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Σ-NUCLEUS POTENTIAL STUDIED WITH THE (π^-, K^+) REACTION*

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We calculate in the impulse approximation the kaon spectrum from the (π^-, K^+) reaction on ^{28}Si. The strength V_0 of the real part of the single particle potential of the Σ hyperons produced in the reaction is treated as a free parameter, and the strength W_0 of the imaginary (absorptive) part is determined by the ΣN cross sections for the $\Sigma\Lambda$ conversion and also for the elastic ΣN scattering (this elastic scattering introduces a strong dependence of W_0 on the Σ momentum). By fitting to the kaon spectrum measured at KEK, we obtain a repulsive $V_0 \simeq$ 40–60 MeV. This result is much closer to previous estimates of V_0 than the results of other analyses of the KEK experiments.

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1. Introduction

Our knowledge of the single particle (s.p.) potential $U_\Sigma = V_\Sigma - iW_\Sigma$ of the Σ hyperon in nuclear matter comes from the analyses of Σ atoms [1,2], of the strangeness exchange (K^-, π) reactions [3,4], and of the hyperon nucleon scattering data to which the hyperon–nucleon interaction potential may be fitted [5–8] (and with this potential one may calculate U_Σ [9–11]). All these analyses lead to the conclusion that the ΣN interaction is well represented by the Nijmegen model F of the baryon–baryon interaction [6] which leads to a repulsive V_Σ with the strength of about 25 MeV at the equilibrium density of nuclear matter [11,12].

Recently, a new source of information on U_Σ became available: the final state interaction of Σ hyperons in the associated production reaction (π, K^+). The first measurement of the inclusive K^+ spectrum from the

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(π^-, K^+) reaction on the ^{28}Si target was performed in KEK at pion momentum of 1.2 GeV$/c$ [13–15]. The existing impulse approximation analyses [13, 15, 17] of this KEK experiment imply a repulsive V_Σ with a surprisingly great strength of about 100 MeV[1], which is inconsistent with the previous estimates. In our present approach we try to reduce this inconsistency.

In the present discussion we apply the simple impulse approximation applied in [17], and described in more detail in [18][2]. We use for W_Σ the strength determined by the ΣN cross sections for the $\Sigma\Lambda$ conversion and for the elastic ΣN scattering, and determine V_Σ by fitting our calculated kaon spectrum to the measured spectrum presented in [15]. The resulting repulsive V_Σ turns out to have a strength of about 40–60 MeV which is much less than the strength of about 100 MeV suggested in Refs. [13, 15, 17].

The essential point in our approach is the inclusion of the elastic ΣN cross section in determining W_Σ which leads to W_Σ strongly depending on Σ momentum $\hbar k_\Sigma$. Let us notice that in the previous analyses the imaginary potential W_Σ was assumed to be independent of Σ momentum.

2. Determination of W_Σ

Here we use the expression for W_Σ in terms of the total cross sections for the $\Sigma\Lambda$ conversion process $\Sigma N \to \Lambda N'$ and for the ΣN total elastic (including charge exchange) scattering:

$$W_\Sigma = W_c + W_e\,, \tag{1}$$

$$W_c = \rho\frac{\hbar^2}{4\mu_{\Sigma N}}\langle k_{\Sigma N} Q_\Lambda \sigma(\Sigma^- p \to \Lambda n)\rangle\,, \tag{2}$$

$$\begin{aligned} W_e \;=\; & \rho\frac{\hbar^2}{4\mu_{\Sigma N}}\langle k_{\Sigma N} Q_\Sigma[\sigma(\Sigma^- n \to \Sigma^- n) \\ & +\sigma(\Sigma^- p \to \Sigma^- p) + \sigma(\Sigma^- p \to \Sigma^0 n)]\rangle\,, \end{aligned} \tag{3}$$

where $\langle\rangle$ denotes the average value in the Fermi sea, $\hbar k_{YN}$ is the relative YN momentum ($Y = \Sigma, \Lambda$), μ_{YN} is the YN reduced mass, and Q_Y is the exclusion principle operator in the YN channel (a projection operator onto nucleon states above the Fermi sea).

[1] A different result, a repulsive V_Σ with a strength of about 30–50 MeV, is reported by Kohno *et al.* [16] who applied a semiclassical distorted wave method which, however, involves various simplified treatments and its accuracy is hard to estimate.

[2] Notice that the conclusions of [18] are not correct because they were reached by a comparison with the KEK results presented in [13] in a figure which contained an error (corrected in [14]).

Relations (1)–(3) were derived in Ref. [19] (see also [9]) within the low order Brueckner (LOB) theory with two coupled channels YN, by applying the optical theorem, in which terms with squares of the reaction matrix were approximated by the corresponding cross sections ($|\mathcal{K}_{\Sigma N,\Sigma N}|^2 \rightsquigarrow \Sigma N$ elastic cross section, $|\mathcal{K}_{\Lambda N,\Sigma N}|^2 \rightsquigarrow \Sigma\Lambda$ conversion cross section).

In applying expressions (2)–(3), we used for the $\Sigma\Lambda$ conversion cross section the parametrization of Gal *et al.* [20], and for the cross sections appearing in (3) we used the cross sections tabulated by Rijken [21].

Our results obtained for W_c, W_e, W_Σ for nuclear matter (with $N = Z$) at equilibrium density $\rho = \rho_0 = 0.166$ fm^{-3} are shown in Fig. 1. With increasing momentum k_Σ the $\Sigma\Lambda$ conversion cross section decreases, on the other hand the suppression of W_c by the exclusion principle weakens. As the net result W_c does not change very much with k_Σ. The same two mechanisms act in the case of W_e. Here, however, the action of the exclusion principle is much more pronounced: at $k_\Sigma = 0$ the suppression of W_e is complete. At higher momenta, where the Pauli blocking is not important, the total elastic cross section is much bigger than the conversion cross section, and we have $W_e \gg W_c$, and consequently $W_\Sigma \gg W_c$. Notice that the action of the absorptive potential W_Σ on the Σ wave function (decrease of this wave function) is similar as the action of a repulsive V_Σ. We expect, therefore, to achieve with strong absorption the same final effect with a relatively weaker repulsion.

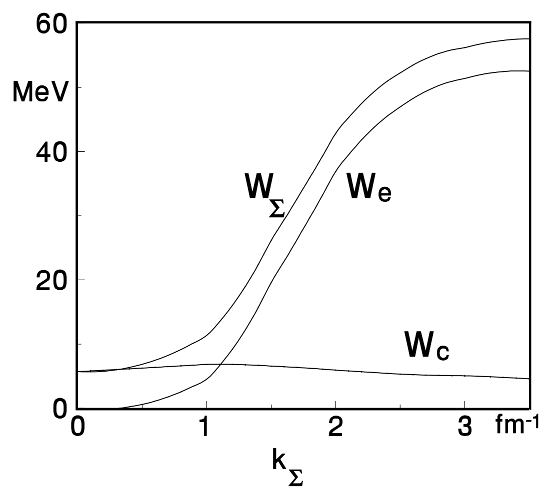

Fig. 1. The absorptive potential W_Σ in nuclear matter of density $\rho = 0.166$ fm^{-3}. See text for explanation.

3. The (π^-, K^+) reaction on the ^{28}Si target in the impulse approximation

We consider the (π^-, K^+) reaction in which the pion π^- with momentum $\boldsymbol{k}_\pi$ hits a proton in the ^{28}Si target in the state ψ_P and emerges in the final state as kaon K^+ moving in the direction $\hat{k}_K$ with energy E_K, whereas the hit proton emerges in the final state as a Σ hyperon with momentum $\boldsymbol{k}_\Sigma$. We apply the simple impulse approximation described in [18], with K^+ and π^- plane waves, and obtain:

$$d^3\sigma/d\hat{k}_\Sigma d\hat{k}_K dE_K \sim |\int d\boldsymbol{r}\exp(-i\boldsymbol{q}\boldsymbol{r})\psi_{\Sigma,\boldsymbol{k}_\Sigma}(\boldsymbol{r})^{(-)*}\psi_P(\boldsymbol{r})|^2\,, \tag{4}$$

where the momentum transfer $\boldsymbol{q} = \boldsymbol{k}_K - \boldsymbol{k}_\pi$, and $\psi_{\Sigma,\boldsymbol{k}_\Sigma}(\boldsymbol{r})^{(-)}$ is the Σ scattering wave function which is the solution of the s.p. Schrödinger equation with the s.p. potential

$$U_\Sigma(r) = (V_0 - iW_0)\theta(R - r)\,, \tag{5}$$

where for W_0 we use the nuclear matter results for W_Σ discussed in Section 2, and presented in Fig. 1, and where V_0 is treated as a free parameter (notice that for a repulsive interaction $V_0 > 0$).

For the proton s.p. potential $V_P(r)$, which determines ψ_P, we also assume the square well form with the radius R (and with a spin-orbit term). The parameters of $V_P(r)$ are adjusted to the proton separation energies.

In the inclusive KEK experiments [13–15] the energy spectrum of kaons at fixed $\hat{k}_K$ was only measured. To obtain this energy spectrum, we have to integrate the cross section (4) over $\hat{k}_\Sigma$.

We present our results for the inclusive cross section as a function of B_Σ, the separation (binding) energy of Σ from the hypernuclear system produced. B_Σ is related to Σ momentum k_Σ[3]. Thus the value of k_Σ which we need to calculate W_0 (see Section 2) is determined by B_Σ.

Our results for kaon spectrum from (π^-, K^+) reaction on ^{28}Si at $\theta_K = 6°$ at $p_\pi = 1.2$ GeV/c together with the experimental results of Ref. [15] are shown in Fig. 2. In calculating the solid (broken) curves we used for W_0 the nuclear matter results for $W_\Sigma(W_c)$ [see Eqs. (1)–(3)]. Curves A, B, C, and D were obtained with V_0 = 25 MeV, 40 MeV, 60 MeV, and 100 MeV respectively. We see that the best fit to the data points is obtained for V_0 of about 40–60 MeV with W_0 determined by $W_\Sigma = W_c + W_e$. Inclusion into the absorptive potential of the contribution W_e of the elastic ΣN scattering is essential for obtaining this result for V_0.

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[3] In the simplest case when the kaon hits the least bound proton in the silicon target (proton in the $d_{5/2}$ state), we have $-B_\Sigma = \hbar^2 k_\Sigma^2/2M_\Sigma$.

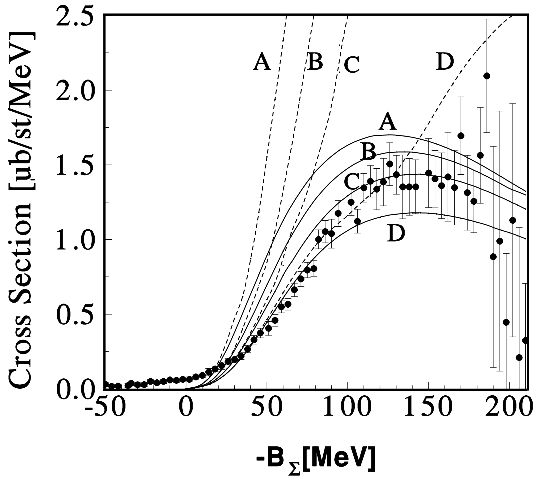

Fig. 2. Kaon spectrum from (π^-, K^+) reaction on ^{28}Si at $\theta_K = 6°$ at $p_\pi = 1.2$ GeV/c. See the text for explanation.

4. Conclusions

Our resulting strength of about 40–60 MeV of the repulsive V_Σ is much closer to the strength of about 25 MeV determined in the analyses of the strangeness exchange reactions and Σ atoms referred to in Section 1, than the strength of about 100 MeV suggested in Refs. [13,15,17]. Thus our treatment of the absorptive potential, $W_\Sigma = W_c + W_e$, appears to be the proper step to achieve consistent results for V_Σ determined in different phenomena.

Notice that our W_Σ depends on the Σ momentum k_Σ, and at low k_Σ we have $W_\Sigma \simeq W_c$. Consequently, in the analyses of Σ atoms and the strangeness exchange reactions in which mainly low Σ momenta are relevant, the approximation $W_\Sigma \simeq W_c$ appears reasonable.

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