NUCLEAR LEVEL DENSITY PARAMETER*

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The nuclear level-density parameters obtained in the Yukawa folded potential approach are presented. The particle-number averaging ($\mathcal{N}$ averaging) Strutinsky shell-correction method is used to extract the shell-correction energy and its change with temperature and the macroscopic nuclear energy for 130 spherical even-even nuclei. A liquid-drop type expression is proposed for the level-density parameter. The best agreement is achieved between our results and the phenomenological formula of von Egidy based on a back-shifted Fermi gas model.

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Nuclear level densities are very useful to understand properties of excited nuclei and to describe fission dynamics, but are also relevant in transport theories and in astrophysical applications. Since the Bethe model [1] of 1936 some more or less successful phenomenological expressions based *e.g.* on the Fermi-gas or the so-called *back shifted* Fermi-gas model were proposed to reproduce the existing data and predict the not yet measured cases. Theoretical calculations within the shell model [2] and the Monte Carlo [3] methods, which generally include pairing correlations, and take the influence of spin and parity into account, have been quite successful in this context. The agreement of theoretical predictions with the experimental data constitutes in general a stringent test of nuclear forces as well as of the parameters used in the calculations. We have performed such an investigation in the framework of the macroscopic–microscopic model searching to

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establish an analytical expression of the level-density parameter depending on mass number A, isospin parameter $I = (N - Z)/A$ and nuclear deformation like in Ref. [4]. The deformation dependence has been analysed using the Yukawa folded (YF) [5] single-particle (s.p.) potential. In a previous study the self-consistent mean-field method with the Skm* Skyrme interaction [6] was used [7], another [8] was based on the relativistic mean-field theory (RMFT) [9] with the NL3 parameter set [10]. These mean fields give access to s.p. levels of nuclei at zero temperature (nuclear ground state). Using the $\mathcal{N}$-averaging [11] Strutinsky shell-correction method we have removed the shell effects from the nuclear ground-state energies. Then an $\mathcal{N}$-averaging was used to evaluate the shell correction energy at finite temperature [12]. Both self-consistent approaches yield too small level densities. The results obtained for spherical nuclei with the Yukawa folded mean-field potential are, however, much closer to the experimental data. They are quite close to the phenomenological Thomas–Fermi formula [13], but agree, in fact, very nicely with the estimates of the back-shifted Fermi-gas formula of von Egidy and collaborators [14] based on most recent experimental data as demonstrated in Fig. 1.

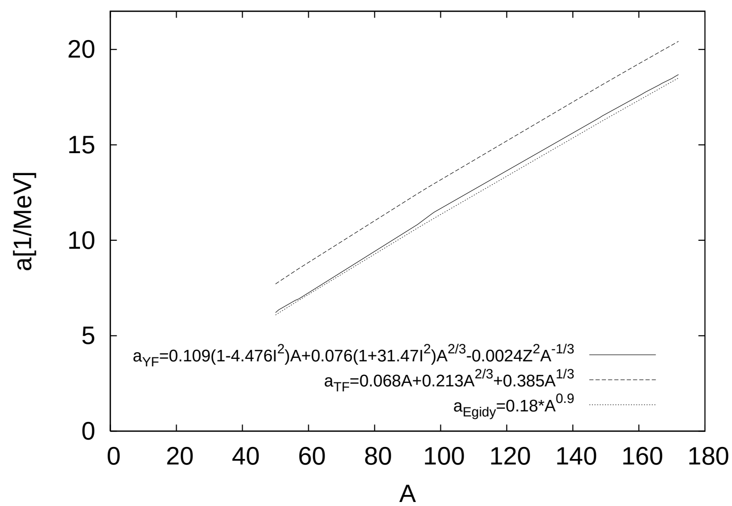

Fig. 1. Level-density parameters a as function of mass number A obtained with the Yukawa folded mean field (solid line) and the experimental data analysed in terms of a Thomas Fermi (dashed line) [13], and Fermi gas model(dotted line) [14].

The nuclear shell structure is washed out with increasing excitation energy. The total s.p. energy of an $\mathcal{N}$ fermion system at finite temperature is

$$E(\mathcal{N}, T) = 2 \sum_{\nu} e_{\nu} n_{\nu} \,, \tag{1}$$

with Fermi-function s.p. level occupation numbers of energy e_ν in the form

$$n_\nu = \left[1 + \exp\left(\frac{e_\nu - \lambda}{T}\right)\right]^{-1} \tag{2}$$

and Fermi energy λ determined by the particle-number condition $2\sum_\nu n_\nu = \mathcal{N}$ where in the nuclear case $\mathcal{N}$ stands for the proton or neutron number. In a grand-canonical description the variational quantity is the Helmholtz free energy

$$F(\mathcal{N},T) = E(\mathcal{N},T) - S(\mathcal{N},T)\cdot T\,, \tag{3}$$

where E is the intrinsic energy of an nucleus and the entropy S is given by

$$S(\mathcal{N},T) = \sum_{\nu=1}^{\infty}\left[-n_\nu\, ln(n_\nu) - (1-n_\nu) ln(1-n_\nu)\right]\,. \tag{4}$$

For a nucleus with N neutrons and Z protons the Helmholtz free energy F needs to be determined separately for neutrons and protons. In each case the average free energy $\widetilde{F}(\mathcal{N},T)$ can be determined through the $\mathcal{N}$-averaging [11] Strutinsky shell-correction method as

$$\widetilde{F}(\mathcal{N};T) = \sum_{N=\mathcal{N}_{\min}}^{\mathcal{N}_{\max}} \frac{2}{3\,N^{2/3}}\, F(N;T)\; j\left(\frac{\mathcal{N}^{1/3} - N^{1/3}}{\gamma}\right)\,, \tag{5}$$

where j is the 6th order correctional polynomial

$$j(u) = \frac{1}{\gamma\sqrt{\pi}}\; e^{-u^2}\left(\frac{35}{16} - \frac{35}{8}u^2 + \frac{7}{4}u^4 - \frac{1}{6}u^6\right)\,. \tag{6}$$

The limits in Eq. (5) are $\left(\mathcal{N}^{1/3} \pm 3\gamma\right)^3$ which guarantees that a large enough number of s.p. levels is included to evaluate the smooth part of the energy with the accuracy of the order of 0.01 MeV. The total nuclear average free energy $\widetilde{F}_{\rm tot}({\rm N},{\rm Z},T)$ is then the sum of the neutron and proton contributions. Having determined $\widetilde{F}_{\rm tot}({\rm N},{\rm Z},T)$ for each nucleus for temperatures $T = 1,2,3,4,5\,{\rm MeV}$, it turns out that its temperature dependence is quadratic

$$\widetilde{F}_{\rm tot}({\rm N},{\rm Z},T) \approx \widetilde{F}_{\rm tot}({\rm N},{\rm Z},0) - aT^2\,, \tag{7}$$

which allows through the knowledge of $\widetilde{F}_{\rm tot}({\rm N},{\rm Z},T)$ to determine the level density parameter $a(Z,A)$ for each nucleus. The sample of $a(Z,A)$ values is then fitted to the following liquid-drop (LD) type expression

$$a(Z,A) = a_{\rm vol}\,(1 + k_{\rm vol}\,I^2)\,A + a_{\rm sur}\,(1 + k_{\rm sur}\,I^2)\,A^{2/3} + a_{\rm Coul}\,Z^2\,A^{-1/3}\,, \tag{8}$$

where the values of the LD volume, surface and Coulomb coefficients (in units MeV^{-1}) are displayed in Fig. 1. The level density parameters obtained in this way for 130 spherical even-even nuclei from our Yukawa folded s.p. potential turn out to be in very good agreement with the most recent experimental data interpreted by von Egidy [14] in terms of a so-called back-shifted Fermi-gas model. We would like to insist, however, on the fact that the analytical form proposed in Ref. [14] and displayed in Fig. 1, does not allow any straightforward generalisation to deformed nuclei. The above presented LD type approach, on the contrary, is very easily generalised to deformed nuclei, simply through shape functions $B_{\rm sur}$ and $B_{\rm Coul}$ of surface and Coulomb term as demonstrated in Ref. [4].

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