}\partial_r + \frac{N^2+l^2}{r^2} + 2\left(3f^2-1\right), \tag{31a}$$

$$\hat{\boldsymbol{D}}_g = -\partial_{rr} - \frac{1}{r}\partial_r + \frac{N^2+l^2}{r^2} + 2\left(f^2-1\right). \tag{31b}$$

The physical interpretation is now clear: a_l describes a field which far away from the vortex core looks like a field with mass $m_a^2 = 4$ and g_l describes a massless field (the Goldstone's mode). The massive field is responsible for amplitude change and the Goldstone's mode changes only the phase of the field.

It is possible that with this conditions the negative radiation pressure can appear. And indeed, our numerical calculations for the full $2+1d$ nonlinear equation ([5]) show that the vortex starts to move towards the source of radiation when hit with, what we call, the amplitude wave. The phase (Goldstone's) wave pushes the vortex away. The initial conditions are respectively:

$$\phi_{\rm amp}(x,y,t=0) = f(r)(1+A\cos(ky+\omega t))e^{{\rm i}\theta}, \tag{32}$$

$$\partial_t\phi_{\rm amp}(x,y,t=0) = -A\omega f(r)\sin(ky+\omega t)e^{{\rm i}\theta} \tag{33}$$

for the amplitude wave and

$$\phi_{\rm gold}(x,y,t=0) = f(r)(1+A{\rm i}\cos(ky+\omega t))e^{{\rm i}\theta}, \tag{34}$$

$$\partial_t\phi_{\rm gold}(x,y,t=0) = -A{\rm i}\omega f(r)\sin(ky+\omega t)e^{{\rm i}\theta} \tag{35}$$

for the Goldstone's mode.

In order to calculate the force which radiation exerts on the vortex we first try to construct a travelling wave plus outgoing scattered wave as a combination of solutions of the linearized equation for different angular momenta. This procedure is very similar to finding a scattering cross-section. However, the linearized equations behave very badly both at the origin and in the limit of large r. Our preliminary results suggest that the partial cross-section for small angular momenta for scattering amplitude to Goldstone's mode ($\sigma_{\rm ag}$) are larger than cross-sections for scattering amplitude to amplitude mode ($\sigma_{\rm aa}$). This fact would support our hypothesis that the surplus of momentum is created due to the transfer of energy from massive field to massless Goldstone's mode. However, for large angular momenta ($l > 10$) the cross-sections $\sigma_{\rm ag}$ tend to zero very quickly whereas $\sigma_{\rm aa}$ decrease very slowly. This would suggest that the total cross-section for amplitude-amplitude scaterring is dominant. For large angular momenta, eigenfunctions are mostly flat for small values of r (similar to Bessel functions with large index), therefore their influence can seen only far away from the vortex. This gives the following picture. The vortex core experiences the negative radiation pressure but the "asymptotic cloud" undergoes the positive radiation pressure. This creates an extra force (or a stress). After a while the positive part prevails and the whole vortex is pushed by radiation. This is possible because the vortex in Goldstone's model is a global one and its field approaches vacuum very slowly, and the energy density is never negligible. In fact the total mass of a global vortex is infinite. The situation described above can be seen in a trajectory of topological zero of a global vortex which is depicted in Fig. 4. One can see that at the beginning the topological zero (and so the maximum of energy density) moves towards the source of radiation (NRP) and after a (quite long) time the vortex core is pushed by radiation. This is a very complicated process. The vortex does not interact with the radiation as a rigid body.

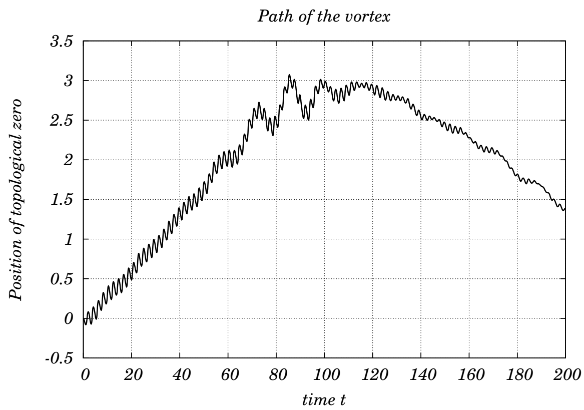

Fig. 4. The path of topological zero for Goldstone's model. The vortex core starts to move towards the source of radiation. The increasing oscillations of its motion indicate interaction between the core and the asymptotic cloud. Next the vortex changes its direction.

Another very often studied model, also with vortices, is the Abelian Higgs model. Here, complex scalar field is coupled to a vector field. Vortices in this model are much more compact. They approach the vacuum exponentially fast. Therefore, large angular momentum cross-sections should disappear quickly enough and the pure NRP should be visible. This fact was confirmed by numerically integrating the appropriate system of partial differential equations. The negative radiation pressure was observed when the amplitude or vector wave hit the vortex. When the phase wave hit the vortex positive radiation pressure was observed.

Another interesting model revealing the negative radiation pressure of the same type is halfcompacton described by Lagrangian ([6]):

$$\mathcal{L} = \tfrac{1}{2}\dot{\phi}^2 - \tfrac{1}{2}\phi'^2 - \tfrac{1}{2}|1-\phi|(\phi+1)^2\,. \tag{36}$$

Small perturbation around $\phi = -1$ vacuum can be described by simple Klein–Gordon equation with finite mass. The perturbation around the second vacuum $\phi = 1$ cannot be described by any linear equation. The second derivative of the potential is infinite, and hence the mass of the field is, formally, infinite as well [7]. The defects interpolating between these two vacua are called halfcompactons since they approach vacuum $\phi = 1$ at finite distance.

If a wave from the strange vacuum hits the halfcompacton it transforms into waves with finite mass on the other side of the defect. This is very similar to the mechanism described upon our toy model. In this model the interaction is not symmetric and when the halfcompacton is hit with the wave coming from the normal vacuum there is only positive radiation pressure. This asymmetry leads to the fact that radiation can push the halfcompacton quite easily only towards the strange vacuum.

5. Conclusions

In the present paper we have shown the mechanism responsible for the negative radiation pressure in the case of two interacting fields. When a topological defect is hit with the more massive field, in favourably conditions, the scattered wave can carry away more momentum the initial wave brought in. We have introduced a simple toy model where we could study this phenomenon both analytically and numerically. The mechanism is more general and can be present in many other field theories. Two such examples are Goldstone's model and Abelian Higgs model. We have performed the numerical calculations for the full partial differential equations and we have found NRP in both cases. However, the Goldstone's model, although described by simpler equations has one difficulty. The vortices described by this model approach vacuum very slowly and they possess infinite energy. This results in a complicated interaction between the vortex and the radiation. The vortex core experiences the NRP while the "asymptotic cloud" feels the positive radiation pressure. This creates an extra force and after a while the positive radiation pressure becomes dominant and the whole vortex is pushed by the radiation. Abelian Higgs vortices are much more localized objects and in certain conditions they experience a simple negative radiation pressure.

The presented mechanism of the NRP should be quite general and if only the reflections from the defect is not too large, should be observed in many physical systems.

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