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MULTI-GLUON FIELD APPROACH OF QCD*

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The decay of 2-gluon colour singlets in quarks: $2g \to q\bar{q}+2q2\bar{q}$ has been simulated with the Monte-Carlo method, taking into account an effective 1-gluon exchange interaction between the emitted quarks, which was folded with a 2-gluon density determined self-consistently. 2-gluon densities were found with different radii, which correspond to 0^{++} glueballs of the size of light $q\bar{q}$, $s\bar{s}$, $c\bar{c}$, $b\bar{b}$ and heavier $q\bar{q}$ systems. Binding potentials between the two gluons have been deduced, which are consistent with the confinement potential from lattice results. However, self-consistency for the deduction of 2-gluon densities requires **massless** (or very light) quarks for all flavours. The masses are given by the binding energies of quarks and gluons, yielding excitation spectra of 0^{++} glueballs and Φ, J/Ψ and Υ states consistent with observation. The sum of q–q potentials yields a strong coupling $\alpha_{\rm s}$ consistent with the available data up to large momenta.
The nucleon is described by a gluonium state coupled to 3 valence quarks, yielding ground state and radial excitations consistent with experiment. Finally, we discuss the compressibility of the nucleon and relate it to that of nuclear matter.

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1. Introduction

The two key problems in the understanding of the strong interaction are the confinement of quarks and gluons and the origin of mass, both related to the non-perturbative structure of quantum chromodynamics (QCD). A linearly rising confinement potential between quarks has been derived in

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potential models [1, 2] and lattice QCD simulations [3, 4], but its origin is not well understood. The mass term in the QCD Lagrangian is also not understood, but for the generation of mass a coupling of the quarks to a scalar Higgs background field has been proposed. Finite quark masses give rise to the axion problem, which has not been solved.

For the description of QCD in the non-perturbative regime mainly two non-perturbative methods have been applied, solutions of Dyson–Schwinger equations [5] and lattice QCD [6], which solves the QCD equations by path integral methods on a space-time lattice.

2. Deduction of 2-gluon densities

In this paper a new phenomenological method is presented, which starts from the conjecture that the non-Abelian structure of QCD may generate bound 2-gluon systems, which decay into $q\bar{q}$ pairs. For the description of such bound states Φ we write the radial wave functions in the form $\psi_\Phi(\vec{r} = \vec{r}_1 - \vec{r}_2) = [\psi_1(\vec{r}_1)\ \psi_2(\vec{r}_2)]$, where $\psi_j(\vec{r}_j)$ are the radial wave functions of the two gluons. To investigate the properties of such 2-gluon systems we studied the decay $2g \to q\bar{q} + 2q2\bar{q}$ with an attractive interaction between the emitted quarks.

Assuming an effective 1-gluon exchange interaction $V_{1g}(R) = -\alpha_s/R$ between the emitted quarks with relative distance $R = |\vec{r}_i - \vec{r}_j|$, the decay from a 2-gluon system $2g \to (q\bar{q})^n$ requires a modification of the free q–q interaction by the density of the 2-gluon system, which may be expressed by a folding integral

$$V_{qq}(R) = \int d\vec{r}\ \rho_\Phi(\vec{r}\ )\ V_{1g}(\vec{R} - \vec{r}\ )\,, \tag{1}$$

where $\rho_\Phi(\vec{r}\ )$ is the 2-gluon density $\rho_\Phi(\vec{r}\ ) = |\psi_\Phi(\vec{r}\ )|^2$.

It is interesting to note, that for a spherical density the Fourier transform of Eq. (1) to momentum (Q) space yields

$$V_{qq}(Q) = -\frac{4\pi\alpha_s}{Q^2}\ \rho_\Phi(Q)\,, \tag{2}$$

where $\rho_\Phi(Q) = 4\pi \int r^2 dr\ j_0(Qr)\ \rho_\Phi(r)$. Comparing this with the standard 1-gluon exchange force yields a Q-dependent strong coupling $\alpha_s(Q) = \alpha_s\ \rho_\Phi(Q)$, which is qualitatively consistent with the known fact of a "running" of $\alpha_s(Q)$ and the condition $\alpha_s(Q) \to 0$ for $Q \to \infty$ (asymptotic freedom).

Further, finite size of the decaying 2-gluon system have been taken into account, as well as the fact that the decay into $2q2\bar{q}$ favours relative angular momentum $L = 0$ between the emitted quarks, whereas for the decay in $q\bar{q}$

the outgoing quarks are in a relative p-state ($L = 1$). Details are given in Ref. [7]. By relativistic Fourier transformation [8] the effective interaction (1) can be transformed to momentum space

$$V_{qq}(Q') = 4\pi \int R^2 dR \; j_0(Q'R) \; V_{qq}(R) \,, \tag{3}$$

with $Q' = Q \sqrt{1 + [Q^2/4m_\Phi^2]}$ and m_Φ being the mass of the 2-gluon system.

Monte-Carlo simulations of gluon–gluon scattering have been performed in fully relativistic kinematics, in which the 2 gluons in the final state can decay in $q\bar{q}$ and $2q2\bar{q}$ (using massless quarks). The potential $V_{qq}(\Delta p)$ (3) has been used as a weight function between the outgoing quarks (with the relative momenta $\Delta\vec{p} = \vec{p}_i - \vec{p}_j$). Resulting gluon momentum distributions $d_{q\bar{q}}(Q)$ and $d_{2q2\bar{q}}(Q)$ for decay into $q\bar{q}$ and $2q2\bar{q}$ were generated. Their sum $D_\Phi(Q') = d_{q\bar{q}}(Q') + d_{2q2\bar{q}}(Q')$ can be related to the radial density $\rho_\Phi(r)$ of the 2-gluon system

$$D_\Phi(Q') = 4\pi \int r^2 dr \; j_0(Q'r) \; \rho_\Phi(r) \,, \tag{4}$$

with Q' as in Eq. (3).

The condition that $\rho_\Phi(r)$ in the interaction (3) and in Eq. (4) should be the same allowed us to determine this density. Resulting momentum distributions $d_{q\bar{q}}(Q)$ and $d_{2q2\bar{q}}(Q)$ for a self-consistent solution with $\langle r^2 \rangle \approx$ 0.5 fm^2 are given in the upper part of Fig. 1. We see that the sum $D_\Phi(Q)$ is in reasonable agreement with $\rho_\Phi(Q)$ from the Fourier transformation of $\rho_\Phi(r)$ inserted in Eq. (3) (dot-dashed line), which is quite well approximated by a radial dependence $\psi_\Phi(r) = \psi_0 \; \exp[-(r/a)^\kappa]$ with values of κ of about 1.5. The resulting density is given in the lower part of Fig. 1, which indicates clearly that a self-stabilized 2-gluon field is generated. The mass m_Φ in the relation between Q and Q' has been used as a fit parameter; for a 2-gluon system with a mean square radius of about 0.5 fm^2 this yields $m_\Phi \sim$ 0.68 GeV. This is consistent with the gluon pole mass of 0.64 ± 0.14 GeV deduced in Ref. [9]. We shall see, that the extracted mass can be understood as binding energy of the 2-gluon system including relativistic mass corrections.

The extracted 2-gluon density should give a significant contribution to the gluon 2-point functions extracted from lattice QCD simulations in form of gluon field correlators [10,11] and the QCD gluon propagator (see *eg.* [12]). In the upper part of Fig. 2 we make a comparison of our results with the 2-gluon field correlator $C_\perp(r)$ of Di Giacomo *et al.* [11], in the lower part with the gluon propagator of Bowman *et al.* [13].

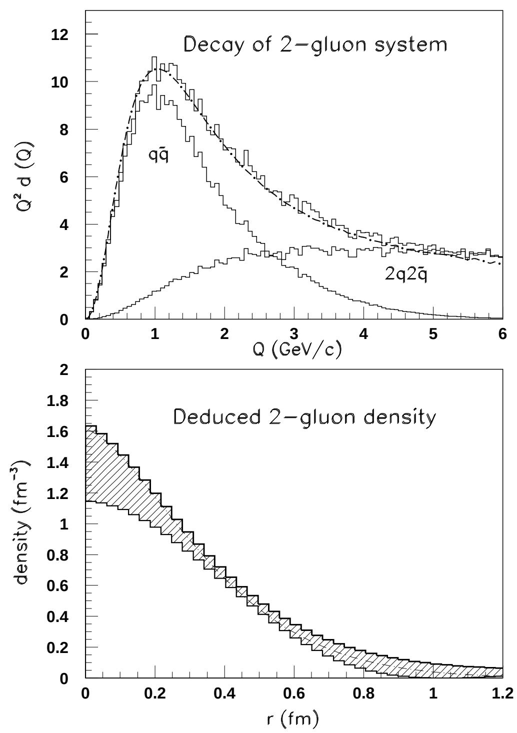

Fig. 1. Upper part: Resulting 2-gluon momentum distributions (multiplied by Q^2) for decay in $q\bar{q}$ and $2q2\bar{q}$ and sum. Lower part: Deduced 2-gluon density with estimated error band.

We get a good agreement with the lattice data (note that in Fig. 2 the gluon propagator is multiplied by Q^2) but we have to add a second 2-gluon component of smaller size, given by the dot-dashed lines.

3. Bindung potential of gluons and 0^{++} glueball states

A finite 2-gluon density, as shown in Fig. 1, may be interpreted as a bound state of the two gluons (glueball). Therefore, from the 2-gluon density the binding potential of the 2-gluon system can be obtained by solving a three-dimensional reduction of the Bethe–Salpeter equation in form of a relativistic Schrödinger equation

$$-\left(\frac{\hbar^2}{2\mu_\Phi}\ \left[\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right]-V_\Phi(r)\right)\psi_\Phi(r)=E_i\psi_\Phi(r)\,, \tag{5}$$

where $\psi_\Phi(r)$ is the 2-gluon wave function and μ_Φ a relativistic mass parameter which is related to m_Φ by $\mu_\Phi=\frac{1}{4}\ m_\Phi+\delta m$, where δm is a relativistic correction.

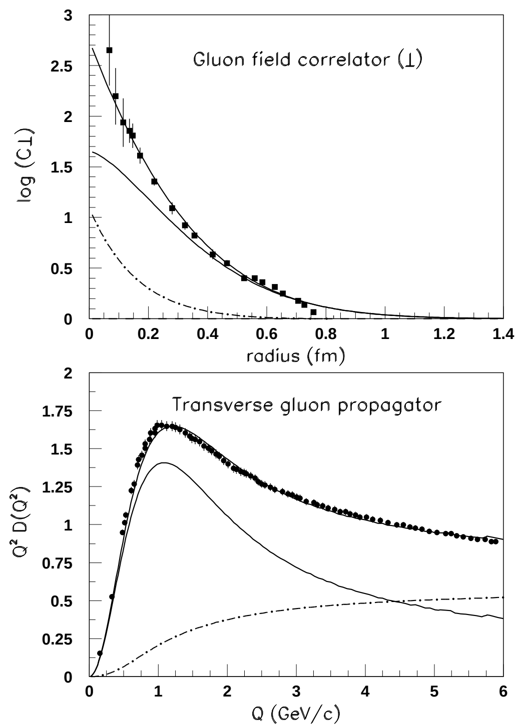

Fig. 2. Gluon field correlator $\log(C_\perp)$ from Ref. [11] (upper part) and gluon propagator from Ref. [13] (lower part) from lattice QCD simulations in comparison with our results. The lower solid lines correspond to the density $\rho_\Phi(r)$ and its Fourier transform $\rho_\Phi(Q)$, respectively, and the dot-dashed lines to an additional vector component. The sum of both contributions yields a good description of both lattice data.

Slightly different solutions of the binding potential were obtained, which are given by the dot-dashed and dashed lines in the upper part of Fig. 3. Because of the relation $2g \to (q\bar{q})^n$ this potential can also be considered as confinement potential between the emitted quarks. This is in a surprising agreement with the $1/r+$ linear form expected from potential models [1, 2] and consistent with the confinement potential from lattice QCD [4]. It is important to note that our potential reproduces the $1/r+$ linear form without any assumption on its distance behavior; this is entirely a consequence of the deduced radial form of the 2-gluon wave function.

Bound state energies $E_i = 0.68\pm$ 0.10 GeV, 1.70± 0.15 GeV, and 2.58± 0.20 GeV have been extracted. Further, in the q–q potential (3) we find one bound state with an energy in the order of -10 MeV. This very low binding energy indicates clearly that the glueball states must have a large width,

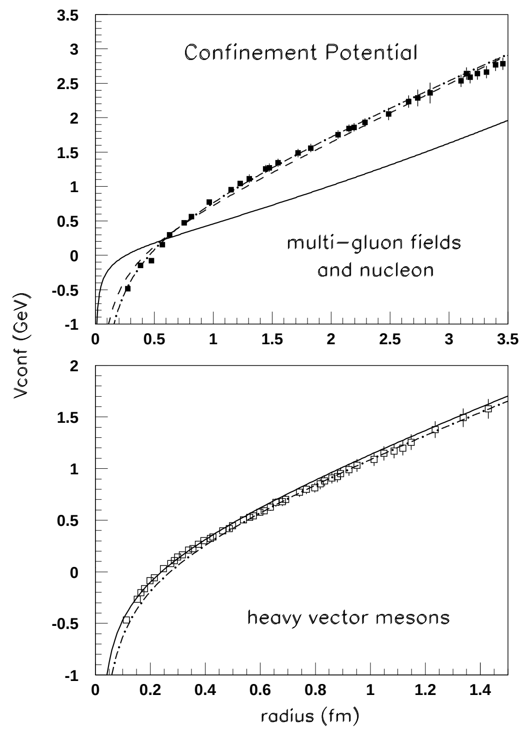

Fig. 3. Upper part: Deduced 2-gluon binding potentials (dot-dashed and dashed lines) in comparison with the confinement potential from lattice gauge calculations [4] (upper part). The solid line corresponds to the binding potential for the nucleon discussed below. Lower part: Potentials deduced for heavy vector ($s\bar{s}$, $c\bar{c}$, and $b\bar{b}$) systems discussed below in comparison with lattice QCD results [3].

since we expect $\Gamma \sim 1/E_0$. This is consistent with the general expectation for the width of glueball states ($\Gamma \geq 500$ MeV). From these results we may conclude that the glueball ground state with $E_0 = 0.68\pm\ 0.10$ GeV and a large width may be identified with the scalar $\sigma(600)$.

Glueball masses have been deduced also from lattice simulations [3, 14] in which a glueball mass below 1 GeV has not been found. However, in these simulations 0^{++} glueball masses have been extracted at about 1.7 and 2.6 GeV which correspond very nicely to the first and second radial excitation. Our evidence for a low lying glueball is supported by QCD sum rule estimates [15] which also require the existence of a low lying gluonium state below 1 GeV.

4. Heavy flavour neutral systems and $\alpha_s(Q)$

Self-consistent 2-gluon densities have been deduced also for smaller systems corresponding to the size of $s\bar{s}$, $c\bar{c}$, and $b\bar{b}$ mesons. Resulting momentum distributions and the corresponding Fourier transformed densities are given in Fig. 4, which are in a good agreement.

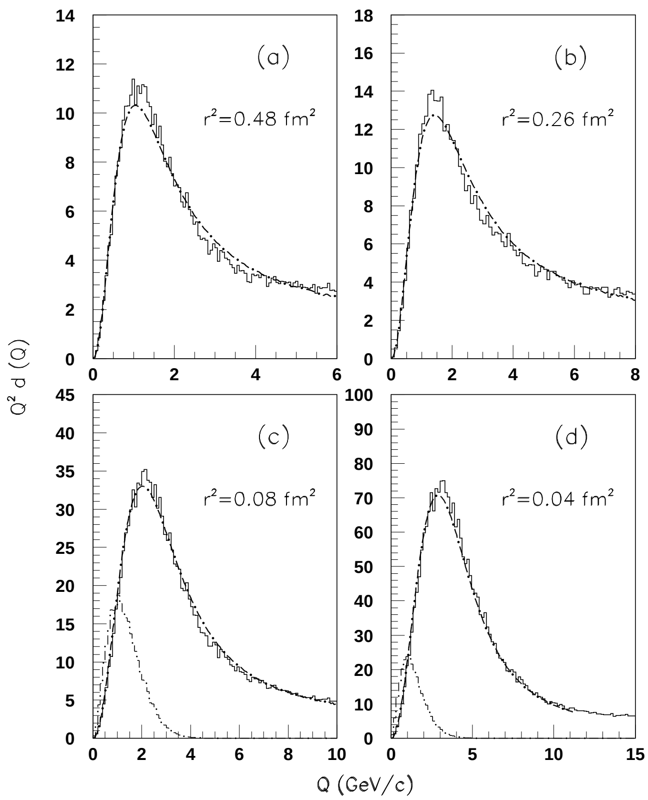

Fig. 4. Momentum distributions (histograms) and Fourier transforms of the 2-gluon density (dot-dashed lines) for 4 different densities investigated, corresponding to light $q\bar{q}$ system (a), $s\bar{s}$ (b), $c\bar{c}$ (c), and $b\bar{b}$ (d). The dot-dashed histograms are simulations assuming c and b quark masses of 1.4 and 4.5 GeV, respectively.

It is interesting to investigate the effect of quark masses in our simulations. Using quark masses of 1.24 and 4.5 GeV for c and b quarks respectively, yields the lower dot-dashed histograms indicating that self-consistent solutions are not possible. Thus, for all systems the intrinsic quark masses have to be zero (or very small). The 2-gluon binding potential is given in the lower part of Fig. 3 in a good agreement with the confinement potential from the lattice QCD [3].

Since all intrinsic quark masses have to be small in our approach, the masses of the different systems have to be explained in a different way. Whereas the binding potential (of 2-gluons) gives rise to binding energies in the order of 1 GeV, the binding potential of quarks (1) depends strongly on the size of the 2-gluon densities. Therefore, the binding of quarks can be much larger. This is shown for the different systems in Fig. 5. For the heavy

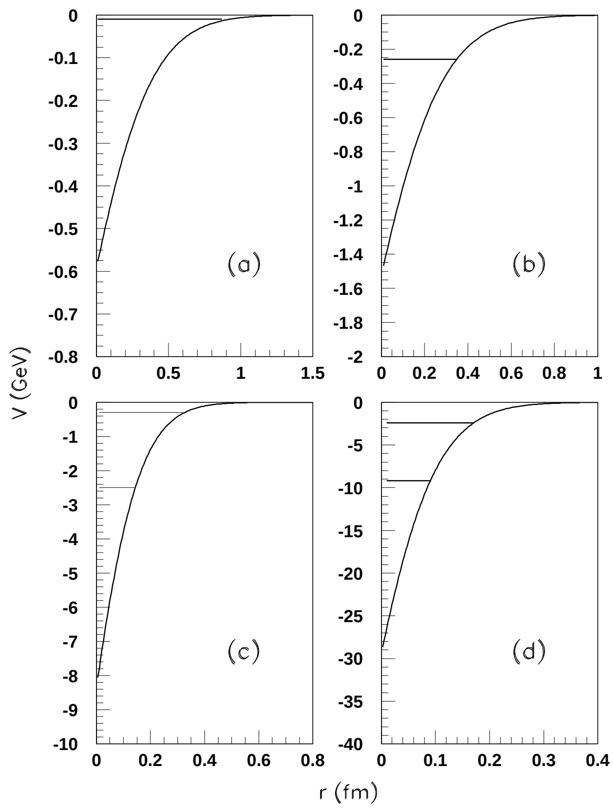

Fig. 5. Folding potential (1) for the four different cases (a)–(d) corresponding to light glueballs, $s\bar{s}$, $c\bar{c}$ and $b\bar{b}$ with the binding energies indicated.

systems (which are of small size) the binding energies are in the order of 2.4 and 9.0 GeV respectively, which shows that indeed the masses of all systems can be explained by the binding of quarks and gluons. The resulting energies of ground and radial Φ, Ψ and Υ states are in a good agreement with the experimental spectra. Details of these calculations will be given elsewhere.

The Fourier transformed potential (2) is directly related to the strong coupling $\alpha_{\rm s}(Q)$. Using the different 2-gluon density distributions $\rho_i(Q)$ deduced from Fig. 4 we obtain $\alpha_{\rm s} = \Sigma_i \; a_i \rho_{\Phi_i}(Q)$. This gives a quantitative description of $\alpha_{\rm s}$ up to large momenta, see Fig. 6.

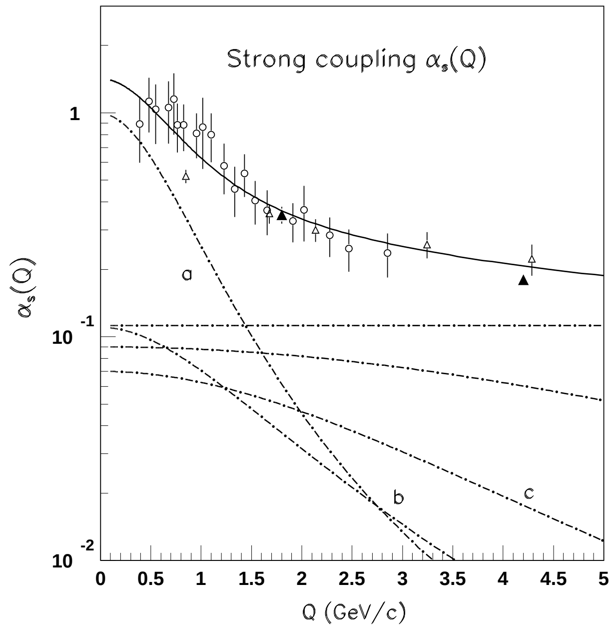

Fig. 6. Strong coupling $\alpha_{\rm s}$ from lattice QCD [16] and experiment [17] (triangles) in comparison with our results, given by the solid line. The contributions of the different 2-gluon densities $\rho_{\Phi_i}(Q)$ are also given.

5. Nucleon structure

Baryons may be described in our approach assuming the decay $4g \rightarrow 5(q\bar{q}) \rightarrow (3q\ q\bar{q}) + (3\bar{q}\ q\bar{q})$, which means $4g \rightarrow$ (baryon + antibaryon). Thus, we describe the nucleon by 3 valence quarks coupled to a 2-gluon field yielding $\rho_N(r) = 4\pi \int \rho_{3q}(r')\rho_\Phi(r-r')dr'$. The resulting binding potential is given in the upper part of Fig. 3 by the solid line, which is more shallow than the confinement potential but the attraction between the emerging quarks is increased by a factor 9. The binding energies of the nucleon g.s. and radial excitations are 0.94 GeV, 1.42± 0.07 GeV, and 1.82± 0.12 GeV in good agreement with experiment.

5.1. Compressibility: from nucleon to nuclear matter

Finally, we discuss the nucleon compressibility which may be linked to that of nuclear matter. From the excitation of the first radial state, the "breathing mode" of the nucleon, the compressibility K_N has been extracted by operator sum rules [18, 19] yielding values of about 1.3 GeV. The breathing mode has also been investigated in high energy p–p and π–p scattering [20]. From a comparison of transition densities deduced from inelastic p–p and e–p scattering strong multi-gluon contributions were extracted which were about a factor 4 stronger than those of the valence quarks. This is in a good agreement with the present results. From the multi-gluon potentials deduced in Ref. [20] we may derive the nucleon compressibility directly. Calculating a potential density for the nucleon given by $V\rho_N(r) = \frac{1}{2}\, V_{NN} \int \rho_N(r_N) t_{NN}(r - r_N) dr_N$ and adding a kinetic energy term T we obtain the energy density $E\rho_N(r) = -(V+T) \cdot \rho_N(r)$. The compressibility is then given by

$$K_N = r^2 \frac{d^2 E\rho_N(r)}{dr^2}|_{r=r_0} \,. \tag{6}$$

From the analysis of high energy p–p scattering [20] the multi-gluon potential is well determined. So, we can determine the compressibility. This is given on the left side of Fig. 7. Indeed, we obtain a compressibility of about 1.3 GeV consistent with the value obtained from sum rules [19].

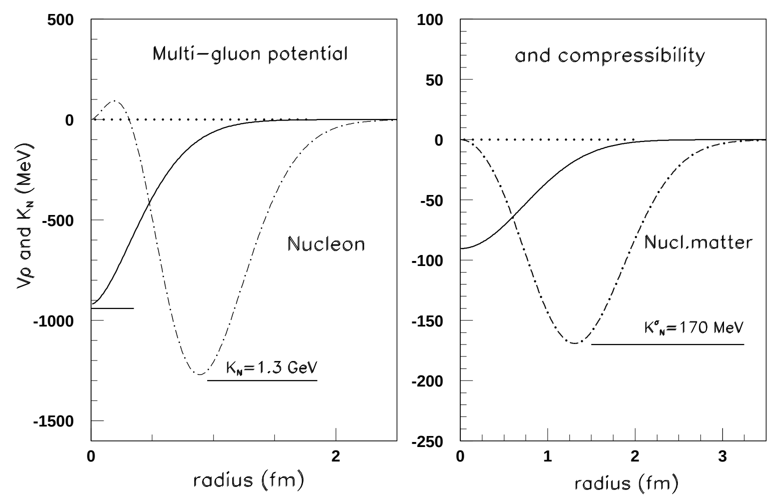

Fig. 7. Left: Nucleon potential density (solid line) and derived compressibility function (6) (dot-dashed lines). Right: The same for the scalar nucleon potential, important for nuclear matter.

For the case of nuclear matter we assume that the dominant contribution is due to compressibility of the nucleons (the compressibility due to the binding of nucleons should not be much larger than their binding energy). Then the compressibility is related to the central (scalar) nucleon potential which has a mean square radius ≥ 1.5 fm^2. Using the corresponding density the derived compressibility is in the order of 160–170 MeV, this is shown on the right side of Fig. 7. This value is rather close to the compressibility of nuclear matter K_∞ of about 220–250 MeV deduced from the study of the giant monopole resonance in heavy nuclei.

6. Conclusion

The present solution of the confinement problem based on our phenomenological description of 2-gluon fields differs entirely from earlier suggestions that confinement could arise from complicated non-perturbative field configurations (magnetic monopoles, flux tubes, vortices or strings) in the Abelian projection of QCD which had severe problems, *eg.* with Casimir scaling. Further, none of these models could explain the generation of mass and the complex Yang–Mills gluon structure found in lattice QCD calculations. In our description all these problems are tied together and are well described. The masses of hadrons are explained by binding effects, and all intrinsic quark masses have to be consistent with zero. Thus, a scalar Higgs field in which the quark masses are generated is not needed. Also the axion problem does not exist when quark masses are zero.

It is of large interest that in our approach in which many of the properties of hadrons are well described, including the problem of the light pion mass and the non-existence of chiral symmetry, quarks arise only from the decay of multi-gluon systems. This has important consequences for our understanding of the origin of our universe and of baryogenesis.

Finally, by the discussion of the compressibility a first example is given which shows that the properties of hadrons are strongly tied to those of nuclear systems.

REFERENCES

[1] R. Barbieri, R. Kögerler, Z. Kunszt, R. Gatto, *Nucl. Phys.* **B105**, 125 (1976).

[2] S. Godfrey, N. Isgur, *Phys. Rev.* **D32**, 189 (1985); D. Ebert, R.N. Faustov, V.O. Galkin, *Phys. Rev.* **D67**, 014027 (2003) and references therein.

[3] G.S. Bali, K. Schilling, A. Wachter, *Phys. Rev.* **D56**, 2566 (1997).

[4] G.S. Bali, B. Bolder, N. Eicker, T. Lippert, B. Orth, K. Schilling, T. Struckmann, *Phys. Rev.* **D62**, 054503 (2000).

[5] P. Maris, C.D. Roberts, *Intern. J. Mod. Phys.* **E12**, 297 (2003) [nucl-th/0301049] and references therein.

[6] M. Creutz, *Quantum Fields on the Computer, Adv. Series on Directions in High Energy Phys. 12*, World Scientific, Singapore 1992 and references therein.

[7] H.P. Morsch, P. Zupranski, hep-ph/060919.

[8] J.J. Kelly, *Phys. Rev.* **C66**, 065203 (2002) and references therein.

[9] C. Alexandrou, P. de Forcrand, E. Follana, *Phys. Rev.* **D63**, 094504 (2001).

[10] A.Di Giacomo, H.G. Dosch, V.I. Shevchenko, Yu.A. Simonov, *Phys. Rep.* **372**, 319 (2002).

[11] A. Di Giacomo, E. Meggiolaro, H. Panagopoulos, *Nucl. Phys.* **B483**, 371 (1997). For our comparison we used $\log C_{\perp}(r)$.

[12] J.E. Mandula, *Phys. Rept.* **315**, 273 (1999) [`hep-lat/9907020`].

[13] P.O. Bowman, U.M. Heller, D.B. Leinweber, M.B. Parappilly, A.G. Williams, *Phys. Rev.* **D70**, 034509 (2004).

[14] C. Morningstar, M. Peardon, *Phys. Rev.* **D56**, 4043 (1997); *Phys. Rev.* **D60**, 034509 (1999) and references therein.

[15] S. Narison, *Nucl. Phys.* **B509**, 312 (1998); *Phys. Rev.* **D73**, 114024 (2006).

[16] S. Furui, H. Nakajima, *Phys. Rev.* **D69**, 074505 (2004); *Phys. Rev.* **D70**, 094504 (2004).

[17] S. Eidelman *et al.*, *Phys. Lett.* **B592**, 1 (2004); `http://pdg.lbl.gov/`

[18] H.P. Morsch, *et al.*, *Phys. Rev. Lett.* **69**, 1336 (1992).

[19] H.P. Morsch, *Z. Phys.* **A350**, 61 (1994).

[20] H.P. Morsch, P. Zupranski, *Phys. Rev.* **C71**, 065203 (2005).