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THE PROBABILITY OF MUON STICKING AND X-RAY YIELDS IN THE MUON CATALYZED FUSION CYCLE IN A DEUTERIUM AND TRITIUM MIXTURE

M.R. Pahlavani[a†], S.M. Motevalli[a,b‡]

[a]Department of Physics, Faculty of Science, Mazandaran University
PO Box 47415-416, Babolsar, Iran

[b]Department of Physics, Islamic Azad University, Ayatollah Amoli Branch
PO Box 678, Amol, Mazandaran, Iran

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The muon catalyzed fusion cycle in mixtures of deuterium and tritium is of particular interest due to the observation of high fusion yields. In the D–T mixture, the most serious limitation to the efficiency of the fusion chain is the probability of muon sticking to the α-particle produced in the nuclear reaction. An accurate kinetic treatment has been applied to the muonic helium atoms formed by a muon sticking to the α-particles. In this work accurate rates for collisions of $\alpha\mu^+$ ions with hydrogen atoms have been used for calculation of muon stripping probability and the intensities of X-ray transitions by solving a set of coupled differential equations numerically. Our calculated results are in good agreement with experimental data available in literature.

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1. Introduction

The feasibility of cold nuclear fusion using muons is well documented today. The catalysis of nuclear reactions by negative muons in the cold mixture of deuterium and tritium is known as muon catalyzed fusion (μCF) [1-10]. Study of the muon catalyzed fusion reactions is of great interest and carried out in many laboratories of the world recently [11-19]. Muons can be created by the decay of pion which is generated in the collision of intermediate-energy proton with target nuclei. In the muon catalyzed fusion process, after injection of muon in to deuterium and tritium mixture, either

† m.pahlavani@umz.ac.ir
‡ motavali@umz.ac.ir

$d\mu$ or a $t\mu$ atom is formed, with a probability proportional to the relative concentrations of D and T in the mixture. These atoms are formed in exited states [20,21] and then, due to cascade processes, de-excite to ground states. The following reactions illustrate direct formation of muonic $d\mu$ and $t\mu$ atoms

$$\mu^- + \mathrm{D} \;\rightarrow\; d\mu + e^-\,(\lambda_d)\,, \tag{1}$$
$$\mu^- + \mathrm{T} \;\rightarrow\; t\mu + e^-\,(\lambda_t)\,, \tag{2}$$

where e^- denotes an electron and λ_d and λ_t are the rate of reactions (1) and (2). The probability of formation of the $d\mu$ atom that will reach its $1s$ ground state is quantified by the parameter q_{1s}, which is a function of target density, ϕ and tritium concentration, C_t. Also it is very sensitive to the $d\mu$ kinetic energy distribution [22–24]. The difference between binding energies of $t\mu$ and $d\mu$ is about 48.1 eV [24]. Therefore, the transfer of a muon from $d\mu$ to a triton is favorable for all temperatures in the given processes

$$d\mu + t \rightarrow t\mu + d + 48.1\,\mathrm{eV}\;(\lambda_{dt})\,, \tag{3}$$

with a rate of $\lambda_{dt} = 2.8 \times 10^8\phi$ [25-28]. The muon mass is about 206.77 times larger than the mass of electron. Consequently, the size of a muonic hydrogen atom is smaller than the one of the electronic hydrogen by the same rate approximately. These small muonic atoms can approach other hydrogen nuclei experiencing reduced Coulomb barrier and then induce d–t fusions. The process in which a muonic molecule is formed is the most important step in the μCF. The formation of muonic molecules of hydrogen isotopes and their nuclear reactions have been the subject of many experimental and theoretical studies [25-27]. In collisions of $t\mu$ muonic atoms with D_2 and DT molecules, the muonic molecules $dt\mu$ are formed during a time interval $\tau_{dt\mu} \leq 10^{-8}\,$sec [29,30] according to the following resonance reactions

$$t\mu + \mathrm{D}_2 \;\rightarrow\; [(dt\mu)_{J\nu}d2e]\,(\lambda_{dt\mu-d})\,, \tag{4}$$
$$t\mu + \mathrm{DT} \;\rightarrow\; [(dt\mu)_{J\nu}t2e]\,(\lambda_{dt\mu-t})\,, \tag{5}$$
$$\lambda_{dt\mu} \;=\; \lambda_{dt\mu-d}C_d + \lambda_{dt\mu-t}C_t\,, \tag{6}$$

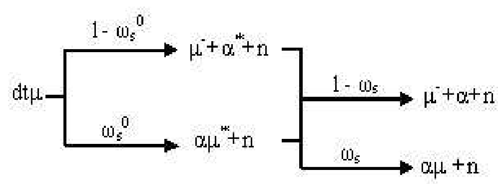
in the excited rotational–vibrational ($J\nu$) state with quantum number $J = \nu = 1$, where C_d and C_t are concentrations of deuterium and tritium nuclei, respectively. A strong resonance effect appear due to a degeneracy in the excited state of the $dt\mu$ and the electron molecule complex. The rate of formation of the $dt\mu$ molecules has been found to depend strongly on temperature, density and on whether collision of the $t\mu$ atom occur with a D_2 or a DT molecule [24,31,32]. In the absence of the effect of helium atoms and other impurities, we have

$$C_d + C_t = 1\,. \tag{7}$$

In fact, the radius of a muonic hydrogen ion ($dt\mu$) is much smaller (about $\sim$ 200 times) than a usual electron molecule, therefore the nuclei may tunnel the coulomb barrier with a high probability and fuse with a rate of $\approx 10^{12}\,\mathrm{sec}^{-1}$ [33]. In d–t fusion, α-particle ($^4\mathrm{He}^{++}$) and neutron (n) are produced. The d–t fusion reaction takes place in $\sim 10^{-12}$ sec which is much shorter than the muon lifetime ($\tau_\mu = 2.197\ \mu$sec). Most of the negative muons are liberated to participate in the next μCF $d + t + \mu^- \to n(14\,\mathrm{MeV}){+}^4\mathrm{He}\ (3.6\,\mathrm{MeV}){+}\mu^-$ cycle. This chain reaction (μCF cycle) repeats until the muon is lost from the cycle due to capture by a heavier nucleus or decay.

2. Theoretical calculations

The sticking of muons to alpha particles after fusion, is an unwanted process and eliminate muons from the chain of fusion reactions. This process is the main loss mechanism in the μCF. The probability of forming an $\alpha\mu$ ion is called initial sticking probability $\omega_s^0(= 0.912\%)$ [34]. After muon catalyzed D–T fusion the muon follows one of three courses: immediate freedom, a short confinement with liberation via subsequent collisions or a life sentence as exhibited by the following diagram:



where $\alpha\mu^*$ ions are formed with an energy of $E^{\mathrm{in}}_{\alpha\mu} = 3.47\,\mathrm{MeV}\,(v^{\mathrm{in}}_{\alpha\mu} = 5.83\,\mathrm{a.u.})$ then are slowed down toward thermal energy by collision with the surrounding D_2 and DT molecules. During the same time, as long as the kinetic energy exceeds the appropriate threshold ($E^{\mathrm{T}}_{\alpha\mu} \approx 10\,\mathrm{keV}$), the $\alpha\mu$ ion can be stripped as a result of collisions. This process is referred to as reactivation and final sticking fraction, ω_s that conventionally related to the initial sticking fraction by $\omega_s = (1 - R)\omega_s^0$. The reactivation coefficient, R depends upon the stopping power of the media and several important cross sections. Stripping process can occur through several channels. Collisions of the $(\alpha\mu)_{1s}$ ions with the surrounding D_2 and DT molecules during the slowing down process can result in $\alpha\mu^+$ charge transfer, ionization or excitation of the discrete $\alpha\mu^*$ levels. Stripping (charge transfer plus ionization) can also happen from the $\alpha\mu^*$ which is the results of the sticking or collisional excitation processes. The important processes induced by $\alpha\mu^+$ are presented below:

(a) Coulomb excitation:

$$\alpha\mu^+(nlm) + p \longrightarrow \alpha\mu^+(n^{'}l^{'}m^{'}) + p\,, \quad n^{'} > n\,, \tag{8}$$

(b) Coulomb de-excitation:

$$\alpha\mu^+(n^{'}l^{'}m^{'}) + p \longrightarrow \alpha\mu^+(nlm) + p\,, \quad n^{'} > n\,, \tag{9}$$

(c) Ionization:

$$\alpha\mu^+(nlm) + p \longrightarrow \alpha + p + \mu^-\,, \tag{10}$$

(d) Charge transfer:

$$\alpha\mu^+(nlm) + p \longrightarrow \alpha + p\mu\,, \tag{11}$$

(e) Stark mixing:

$$\alpha\mu^+(nlm) + \mathrm{H} \longrightarrow \alpha\mu^+(nl^{'}m^{'}) + \mathrm{H}\,, \quad l \neq l^{'}\,, \tag{12}$$

(f) Radiative:

$$\alpha\mu^+(n^{'}l^{'}m^{'}) \longrightarrow \alpha\mu^+(nlm) + \gamma\,, \quad n^{'} > n\,, \tag{13}$$

(g) Auger de-excitation:

$$\alpha\mu^+(n^{'}l^{'}m^{'}) + \mathrm{H} \longrightarrow \alpha\mu^+(nlm) + p + e^-\,, \quad n^{'} > n\,. \tag{14}$$

Experiments on muonic system are very difficult to perform due to the muon's short life time. Therefore, one possibility is to do experiments on electronic systems and then scale them to muonic systems. This scaling should be done carefully by taking into account appropriate threshold energy, momentum transfer *etc.* Such scaling has been derived in the frame of Born approximation [35] and is given by

$$\begin{aligned} \sigma_\mu(v) &= \frac{m_e}{m_\mu}\left[\frac{1}{(1+\epsilon)^2}\right]\left[\sigma_e\left(\frac{v}{1+\epsilon}\right) - \epsilon^2\sigma_e\left(\frac{\epsilon v}{1+\epsilon}\right)\right]\,, \\ \epsilon &= 2x - 1 - \left[(2x-1)^2 - 1\right]^{0.5}\,, \end{aligned} \tag{15}$$

where σ_μ and σ_e are muonic and electronic cross sections, respectively. x is equal to the ratio of collision energy to muon threshold energy and m_μ and m_e are reduced masses of muonic and electronic atoms, respectively. The muon stripping reaction is either ionization or charge transfer and can occur only before the $\alpha\mu^+$ is slowed down from its initial velocity (5.83 a.u.) to a velocity ≤ 1 a.u.. The kinetic of reactivation is described by the various rates in a set of coupled differential equations. The fraction of stripped muonic helium ions in terms of population probabilities can be written as

$$\frac{dP_{\rm st}(t)}{dt} = \sum_i \lambda_{\rm st}^{(i)}(v(t))P_i(t)\,, \tag{16}$$

where $\lambda_{\rm st}^{(i)}(v(t))$ are velocity-dependent stripping rates from the individual energy levels and $P_i(t)$ are the time-dependent population probabilities for the state i of muonic helium ion. The time-dependent population probabilities for the state i of the muonic helium ion are determined by

$$dP_i(t)/dt = \lambda_{\rm pop}^{(i)} - P_i(t)\lambda_{\rm depop}^{(i)}\,, \tag{17}$$

where $\lambda_{\rm pop}^{(i)}$ and $\lambda_{\rm depop}^{(i)}$ are the rates of populating and de-populating probability of state i, respectively. These rates can be given by the following relations:

$$\begin{aligned} \lambda_{\rm pop}^{(i)} &= \sum_{i'(n_{i'}>n_i)} \left(\lambda_{\rm Au}^{(i'\to i)} + \lambda_{\rm ra}^{(i'\to i)} + \lambda_{\rm de-ex}^{(i'\to i)}\right) P_{i'} \\ &\quad + \sum_{i'(n_{i'}<n_i)} \lambda_{\rm ex}^{(i'\to i)} P_{i'} + \sum_{i'(n_{i'}=n_i)} \lambda_{\rm Stark}^{(i'\to i)} P_{i'}\,, \end{aligned} \tag{18}$$

$$\begin{aligned} \lambda_{\rm depop}^{(i)} &= \lambda_{\rm st}^{(i)} + \sum_{i'(n_{i'}<n_i)} \left(\lambda_{\rm Au}^{(i\to i')} + \lambda_{\rm ra}^{(i\to i')} + \lambda_{\rm de-ex}^{(i\to i')}\right) \\ &\quad + \sum_{i'(n_{i'}>n_i)} \lambda_{\rm ex}^{(i\to i')} + \sum_{i'(n_{i'}=n_i)} \lambda_{\rm Stark}^{(i\to i')}\,, \end{aligned} \tag{19}$$

where $\lambda_{\rm Au}$, $\lambda_{\rm ra}$, $\lambda_{\rm de-ex}$, $\lambda_{\rm ex}$, $\lambda_{\rm Stark}$ and $\lambda_{\rm st}$ are the Auger de-excitation, radiative, Coulomb de-excitation, Coulomb excitation, Stark mixing and striping rates, respectively. In general, λ is given by

$$\lambda = N\sigma v\,[\sec^{-1}]\,, \tag{20}$$

where N, v and σ are density of surrounded media, relative velocity and cross section for all processes under consideration, respectively. The excitation rates were obtained using Born approximation given by Bracci and Fiorentini [6]. Velocity dependent de-excitation rates are determined by substitution of $\lambda_{\rm ex}$ in $\lambda_{\rm de-ex}^{n\to 1} = \lambda_{\rm ex}^{1\to n}/n^2$. The ionization rates were obtained by Eikonal Initial State-Continuum Distorted Wave (EIS-CDW) method given by Igarashi and Shirai [7]. The charge transfer rates are obtained by Bracci and Fiorentini [6]. The Stark mixing and Auger rates were calculated using the formulas given by Leon and Bethe [8, 10]. The radiative rates were obtained by scaling the results of hydrogen atom following Bethe and Salpeter [9]. Summation over $n_{i'} = n_i$ for the Stark mixing term implied over all angular momentum states with the same principle quantum number. The time and velocity dependence in Eq. (16) are coupled through the energy-loss equation for muonic helium ion given by

$$\frac{dE_{\alpha\mu}}{dt} = -v_{\alpha\mu}S(E_{\alpha\mu}) = -\left(\frac{2E_{\alpha\mu}}{m_{\alpha\mu}}\right)^{1/2} S(E_{\alpha\mu}) \,, \tag{21}$$

where $S = -dE/dx$ is the stopping power of the surrounding media and $m_{\alpha\mu}$ is the mass of muonic helium ion. This coupling reveals the central role for the stopping power in the reactivation process. Since the stopping power depends only on charge and velocity (not on mass) of the particle, it is possible to use the proton stopping power instead of $\alpha\mu^+$. Stopping power due to ionization-excitation for proton, deuteron, triton and α particles are given by [36]

$$\begin{aligned} S &= \frac{dE}{dx}\left[\frac{\mathrm{MeV}}{\mathrm{m}}\right] = 4\pi r_0^2 z^2 \frac{mc^2}{\beta^2}\phi Z\left[\ln\left(\frac{2mc^2}{I}\beta^2\gamma^2\right) - \beta^2\right], \\ \gamma &= \frac{1}{(1-\beta^2)^{1/2}}\,, \qquad \beta = \frac{v}{c}\,, \end{aligned} \tag{22}$$

where r_0 is classical electron radius, c is speed of light in vacuum, z is charge of the incident particle, ϕ is number of atoms/m^3 of target, Z is atomic number of the material and I is mean excitation potential of the target. The initial conditions are: $E_{\alpha\mu}(0) = E^{\mathrm{in}}_{\alpha\mu} = 3.47\,\mathrm{MeV}$, $P_{\mathrm{st}}(0) = 0$ and the initial values of populated levels are determined by the initial sticking, $P_i(0) = \omega_s^0(i)/\omega_s^0$. The initial sticking probability, ω_s^0 and fractions of the nl states are listed in Table I. The populations $P_i(t)$ for $n = 1, 2, \ldots, 6$ and the l sublevels are treated in detail for $n < 4$. The reactivation coefficient R is equivalent to the stripping fraction $P_{\mathrm{st}}(t)$ at $t = \infty$.

The intensity of X-ray transition in muonic helium ion is another quantity which can be measured experimentally and calculated along with reactivation coefficient (R). Muons in excited levels of the $\alpha\mu^+$ may de-excite under X-ray emission. The X-ray spectrum depends not only on the initial sticking in the atomic levels and the reactivation of the muon but also on intra-atomic transitions due to inelastic collisions, internal and external Auger effect and Stark mixing. The photon intensity per sticking event is calculated using

$$d\gamma_{n\longrightarrow n'}/dt = \sum_{i'(n_{i'}=n')} \sum_{i(n_i=n)} \lambda_{\mathrm{ra}}^{(i'\to i)} P_{i'} \,. \tag{23}$$

The number of X-ray photons emitted per fusion is the most useful quantity that can be measured experimentally. The X-ray yields for the $n' \longrightarrow n$ transition is given by

$$Y(n' \longrightarrow n) = \gamma_{n'\longrightarrow n}\omega_s^0 \,. \tag{24}$$

TABLE I

The fractions of the nl states, $\omega_s^0(i)(\%)$ and the initial sticking probability, $\omega_s^0(\%)$ [34].

nl	Fractions of the nl states, $\omega_s^0(i)(\%)$
$1s$	0.7035
$2s$	0.1007
$2p$	0.0245
$3s$	0.0306
$3p$	0.0088
$3d$	0.0002
$n=4$	0.0171
$n=5$	0.0089
All others	0.0179
Total	0.9121

The K series corresponds to the $n \to 1$ transitions. The intensities of K_α, K_β and K_γ are measurable quantities and can be obtained from further investigation of the muonic atom process of the $\alpha\mu^+$ excited states. Doppler broadening will give us information on the $\alpha\mu^+$ ion velocity during the X-ray emission. For the $K_\alpha(n = 2 \to n = 1)$ X-ray, a central energy of 8.2 keV with a Doppler broadening of 0.5 keV FWHM is expected [45]. The K_α X-ray yield is given as $Y(K_\alpha) = \gamma_{K_\alpha}\omega_s^0$, where γ_{K_α} is the number of X-rays emitted per $\alpha\mu^+$ ion and can be obtained in the same way used to calculate reactivation coefficient (R).

The calculation for muon stripping probability from $\alpha\mu^+$ and the intensity of X-ray transitions have been done by solving a set of coupled differential equations numerically. The stopping power of muonic helium ion as a function of velocity in different fuel densities have been shown in Fig. 1. As it is clear from Fig. 1, the stopping power increases slowly with decreasing velocity to reach a maximum then decreases rapidly. Also the stopping power increases with increasing density. The time-dependent population probabilities $P_i(t)$ for $1s, 2s, 2p, 3s, 3p, 3d$ are shown in Fig. 2 for a deuterium–tritium target at density $\phi = 1.2$ L.H.D (L.H.D $\equiv$ Liquid Hydrogen Density $= 4.25 \times 10^{22}$ atoms/cm^3). The initial populations of all excited states are seen to drop to 0 during the stopping time, and only $1s$ orbital stays occupied.

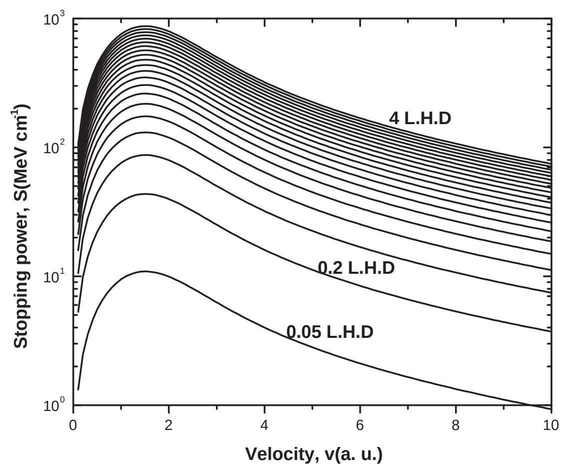


Fig. 1. The stopping power of $\alpha\mu^+$ as a function of velocity for different densities 0.05 and 0.2 to 4 by step of 0.2 L.H.D.

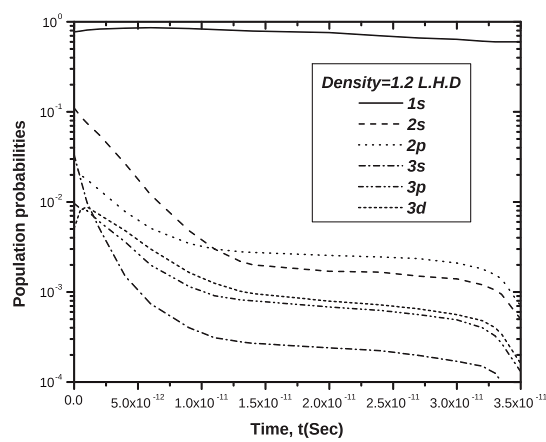


Fig. 2. The population probabilities $P_i(t)$ as a function of time in a D–T target at density $\phi = 1.2$ L.H.D.

The time-dependent stripping fraction, $P_{\rm st}(t)$ and surviving fraction of the initial kinetic energy, E/E_0 are shown in Fig. 3. Slowing down of $\alpha\mu^+$ from $v_{\alpha\mu} = 5.83$ a.u. to $v_{\alpha\mu} \approx 1$ a.u. takes about $t_{\rm stop} \approx 4 \times 10^{-11}$ sec.

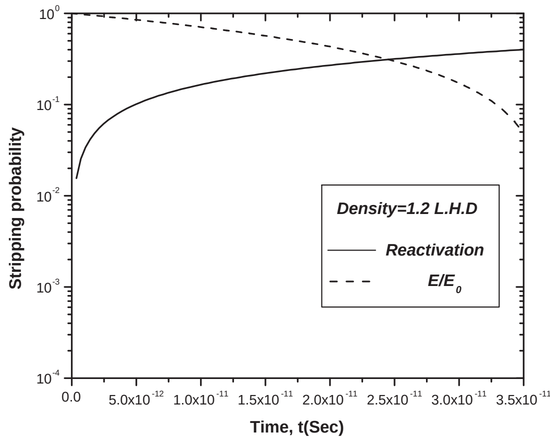


Fig. 3. Stripping fraction, R (heavy solid curve), surviving fraction of initial kinetic energy, E/E_0 (dashed curve) in a D–T target at density $\phi = 1.2\,$L.H.D.

This time is longer than the lifetime of the excited $\alpha\mu^+$ states so that the cascade of $\alpha\mu^+$ actually takes place during the slowing down process. The calculated reactivation coefficient, final sticking and the average number of X-rays per sticking $(K_\alpha, K_\beta, K_\gamma)$ as a function of density are shown in Fig. 4 for $\phi < 4\,$L.H.D. The most K_α radiation actually emitted by $\alpha\mu^+$ atoms that formed in the ground state. If $\alpha\mu^+$ is formed in the $2p$ state more than one $K_\alpha(2p \to 1s)$ X-ray expected per sticking. Our theoretical results for stripping are compared in Table II with other theoretical and

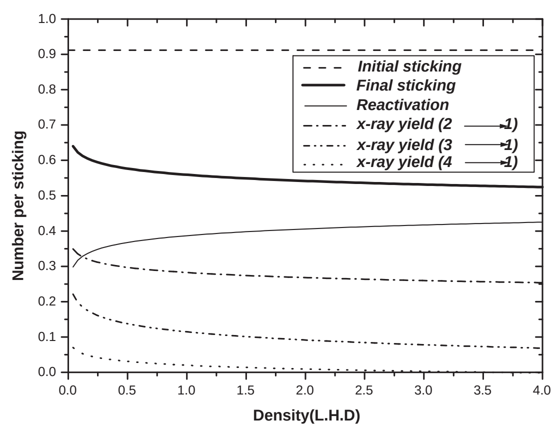


Fig. 4. The density dependence of initial sticking, $\omega_s^0(\%)$, final sticking, $\omega_s(\%)$, reactivation coefficient, R and K-series X-ray per sticking $(K_\alpha, K_\beta, K_\gamma)$ for $dt\mu$ fusion (K_β and K_γ multiplied by factor 3).

experimental data. It is evident that experimental results of the effective sticking probability are smaller than the theoretical calculations, however, our results agree well with experiment.

TABLE II

The reactivation coefficient, R and final sticking, $\omega_s(\%)$ for muonic helium ion in different densities.

Source Density=1.2(L.H.D)	R	$\omega_s(\%)$
Present theory	0.391	0.555
Ref. [37]	—	0.57 ± 0.07
Ref. [38]	0.36	0.57
Ref. [39]	0.248	0.664
Ref. [1]	—	0.59
Experiments		
PSI-Bossy *et al.,* (1987)[40]	—	0.39 ± 0.10
PSI-Breunlich *et al.,* (1987)[28]	—	0.45 ± 0.05
PSI-Petitjean *et al.,* (1993)[12]	—	$0.48 \pm 0.02 \pm 0.04$
LAMPF-Jones *et al.,* (1993)[41]	—	$0.43 \pm 0.05 \pm 0.06$
KEK-Nagamine *et al.,* (1993)[42]	—	0.51 ± 0.004
RIKEN-RAL-Ishida *et al.,* (1999), Liquid [11]	—	0.434 ± 0.030
RIKEN-RAL-Ishida *et al.,* (1999), Solid [11]	—	0.421 ± 0.030
RIKEN-Ishida *et al.,* (2001)[43] Density=1.45(L.H.D)	—	0.532 ± 0.030
Present theory	0.395	0.551
Experiment		
PSI-Petitjean (2001)[44]	—	0.505 ± 0.029

3. Discussion and conclusion

In this investigation, the density dependence of probability of muon reactivation, final sticking coefficient and intensity of X-rays emitted by muonic helium ion have been studied numerically. In order to do this, we consider all reactions that separate muon from muonic helium ion, namely coulomb excitation and de-excitation, ionization, charge transfer, Stark mixing, radiative transitions and Auger de-excitation. Using a set of coupled differential equations, the time dependence of muon reactivation coefficient (R) and surviving fraction of the initial kinematic energy of $\alpha\mu^+$ (E/E_0) in the D–T mixture for different fuel density have been calculated. The measurement of muonic helium ion X-ray provides an independent method to test our knowledge about muon reactivation and sticking. The present calculations for D–T media with a density of $\phi = 1.2$ L.H.D for the K_α X-ray yield per fusion of $Y(K_\alpha) = 0.252\%$ agrees with experimental data of $Y(K_\alpha) = 0.242 \pm 0.017\%$ (liquid) and $Y(K_\alpha) = 0.250 \pm 0.017\%$ (solid) [11]. The intensity ratio of K_β and K_α X-rays emitted by $\alpha\mu^+$, $Y(K_\alpha)/Y(K_\beta) = 0.114$ is still larger than experimental data, $Y(K_\alpha)/Y(K_\beta) = 0.075 \pm 0.012$ (liquid) and $Y(K_\alpha)/Y(K_\beta) = 0.060 \pm 0.012$ (solid) [11]. Results based on our approach shown that the muon reactivation increases when the average number of X-rays per sticking reduces with increasing density. Our calculated results are in good agreement with available experimental data [11, 12, 28, 40–44] at all. The energy required to produce a muon estimated to be about 5000 MeV. Since each deuterium and tritium fusion generates 17.6 MeV, we see that the number of catalysis reactions by a muon should be about 285 to reach the scientific break-even (1/3 of the commercial break-even). The break-even point is reached when the fusion process generates as much energy as was initially put in (*i.e.*, the energy output equals the energy input). The obtained results show that the muon cycle coefficient increases almost slowly with the density of deuterium and tritium mixture. The output energy of the number of catalysis reactions by a muon in it's lifetime ($\tau_\mu = 2.197\,\mu$sec), is much smaller than the input energy required to produce a muon. Therefore, a fusion energy system based on the muon catalyzed fusion in deuterium and tritium fuel seems to be viable at plasma conditions with fuel densities about 100 times of L.H.D.

REFERENCES

[1] J.S. Cohen, G.M. Hale, C-Y. Hu, *Hyperfine Interact.* **101/102**, 349 (1996).

[2] D. Harley, *Phys. Rev.* **A45**, 8981 (1992).

[3] A. Gula, *Acta Phys. Pol. B* **16**, 589 (1985).

[4] J. Gronowski *et al., Acta Phys. Pol. A* **106**, 795 (2004).

[5] A. Adamczak, M.P. Faifman, *Phys. Rev.* **A64**, 052705 (2001).

[6] L.I. Bracci, G. Fiorentini, *Nucl. Phys.* **A364**, 383 (1981).

[7] A. Igarashi, T. Shirai, *Phys. Rev.* **A51**, 4699 (1995).

[8] M.C. Struensee, J.S. Cohen, *Phys. Rev.* **A38**, 44 (1988).

[9] H.A. Bethe, E. Salpeter, *Quantum Mechanics of One- and Two- Electron Atoms*, Academic Press, New York 1957.

[10] M. Leon, H.A. Bethe, *Phys. Rev.* **127**, 636 (1962).

[11] K. Ishida, K. Nagamine *et al., RIKEN Rev.* **20**, 3 (1999).

[12] C. Petitjean *et al., Hyperfine Interact.* **82**, 273 (1993).

[13] V.M. Bystritsky *et al., Phys. Rev.* **A71**, 032723 (2005).

[14] V.M. Bystritsky, *Yad. Fiz.* **58**, 688 (1995) [*Phys. At. Nucl.* **58**, 631 (1995)].

[15] C. Petitjean, *Nucl. Phys.* **A543**, 79c (1992).

[16] T. Matsuzaki *et al., Phys. Lett.* **B557**, 176 (2003).

[17] T. Matsuzaki *et al., Prog. Theor. Phys. Suppl.* **154**, 225 (2004).

[18] V.M. Bystritsky *et al., Eur. Phys. J.* **D8**, 75 (2000).

[19] T. Matsuzaki *et al., Nucl. Instrum. Methods Phys. Res.* **A465**, 365 (2001).

[20] W.H. Breunlich *et al., Annu. Rev. Nucl. Part. Sci.* **39**, 311 (1989).

[21] G.Y. Korenman, *Hyperfine Interact.* **101/102**, 81 (1996); G.A. Fesenko, G.Y. Korenman, *Hyperfine Interact.* **101/102**, 91 (1996).

[22] L.I. Menshikov, L.I. Ponomarev, *Pis'ma Zh. Eksp. Teor. Fiz.* **39**, 542 (1984) [*JETP Lett.* **39**, 663 (1984)].

[23] W. Czaplinski *et al., Phys. Rev.* **A50**, 525 (1994).

[24] V.R. Bom *et al., J. Exper. Theor. Phys.* **100**, 663 (2005).

[25] A.J. Caffery *et al., Muon Catal. Fusion* **1**, 53 (1987).

[26] S.E. Jones *et al., Muon Catal. Fusion* **1**, 21 (1987).

[27] V.M. Bystritsky *et al., Phys. Lett.* **B94**, 476 (1980); *Zh. Eksp. Teor. Fiz.* **80**, 1700 (1981) [*Sov. Phys. JETP* **53**, 877 (1981)].

[28] W.H. Breunlich *et al., Phys. Rev. Lett.* **58**, 329 (1987).

[29] S.E. Jones *et al., Phys. Rev. Lett.* **51**, 1757 (1983).

[30] S. Eliezer, Z. Henis, *Fusion Technology* **26**, 46 (1994).

[31] M.P. Faifman *et al., Hyperfine Interact.* **101/102**, 179 (1996).

[32] P. Ackerbauer *et al., Nucl. Phys.* **A652**, 311 (1999).

[33] L. Bogdanova *et al., Zh. Eksp. Teor. Fiz.* **83**, 1615 (1982) [*Sov. Phys. JETP* **56**, 931 (1982)].

[34] C-Y. Hu, G.M. Hale, J.S. Cohen, *Phys. Rev.* **A49**, 4481 (1994).

[35] D.R. Bates, G. Griffing, *Proc. Phys. Soc.* **A66**, 961 (1953).

[36] N. Tsoulfanidis, *Measurement, "Detection of Radiation"*, second edition, Taylor, Francis publication, USA 1995.

[37] V.E. Markushin, *Muon Catal. Fusion* **3**, 395 (1988).

[38] H. Refelski *et al., Prog. Part. Nucl. Phys.* **22**, 297 (1989).

[39] H. Takahashi, *Muon Catal. Fusion* **1**, 237 (1987).

[40] H. Bossy *et al., Phys. Rev. Lett.* **59**, 2864 (1987).

[41] S.E. Jones, S.F. Taylor, A.N. Andeson, *Hyperfine Interact.* **82**, 303 (1993).

[42] K. Nagamine *et al., Hyperfine Interact.* **82**, 343 (1993).

[43] K. Ishida *et al., Hyperfine Interact.* **138**, 225 (2001).

[44] C. Petitjean, *Hyperfine Interact.* **138**, 191 (2001).

[45] K. Ishida, *et al., J. Phys. G* **29**, 2043 (2003).