# TWO-SOURCE EMISSION OF PROTONS IN $\operatorname{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ REACTIONS AT $1.2 A \mathrm{GeV}$ 

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A two-source model is used in this paper to describe the reaction process of $\operatorname{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ at 1.2 A GeV . The distribution of azimuthal angle between ${ }^{7} \mathrm{Be}$ and $p$, the transverse momentum distribution of $p$, and the total transverse momentum distribution of ${ }^{7} \mathrm{Be}$ and $p$ produced in the reactions are analyzed. It is found that the modelling results describe approximately the fluctuation and mean trend of the experimental data of Stanoeva et al.

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## 1. Introduction

Nucleus-nucleus collisions at high energies are important research fields in particle and nuclear physics. In recent 20-30 years, a lot of experimental [1-5] and theoretical [6-10] work have been finished. Lower end of high energy is a special energy for nucleus-nucleus collisions. At this energy nuclear limiting fragmentation has been shown to also apply [11]. From intermediate energy ( MeV ) region to high energy ( GeV ) region, mechanisms of nuclear reactions are expected to change with incident energy. Especially, at the lower end of high energy $(1-2 A \mathrm{GeV})$, incident projectile nuclei can stop in target emulsion with a length of 10 cm . This is convenient for us to investigate the projectile fragmentation at $1-2 A \mathrm{GeV}$.

Recently, Stanoeva et al. [12] reported the peripheral fragmentation of the ${ }^{8} \mathrm{~B}$ nuclei fragmentation at an energy of 1.2 AGeV in nuclear track emulsion. The purely fragmentation mode of ${ }^{8} \mathrm{~B}$ to ${ }^{7} \mathrm{Be}+p\left[\mathrm{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}\right]$ was studied experimentally by them [12]. It is shown that the distribution of azimuthal angle between ${ }^{7} \mathrm{Be}$ and $p$ in events with a low total transverse momentum does not display an isotropic emission. The transverse momentum distribution of the protons and the total transverse momentum distribution of the ${ }^{7} \mathrm{Be}$ and protons produced in $\mathrm{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ show a no-single source distribution [12].

To explain the azimuthal and transverse momentum distributions in the fragmentation mode of $\operatorname{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$, we analyze the experimental data of Stanoeva et al. [12] in this paper. A two-source model [13-16] suggested by Liu et al. are used.

## 2. The model

The model used in this paper is a two-source model [13-16]. According to the two-source model [13-16], the light nuclear fragments produced in a spectator have two emission sources: a hot source and a cold source. The hot source is the contact layer of the spectator related to the participant, it has a high excitation degree. The cold source is the other part of the spectator, it has a low excitation degree. We assume that the light nuclear fragments are emitted isotropically.

Let the beam direction be the $0 z$ axis and the reaction plane be the $x 0 z$ plane. In the rest frame of the hot (or cold) emission source, the three components $p_{x}^{\prime}, p_{y}^{\prime}$, and $p_{z}^{\prime}$ of momentum $p^{\prime}$ of final-state fragment can be given by the Gaussian distribution with the same standard deviation $\sigma_{\mathrm{H}}$ (or $\sigma_{\mathrm{C}}$ ). For the purpose of convenience, $\sigma_{\mathrm{H}}$ and $\sigma_{\mathrm{C}}$ are written as $\sigma$ in the following discussion.

Considering the interactions among different emission sources, the concerned source will have expansions and movements in the momentum space [17-22]. The simplest relations between the momentum $p_{x, y}^{\prime}$ in source rest frame and the momentum $p_{x, y}$ in final state is linear. We have

$$
\begin{equation*}
p_{x, y}=a_{x, y} p_{x, y}^{\prime}+B_{x, y}=a_{x, y} p_{x, y}^{\prime}+b_{x, y} \sigma, \tag{1}
\end{equation*}
$$

where $B_{x, y}$ represent movements of the emission source. $a_{x, y}$ and $b_{x, y}$ are coefficients describing the expansion and movement of the source, respectively. It seems that Eq. (1) is in contradiction with the Lorentz transformation. We would like to point out that one could understand the current formalism because Eq. (1) represented the relations of "mean" momenta between the cases of laboratory (or center-of-mass) reference frame and source rest frame [17-22]. On the other side, the values of $p_{x, y}$ are less than $0.3 \mathrm{MeV} / c$ in the experimental data of Stanoeva et al. [12]. The relativistic effect may be neglected.

We have two methods to calculate the azimuthal and transverse momentum distributions. In the Appendix, the two methods are given. For the purpose of convenience, we use the Monte Carlo method in this paper. The azimuthal and transverse momentum distributions can be obtained by a statistical method. An isotropic emission gives $a_{x, y}=1$ and $b_{x, y}=0$. The physics condition gives $a_{x, y} \geq 1$. The number and excitation degrees of emission sources do not affect the azimuthal angle. Because the parameters $b_{x, y}$
are normalized to the width of the momenutm distribution, the expression for $\varphi$ does not contain the parameter $\sigma$. This allows us to describe the particle angular distribution in a way that is independent of the temperature of the source. The parameters $B_{x, y}$ describe how much the source is displaced from the beam axis or what the average transverse momentum of the source is, and the parameters $b_{x, y}$ describe only the displacement coefficient of the source [17-22].

The two-source model used in the present work is a generalization of the Goldhaber model of 1970's [23]. In the latter model, the momentum width of the source is related to the Fermi momentum of the fragmenting nucleus. Because the projectile fragments measured in general emulsion experiments at high energies are in a forward cone which is defined by the Fermi momentum ( $0.2 \mathrm{GeV} / c$ per nucleon) over beam momentum (in $\mathrm{GeV} / c$ per nucleon), the value for the momentum width used in the following section, particularly the one for the hot source, also follows this relation.

## 3. Comparison with experimental data

The normalized distribution, $(1 / N)(d N / d \varphi)$, of azimuthal angle between ${ }^{7} \mathrm{Be}$ and $p$ in "white" stars $\mathrm{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ at $1.2 A \mathrm{GeV}$ for $P_{\mathrm{T}}<60 \mathrm{MeV} / c$ per nucleon is given in Fig. 1, where $N$ represents the number of protons (or events), "white" stars mean that the events have no target fragmentation, and $P_{\mathrm{T}}$ denotes the total transverse momentum of ${ }^{7} \mathrm{Be}$ and $p$. Although the number and excitation degrees of emission sources do not affect the azimuthal distribution, the cut condition of $P_{\mathrm{T}}<60 \mathrm{MeV} / c$ per nucleon implies that there is only one emission source and the emission source has a low excitation degree. In Fig. 1, the histogram and curve are the experimental data of Stanoeva et al. [12] and our calculated result, respectively.


Fig. 1. Normalized distribution of azimuthal angle between ${ }^{7} \mathrm{Be}$ and $p$ in "white" stars $\operatorname{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ reactions at 1.2 AGeV . The histogram is the experimental data of Stanoeva et al. [12]. The curve is our calculated result.

In the calculation, we take $a_{x}=1.0, b_{x}=-0.8, a_{y}=1.0$, and $b_{y}=0.0$. In the selection of parameter values, the method of $\chi^{2}$-testing is used. One can see that the model describes the azimuthal distribution in fragmentation process $\mathrm{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ at 1.2 A GeV .

Fig. 2 shows the normalized transverse momentum distribution, $(1 / N)$ $\left(d N / d p_{\mathrm{T}}\right)$, of protons produced in $1.2 A \mathrm{GeV} \mathrm{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ reactions. The histogram is the experimental data of Stanoeva et al. [12]. The circles, squares, and curve are our calculated results for $5 \times 10^{2}, 5 \times 10^{3}$, and $5 \times 10^{5}$ events, respectively. To see a clear display, the horizontal positions of circles and squares have been moved by $\pm 1 \mathrm{MeV} / c$, respectively. In the calculation, we take $\sigma_{\mathrm{H}}=100.0 \mathrm{MeV} / c$ and $\sigma_{\mathrm{C}}=30.0 \mathrm{MeV} / c$. The contributions of the hot and cold sources are taken to be 0.25 and 0.75 , respectively. The values of $a_{x, y}$ and $b_{x, y}$ are the same as those for Fig. 1. One can see that the circles and curve describe approximately the fluctuation and mean trend of the experimental data, respectively.


Fig. 2. Normalized transverse momentum distribution of protons produced in $\operatorname{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ reactions at $1.2 A \mathrm{GeV}$. The histogram is the experimental data of Stanoeva et al. [12]. The circles, squares, and curve are our calculated results.

Fig. 3 is similar to Fig. 2, but it shows the results in the center-of-mass reference frame. The parameter values used for Fig. 3 are the same as those for Fig. 2. The fluctuation and mean trend of the experimental data can be described approximately by the calculated circles and curve, respectively.

In Fig. 4, the normalized total transverse momentum distribution, $(1 / N)$ $\left(d N / d P_{\mathrm{T}}\right)$, of ${ }^{7} \mathrm{Be}$ and $p$ in $\mathrm{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ reactions at $1.2 A \mathrm{GeV}$ is given. According to the conservation of momentum, the transverse momentum of ${ }^{7} \mathrm{Be}$ should be equal to that of $p$, i.e., $p_{\mathrm{T}^{7} \mathrm{Be}}=p_{\mathrm{T}}$. The total transverse momentum of ${ }^{7} \mathrm{Be}$ and $p$ in $\mathrm{MeV} / c$ per nucleon will be $P_{\mathrm{T}}=\frac{1}{7} p_{\mathrm{T}^{7} \mathrm{Be}}+p_{\mathrm{T}}=$ $\frac{8}{7} p_{\mathrm{T}}$. The histogram is the experimental data of Stanoeva et al. [12]. The circles, squares, and curve are our calculated results for $5 \times 10^{2}, 5 \times 10^{3}$, and


Fig. 3. As for Fig. 2, but showing the results in the center-of-mass reference frame.
$5 \times 10^{5}$ events, respectively. To see a clear display, the horizontal positions of circles and squares have been moved by $\pm 1 \mathrm{MeV} / c$, respectively. In the calculation, the parameter values used for Fig. 4 are the same as those for Fig. 2. One can see that the circles and curve describe approximately the fluctuation and mean trend of the experimental data, respectively.


Fig. 4. Normalized total transverse momentum distribution of ${ }^{7} \mathrm{Be}$ and $p$ in $\operatorname{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ reactions at $1.2 A \mathrm{GeV}$. The histogram is the experimental data of Stanoeva et al. [12]. The circles, squares, and curve are our calculated results.

## 4. Discussions and conclusions

In Fig. 1, only the events having no target fragment are studied. These events correspond to peripheral collisions which are not violent. The cut condition of $P_{\mathrm{T}}<60 \mathrm{MeV} / c$ per nucleon implies that there is only one emission source which has a low excitation degree. The excitation degree of
the emission source does not affect the azimuthal distribution. The values of parameters $a_{x, y}$ and $b_{x, y}$ for Fig. 1 show that the emission source has a movement along negative $x$ direction as it evaporating proton.

In figures $2,3,4$, we have used the same parameter values, i.e. $\sigma_{\mathrm{H}}=$ $100.0 \mathrm{MeV} / c$ corresponding to contribution fraction of 0.25 and $\sigma_{\mathrm{C}}=$ $30.0 \mathrm{MeV} / c$ corresponding to contribution fraction of 0.75 . The temperatures of hot and cold sources obtained by $\sigma^{2} / m_{p}$ are 10.7 and 1.0 MeV , respectively, where $m_{p}$ is the mass of proton. One can see that in $\operatorname{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ reaction the excitation degree of emission source is very low. The cold source has a main contribution. If we assume that the hot and cold sources reach an equilibrium sate, the state will have a temperature of 3.4 MeV .

Our investigation is based on the assumption of equilibrium. The calculated results for description of experimental data on $\operatorname{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ reaction published in paper of Stanoeva et al. [12] leads to reproduction of momentum spectra of observed particles as well as distribution of azimuthal angle between ${ }^{7} \mathrm{Be}$ and proton. This description renders that the interacting system reaches an equilibrium state. The emission of proton in $\operatorname{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ reaction is an evaporation process.

The fragmentation process of $\operatorname{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ at an energy of $1.2 A \mathrm{GeV}$ is investigated by a two-source model [13-16]. The analyzed results show that the protons produced in the reactions have two emission sources: a hot source and a cold source. In the two-source model [13-16], the hot source is regarded as the contact layer of the spectator related to the participant, and the cold source is regarded as the other part of the spectator. In the calculation, the fragments or particles are assumed to emit isotropically in the source rest frame, and interactions among different emission sources affect the momentum of the final-state fragments or particles [17-22].

The simplest relation between the fragment momenta in the source rest frame and in the concerned reference frame is assumed to be linear. We have given the azimuthal angle and transverse momentum by two methods: general calculation and Monte Carlo simulation. The mean trend and fluctuation of the experimental data are described approximately by the two methods.

The parameters $a_{x, y}$ and $b_{x, y}$ describe the transverse structure of the emission source. $a_{x, y}>1$ means that there is an expansion of the source along the $x$ or $y$ direction. $b_{x, y}>0$ and $b_{x, y}<0$ mean that there is a movement of the source along the positive and negative $x$ or $y$ directions, respectively. Generally speaking, the expansion and movement of the source are caused by the asymmetry of mechanics. The present work shows that the emission source in $\operatorname{Em}\left({ }^{8} \mathrm{~B}, p^{7} \mathrm{Be}\right) \mathrm{Em}$ reactions at $1.2 A \mathrm{GeV}$ has a movement along negative $x$ direction $\left(b_{x}=-0.8\right)$.

In summary, we have used a two-source model to fit the momentum spectra of proton and ${ }^{7} \mathrm{Be}$ from the fragmentation of ${ }^{8} \mathrm{~B}$ in emulsion. It is shown that the experimental data could be described with appropriate parameters related to the width of momentum distribution in the source, the expansion and motion of the source, and the relative contribution of the two source.

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## Appendix A

Calculation of azimuthal and transverse momentum distributions
According to the knowledge of probability theory and Eq. (1), the distribution of $p_{x, y}$ can be given by

$$
\begin{equation*}
f_{p_{x, y}}\left(p_{x, y}\right)=\frac{1}{\sqrt{2 \pi} \sigma a_{x, y}} \exp \left[-\frac{\left(p_{x, y}-b_{x, y} \sigma\right)^{2}}{2 \sigma^{2} a_{x, y}^{2}}\right] . \tag{A.1}
\end{equation*}
$$

The combined density function of $p_{x}$ and $p_{y}$ is

$$
\begin{align*}
f_{p_{x}, p_{y}}\left(p_{x}, p_{y}\right)= & f_{p_{x}}\left(p_{x}\right) f_{p_{y}}\left(p_{y}\right)=\frac{1}{2 \pi \sigma^{2} a_{x} a_{y}} \\
& \times \exp \left[-\frac{\left(p_{x}-b_{x} \sigma\right)^{2}}{2 \sigma^{2} a_{x}^{2}}-\frac{\left(p_{y}-b_{y} \sigma\right)^{2}}{2 \sigma^{2} a_{y}^{2}}\right] . \tag{A.2}
\end{align*}
$$

Considering the azimuthal angle

$$
\begin{equation*}
\varphi=\arctan \frac{p_{y}}{p_{x}} \tag{A.3}
\end{equation*}
$$

and the transverse momentum

$$
\begin{equation*}
p_{\mathrm{T}}=\sqrt{p_{x}^{2}+p_{y}^{2}}, \tag{A.4}
\end{equation*}
$$

we have the combined density function of $\varphi$ and $p_{\mathrm{T}}$ to be

$$
\begin{align*}
f_{\varphi, p_{\mathrm{T}}}\left(\varphi, p_{\mathrm{T}}\right)= & p_{\mathrm{T}} f_{p_{x}, p_{y}}\left(p_{\mathrm{T}} \cos \varphi, p_{\mathrm{T}} \sin \varphi\right)=\frac{p_{\mathrm{T}}}{2 \pi \sigma^{2} a_{x} a_{y}} \\
& \times \exp \left[-\frac{\left(p_{\mathrm{T}} \cos \varphi-b_{x} \sigma\right)^{2}}{2 \sigma^{2} a_{x}^{2}}-\frac{\left(p_{\mathrm{T}} \sin \varphi-b_{y} \sigma\right)^{2}}{2 \sigma^{2} a_{y}^{2}}\right] . \tag{A.5}
\end{align*}
$$

Thus, the $\varphi$ distribution is

$$
\begin{equation*}
f_{\varphi}(\varphi)=\int_{0}^{\max } f_{\varphi, p_{\mathrm{T}}}\left(\varphi, p_{\mathrm{T}}\right) d p_{\mathrm{T}} \tag{A.6}
\end{equation*}
$$

and the $p_{\mathrm{T}}$ distribution is

$$
\begin{equation*}
f_{p_{\mathrm{T}}}\left(p_{\mathrm{T}}\right)=\int_{0}^{2 \pi} f_{\varphi, p_{\mathrm{T}}}\left(\varphi, p_{\mathrm{T}}\right) \mathrm{d} \varphi \tag{A.7}
\end{equation*}
$$

In the Monte Carlo calculation, let $R_{1}, R_{2}, R_{3}, R_{4}, R_{5}$, and $R_{6}$ denote random variables distributed in $[0,1]$, we have

$$
\begin{equation*}
p_{x, y}^{\prime}=\sqrt{-2 \ln R_{1,3}} \cos \left(2 \pi R_{2,4}\right) \sigma, \tag{A.8}
\end{equation*}
$$

because $p_{x, y}^{\prime}$ obey Gaussian distribution. Considering Eqs. (1), (A.3), and (A.5) the azimuthal angle and transverse momentum can be written as

$$
\begin{equation*}
\varphi=\arctan \frac{a_{y} \sqrt{-2 \ln R_{3}} \cos \left(2 \pi R_{4}\right)+b_{y}}{a_{x} \sqrt{-2 \ln R_{1}} \cos \left(2 \pi R_{2}\right)+b_{x}}, \tag{A.9}
\end{equation*}
$$

and

$$
\begin{equation*}
p_{\mathrm{T}}=\sigma \sqrt{\left[a_{x} \sqrt{-2 \ln R_{1}} \cos \left(2 \pi R_{2}\right)+b_{x}\right]^{2}+\left[a_{y} \sqrt{-2 \ln R_{3}} \cos \left(2 \pi R_{4}\right)+b_{y}\right]^{2}}, \tag{A.10}
\end{equation*}
$$

respectively $[21,22]$.

## REFERENCES

[1] G. Singh, K. Sengupta, P.L. Jain, Phys. Rev. C41, 999 (1990).
[2] M. El-Nadi, N. Ali-Mossa, A. Abdelsalam, Nuovo Cim. A110, 1255 (1997).
[3] D. Kudzia, B. Wilczyńska, H. Wilczyński, Phys. Rev. C68, 054903 (2003).
[4] B. Hong et al. [FOPI Collaboration], Phys. Rev. C71, 034902 (2005).
[5] M. Murray [BRAHMS Collaboration], J. Phys. G 31, S1137 (2005).
[6] X.N. Wang, M. Gyulassy, Phys. Rev. Lett. 68, 1480 (1992).
[7] S.H. Kahana, T.J. Schlagel, Y. Pang, Nucl. Phys. A566, 465c (1994).
[8] Z.W. Lin, S. Pal, C.M. Ko, B.A. Li, B. Zhang, Nucl. Phys. A698, 375 (2002).
[9] H.M. Hofmann, G.M. Hale, Nucl. Phys. A613, 69 (1997).
[10] U. Ornik, R.M. Weiner, G. Wilk, Nucl. Phys. A566, 469c (1994).
[11] F.H. Liu, Phys. Rev. C62, 024613 (2000).
[12] R. Stanoeva, V. Bradnova, P.I. Zarubin, I.G. Zarubina, N.A. Kachalova, A.D. Kovalenko, A.I. Malakhov, P.A. Rukoyatkin, V.V. Rusakova, S. Vokál, G.I. Orlova, N.G. Peresadko, S.P. Kharlamov, E. Stan, M. Haiduc, I. Tsakov, talk given at the Conference on Physics of Fundamental Interactions, Moscow, Russia, 5-9 December 2005, nucl-ex/0605013.
[13] F.H. Liu, Y.A. Panebratsev, Nuovo Cim. A111, 1213 (1998).
[14] F.H. Liu, Y.A. Panebratsev, Phys. Rev. C59, 941 (1999).
[15] F.H. Liu, Chin. J. Phys. 38, 1063 (2000).
[16] F.H. Liu, Chin. J. Phys. 39, 401 (2001).
[17] F.H. Liu, Europhys. Lett. 63, 193 (2003).
[18] F.H. Liu, N.N. Abd Allah, D.H. Zhang, M.Y. Duan, Int. J. Mod. Phys. E12, 771 (2003).
[19] F.H. Liu, N.N. Abd Allah, B.K. Singh, Phys. Rev. C69, 057601 (2004).
[20] F.H. Liu, N.N. Abd Allah, D.H. Zhang, M.Y. Duan, Chin. J. Phys. 42, 152 (2004).
[21] F.H. Liu, J.S. Li, M.Y. Duan, Phys. Rev. C75, 054613 (2007).
[22] F.H. Liu, Chin. Phys. 16, in press (2008).
[23] A.S. Goldhaber, Phys. Lett. B53, 306 (1974).

