Vol. 39 (2008) ACTA PHYSICA POLONICA B No 3

A STUDY OF LEVELS IN ^{149}Nd USING THE (d,p) AND (d,t) REACTIONS

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(Received October 3, 2007)

The excited states in the ^{149}Nd nucleus up to an excitation energy of about 2 MeV were investigated by (d,p) and (d,t) reactions with good energy resolution. The reaction products were analysed in a magnetic spectrograph and detected with photographic emulsion. Triton spectra from the ^{150}Nd$(d,t)^{149}$Nd reaction were measured at thirteen reaction angles from 5° to 125° and the ^{148}Nd$(d,p)^{149}$Nd reaction was studied at three angles, 60°, 90° and 125°, using beams of 12.1 MeV deuterons. The (d,t) angular distributions and ratios of (d,t) and (d,p) cross sections at selected angles were used to determine l values for a number of transitions. The reaction products were analysed with standard DWBA calculations and spectroscopic factors were deduced. The measured triton angular distributions are well described by the DWBA calculations and on these bases it was possible to give unambiguous assignments for a numbers of levels.

PACS numbers: 24.10.Fr, 24.50.+g, 25.50.Gx

1. Introduction

The ^{149}Nd nucleus with 89 neutrons is of special interest since it lies at the lower limit of the deformed region where the transition between spherical and deformation shapes take place. Kenephic and Sheline [1] suggest that in nuclei from the transition region deformed and spherical states may coexist at low excitation energy. In the Burke *et al.* [2] paper about the level structure of the ^{149}Nd nucleus it was concluded that the total observed intensities for each of the l-values 0, 1, 2, 3, 4, 5 and 6 cannot be explained by the extreme single-particle shell model but are consistent with those predicted by the Nilsson model. However, the splitting of the strength among the observed states cannot be explained by the basic Nilsson model.

In previous studies of the neighbouring isotones ^{151}Sm, ^{153}Gd and ^{155}Dy a number of rotational band have been reported [3–5]. The level structure of the ^{149}Nd nucleus is difficult to interpret and this difficulty is due partly to a shortage of experimental data. From the experimental point of view the level structure of the ^{149}Nd nucleus has been studied by means of different kinds of nuclear reactions, especially those induced by light projectiles. In particular, the level scheme of ^{149}Nd has been investigated using the one nucleon transfer reactions (d,t) and $(^3\mathrm{He},\alpha)$ with unpolarised [2, 6] and polarised deuterons [7], the (p,d) reaction [8], decay studies [9–12], and an (n,γ) study [13]. A (d,t) study with a polarised deuteron beam was helpful in establishing spins for several levels in ^{149}Nd [7]. Neutron capture and fusion reaction are generally supposed to populate all levels, whereas one-nucleon transfer reactions induced by light projectiles are supposed to be selective, *e.g.* the (d,t) reaction selectively excites neutron-hole states and the (p,d) reaction neutron–particle states, respectively. The results obtained by studies of the level scheme of ^{149}Nd are summarised in the Nuclear Data Sheets (NDS) compilation [14], where a more complete list of references can be found.

In order to construct a nuclear model for the ^{149}Nd nucleus from the transitional region good experimental data are needed such as energies, spins, particle spectroscopic factors, *etc.*

In the present paper states of the ^{149}Nd nucleus have been populated via the $^{150}\mathrm{Nd}(d,t)^{149}\mathrm{Nd}$ and $^{148}\mathrm{Nd}(d,p)^{149}\mathrm{Nd}$ reactions at 12.1 MeV bombarding energy, which is close to the Coulomb barrier for the process and as indicated in paper [15] produces more distinct l-value patterns in the angular distributions of the tritons from the (d,t) reaction. Accurate measurements of the differential cross sections for transitions to about a few tens of excited states of the ^{149}Nd nucleus, in a high energy resolution experiment, allowed the spin and parity for many levels up to about 1500 keV of excitation energy to be confirmed. In this paper the experiments, results and spin and parity attributions are described and briefly discussed. The present results are in good agreement with previous one-nucleon transfer reactions [2, 5–7].

2. Experimental procedure and results

The experimental procedures were identical to those described earlier [15]. Evaporated neodymium targets isotopically enriched in ^{148}Nd and ^{150}Nd up to about 95% were bombarded by 12.1 MeV deuterons with a beam current of 100–200 nA from the Niels Bohr Institute tandem Van de Graaff accelerator. The targets were approximately 110 μg/cm^2 thick, evaporated onto 10–20 μg/cm^2 thick carbon backings. The reaction products from the $^{149}\mathrm{Nd}(d,p)$ and $^{150}\mathrm{Nd}(d,t)$ reactions were analysed in a single gap Elbek-type [19] magnetic spectrograph and detected in Ilford K2 nuclear emulsion with a thickness of 25 microns. The triton spectra were recorded for thirteen angles in 5° intervals from 5° to 50° and then for 60°, 90° and 125°. The proton spectra were recorded only at three angles, 60°, 90° and 125°. The total integrated deuteron beam current was approximately from 4000 μC to 8000 μC. The overall energy resolution was 12–15 keV (FWHM) in detection of the outgoing tritons and about 2–3 keV better for the protons. After development the nuclear emulsions were scanned manually by a microscope in 0.20 mm strips. The uncertainties in excitation energies are less than 3 keV for low lying states (below 1 MeV) and less than 4–6 keV for states above 1 MeV of excitation. Additionally, the excitation energies were determined through the calibration of the spectra using a polynomial of rank 2, whose parameters were set by reproducing the excitation energies of about 10 levels determined in γ-ray experiments [14] and identified in the present triton and proton spectra. The absolute cross sections were determined by normalisation to the cross section for elastic deuteron scattering. The absolute cross section values are not known to better than ±20% due to uncertainties in the normalisation procedure. The relative values of the cross sections for a state measured at different angles from the same target nucleus should have a better accuracy and are of statistical nature only (about ±10%).

A representative experimental triton spectrum at $\theta_{\text{lab}} = 50°$ for the ^{150}Nd$(d,t)^{149}$Nd reaction is shown in Fig. 1. The proton and triton spectra obtained from the ^{150}Nd$(d,t)^{149}$Nd and ^{148}Nd$(d,p)^{149}$Nd reactions at an angle of 60° are shown in the form of lines in Fig. 2. The vertical lines represent the energies at which proton and triton groups were found and the heights of the lines indicate the absolute cross sections for the states. Fig. 2 reflects the direct correspondence between levels excited by the (d,t) and (d,p) reactions. Generally, the pick-up (d,t) excitation energies of most of the levels correlate well with those observed for the neutron stripping (d,p) reaction. The level energies obtained from average level positions excited by the (d,p) and (d,t) reactions at three different angles (60°, 90° and 125°) are listed in Table I, which also summarises the differential cross sections measured at 60°, 90° and 125° and the suggested assignments for most of the investigated levels, and the extracted average spectroscopic factors. The level assignments were taken from the NDS compilation [14] and were also determined in the present measurements from the triton angular distributions and the (d,p) reaction from the ratios:

$$R = (d\sigma/d\Omega)_{60°}/(d\sigma/d\Omega)_{90°} \quad \text{and} \quad R = (d\sigma/d\Omega)_{60°}/(d\sigma/d\Omega)_{125°}\,. \quad (1)$$

To allow spectroscopic information to be extracted from the experimental data a series of DWBA calculations was performed in the finite-range approximation with the DWUCK code [21]. The deuteron, proton and triton optical model parameters were taken from the literature [15, 20], since these parameters have been successfully applied to the analysis of (d,t) and (d,p) reactions on rare-earth nuclei.

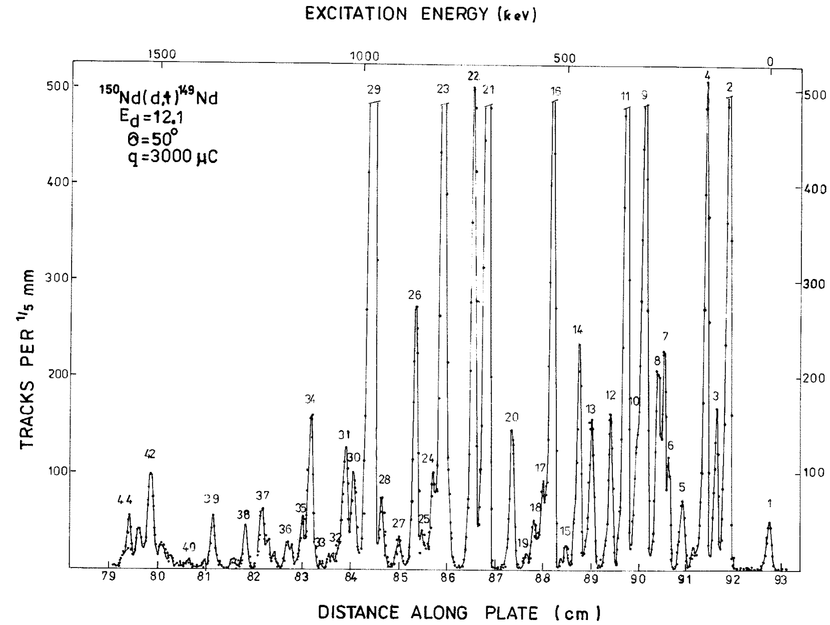
EXCITATION ENERGY (keV)
DISTANCE ALONG PLATE (cm)
TRACKS PER 1/5 mm
150Nd(d,t)149Nd
Ed=12.1
Θ=50°
q=3000 μC

Fig. 1.

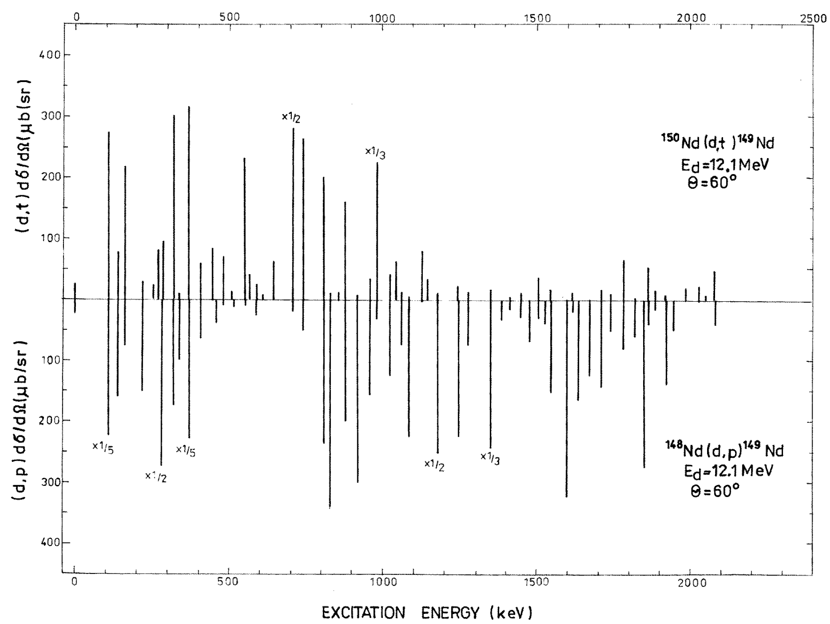
(d,t) dσ/dΩ(μb/sr)
(d,p) dσ/dΩ(μb/sr)
150Nd (d,t)149Nd
Ed=12.1MeV
Θ=60°
148Nd (d,p)149 Nd
Ed=12.1 MeV
Θ=60°
EXCITATION ENERGY (keV)

Fig. 2.

TABLE I

Levels populated in the $^{148}\mathrm{Nd}(d,p)^{149}\mathrm{Nd}$ and $^{150}\mathrm{Nd}(d,t)^{149}\mathrm{Nd}$ reactions. Columns 1 and 2 give the adopted energies, spins and parities of the ^{149}Nd levels [14]; columns 3 and 4 give the average energies observed at 60°, 90° and 125°; in columns 5–10 the cross sections are presented; column 11 gives the deduced momentum transfer l in the present experiment; columns 12 and 13 give the spectroscopic factors; and in column 14 the successive number of levels presented in Fig. 1 are given.

Adopted [14]		Levels populated in present $^{148}\mathrm{Nd}(d,p)$ and $^{150}\mathrm{Nd}(d,t)$ reactions											
E_{exc} (keV)	J^{Π}	Energy average (d,p) (keV)	(d,t) (keV)	$d\sigma/d\Omega(d,p)(\mu\mathrm{b/sr})$ 60°	90°	125°	$d\sigma/d\Omega(d,t)(\mu\mathrm{b/sr})$ 60°	90°	125°	l^*	(d,p) $(2J+1)S_{lj}^{(+)}$	(d,t) $S_{lj}^{(-)}$	peak no*
0.0	$5/2^-$	0.0	0.0	26	9	7	23	8	22	3	0.05	0.02	1
108.521	$7/2^-$	108	108	1115	461	202	275	141	60	3	2.09	0.40	2
138.447	$5/2^-$	138	138	159	91	44	77	35	20	3	0.37	0.12	3
165.087	$1/2^-,3/2^-$	166	165	78	34	12	225	98	36	1	0.05	0.10	4
220.706	$9/2^-$	221	220	155	85	65	27	35	19	5	2.35	0.49	5
258.330	$3/2^-$		256				26	10	6	1		0.01	6
270.859	$(9/2^+)$		268				81	55	20	4		0.22	7
285.481	$1/2^-$	282	285	550	235	101	95	46	20	1	0.37	0.05	8
316.230	$(5/2^-,7/2^-)$												
321.144	$(5/2^-,7/2^-)$	318	320	176	114	57	302	181	56	3	0.42	0.48	9
332.936	$5/2^+$												
340.6	$11/2^+,13/2^+$	341	339	98	70	56	10	43	36	6	3.11	1.14	10
365.930	$3/2^-$	366	366	1153	382	177	316	137	53	1	0.67	0.15	11
403.728	$1/2^-$	407	403	75	34	18	61	20	11	1	0.06	0.03	12
447	$5/2^-,7/2^-$		447				86	57	19	3		0.16	13
459.529	$(3/2^-,5/2^-)$	459		39	22	15				3	0.09		
474.56	(3/2,5/2/7/2)												
482.672	$1/2+$	483	482	7	10		67	27	12	0	0.01	0.02	14
517.44	(3/2,5/2,7/2)	521	510	14	22	11	9	20	16	6	0.65	0.63	15
548.656	$3/2^-$	551	548		6		237	114	44	1	0.01	0.13	16
571.440	$3/2^+$		571				43	30	9	2		0.03	17
591	$(5/2^+)$	594	589	27	32	13	26	16	20	2	0.03	0.03	18
613			611						7	5.6		0.32	19
647	$9/2^-,11/2^-$		645				65	113	71	5		1.77	20
705.128	(3/2,5/2)												
709.044	$(3/2,5/2^-)$	709	706	18	17	7	561	406	185	2	0.02	0.49	21
741.06	$3/2^+$	745	738	49	7		267	193	103	2	0.02	0.25	22
796	$1/2^+$												
814.40	$1/2^+$	807	812	240	125	82	401	258	132	0	0.15	0.23	23
836		830	831	394	199	91	10	10	14	1	0.25	0.02	24
861			860				16	21	13	4		0.25	25
881.36	$3/2^+$	887	880	200	142	66	159	102	57	2	0.17	0.15	26
920.65	(3/2,5/2,7/2)	916	918	303	122	50	5	27	13	1	0.16	0.03	27
956		957	959	156	89	47	34	33	18	3	0.29	0.11	28
985.15	$1/2^+$	992	984	33	18	10	977	580	200	0	0.02	0.53	29
1025	$1/2^-,3/2^-$	1029	1025	172	106	58	44	28	16	1	0.13	0.04	30
1045	$3/2^+,5/2^+$		1044				66	55	28	2		0.08	31
1067		1064	1060	72	69	28		13	10				32
1081		1086		226	91	44	8						33
1129	$3/2^+,5/2^+$		1128				83	85	44	2		0.12	34
1149	$3/2^+,5/2^+$		1147				33	17	13	2		0.04	35
1178													
1189		1181		507	219	99	14						36
1245		1248	1240	227	139	56	23	29	12	1	0.14	0.03	37
1282	$1/2^-$	1283	1287	75	76	32	13	12	7	1	0.07	0.02	38
1359	$1/2^+$	1360	1353	717	337	157	17	10	5	0	0.28	0.01	39
		1386			33	16							
		1415	1413		16	6	5	2	2				40
1465		1446			31	17							
1481		1479			75	47	13						41

Adopted [14]		Levels populated in present ^{148}Nd(d,p) and ^{150}Nd(d,t) reactions											
$E_{\rm exc}$ (keV)	J^{Π}	Energy average (d,p) (keV)	(d,t) (keV)	$d\sigma/d\Omega(d,p)(\mu$b/sr) 60°	90°	125°	$d\sigma/d\Omega(d,t)(\mu$b/sr) 60°	90°	125°	l^*	(d,p) $(2J+1)S_{lj}^{(+)}$	(d,t) $S_{lj}^{(-)}$	peak no*
1505	$1/2^-,3/2^-$	1509	1503		21	15	36		10	1	0.03	0.04	42
1531	$1/2^-,3/2^-$	1533			43	25							
1553	$1/2^-,3/2^-$	1558	1547		146	78	18						44
		1594		325	101	55							
1622	$9/2^-,11/2^-$	1621	1616		20	13	9	22	11				
		1634		185	34	13							
		1671		124	42	16							
1709	$9/2^-,11/2^-$	1709	1702	146	71	34		34	26				
1718	$3/2^+,9/2^+$		1724					12	11				
1736	$3/2^+,5/2^+$	1747	1741	51	31	10	13	15	5				
1785	$3/2^+,5/2^+$	1783	1789	77	31	23	67	57	44				
1797	$3/2^+,5/2^+$												
1817		1818	1821	61	29	25	3	8	8				
		1852		178	83	48							
1858	$3/2^+,5/2^+$	1868	1864		55	17	53	52	24				
1882		1885	1884		13	15	1	12	8				
1909		1925	1918		141	60	5	4	6				
1957	$3/2^+,5/2^+$	1947			49	25							
1980	$3/2^+,5/2^+$		1983				19	10	11				
2029	$3/2^+,5/2^+$		2033				24	26	20				
2055			2052					8	6				
			2065					16	7				
2078	$3/2^+,5/2^+$	2094	2086		43	23	54	54	32				
2102			2103					8	5				
2151		2140			157	75							
2227	$3/2^+,5/2^+$		2237				11	17					
2293			2303				47	48					

The calculations were performed for neutrons transferred to the $3p_{1/2,3/2}$; $2d_{3/2,5/2}$; $2f_{5/2,7/2}$; $1h_{9/2,11/2}$, $1i_{13/2}$, and $3s_{1/2}$ shell model states. The DWBA calculations we used to extract the spectroscopic factors $S_{lj}^{(+)}$ and $S_{lj}^{(-)}$ by normalising the experimental cross sections obtained at 60°, 90° and 125° to the DWBA cross sections:

$$\frac{d\sigma}{d\Omega} = N^{(+)} S_{lj}^{(+)} (2J+1) \sigma_l^{(+)}(\theta) \quad \text{for the } (d,p) \text{ reaction}\,, \tag{2}$$

$$\frac{d\sigma}{d\Omega} = N^{(-)} S_{lj}^{(-)} (2J+1) \sigma_l^{(-)}(\theta) \quad \text{for the } (d,t) \text{ reaction}\,, \tag{3}$$

where $\sigma_l^{(+)}(\theta)$ and $\sigma_l^{(-)}(\theta)$ are the single particle transfer cross sections obtained from the DWBA calculations for the (d,p) and (d,t) reactions, respectively, and $\mathrm{N}^{(\pm)}$ is the normalisation factor assumed to be 1.5 and 3.0 for the (d,p) and (d,t) reactions, respectively. The spectroscopic factors $S_{lj}^{(+)}(2J+1)$ and $S_{lj}^{(+)}$ extracted from the (d,p) and (d,t) reactions are listed in Table I.

The measured triton angular distributions from the ^{150}Nd$(d,t)^{149}$Nd reaction and the results of the calculated DWBA angular distributions (solid and broken lines) for different l transfers are compared in Figs. 3–8. The error bars in the figures refer to statistical errors only.

The DWBA curves are quite different for different l-transfers and the clear structure of the experimental angular distributions is rather well described by the theoretical calculations.

The distributions for transitions with $l = 0$ are shown in Fig. 3. All these distributions have a pronounced minimum at $\theta \approx 20^\circ$ and the shapes are distinctly different from the other distributions. The spin assignments of $1/2^+$ for these levels are certain.

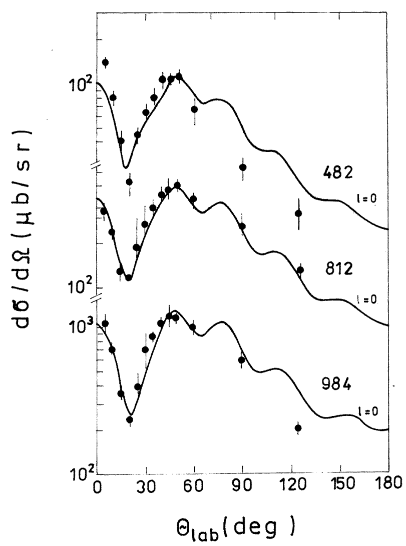

Fig. 3.

There are several states populated in the (d, t) reaction by an $l = 1$ neutron transfer. The corresponding distributions are shown in Figs. 4a and 4b. In nearly all cases the observed intensity at large angles is considerably smaller than the calculated intensity. At forward angles the shapes are almost the same with a small secondary maximum between 15° and 20°. The differences at backward angles are unexplained (but could be due to the inelastic effect, j-dependence effect and other mixing phenomena).

The distributions for $l = 2$ and 3 are shown in Figs. 5a, 5b and 6, respectively, and are quite similar. The maxima seem to occur at slightly lower angles for the $l = 2$ distributions than for the $l = 3$ distributions.

The $l = 4$ distributions shown in Fig. 7 again peak at somewhat larger angle than the $l = 2$ and 3 distributions and also fall off quite steeply at forward angles.

Finally, the distributions with $l = 5$ and 6 are shown in Fig. 8. The cross sections at backward angles are relatively flat.

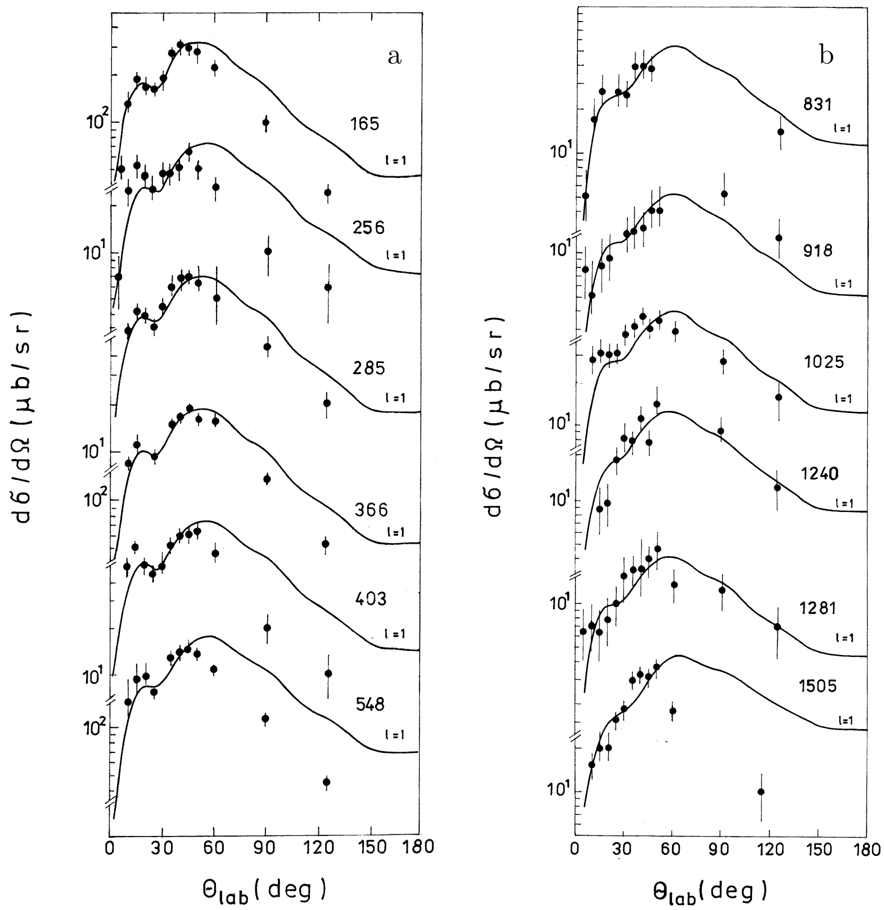

Fig. 4.

An inspection of the angular distributions (Figs. 3–8) shows that the data are well described by the DWBA calculations and that the shapes of the angular distributions are quite characteristic of the different transferred l values, allowing the firm assignment of the transferred angular momentum. In particular, the positions of the first minima for the $l = 0$ transfers are excellently reproduced.

As can be seen from Table I, the present data nicely confirm most previous assignments reported in the adopted level scheme [14] and make some new assignments possible. The results can be summarised as follows:

- Three levels at 482, 812 and 984 keV are populated in the present work by $l = 0$ transitions (Fig. 3) and consequently have spin and parity $1/2^+$, confirming the previous spin assignments.

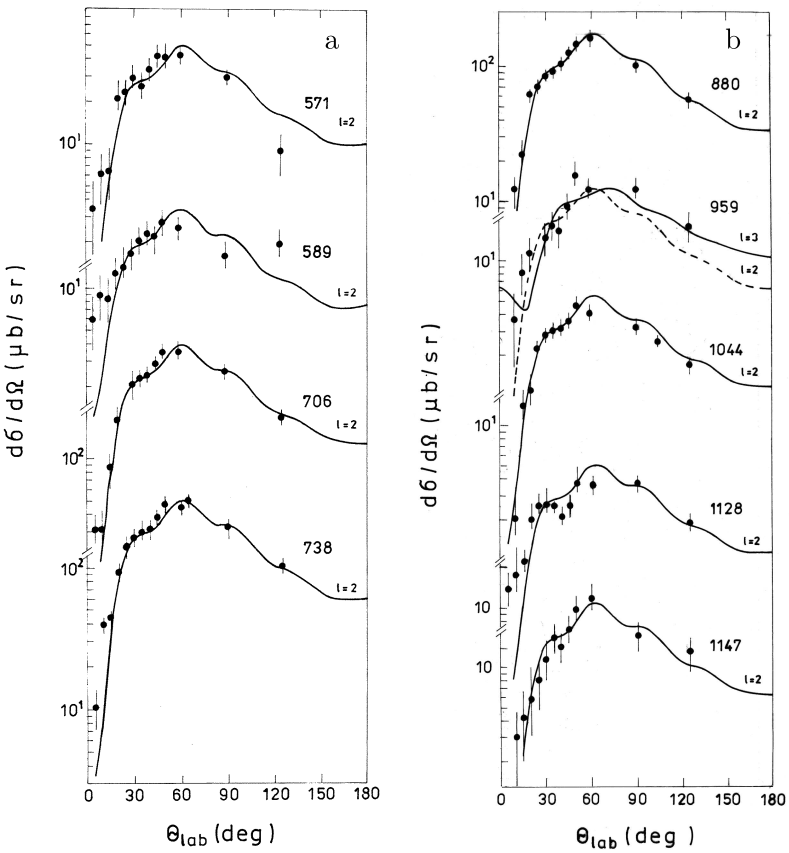

Fig. 5.

- The levels at 165, 256, 285, 366, 403, 548, 1025, 1287 and 1505 keV are populated by $l = 1$ transitions (Figs. 4a, 4b) in the present work and were previously known to be populated by $l = 1$ transitions. The levels at 831, 918 and 1240 keV are reasonably reproduced by $l = 1$ transitions. The previous spin assignments were tentative or unknown and the present results indicate $l = 1$ transfers for these levels.

- For the levels at 571, 589, 738, 880, 1044, 1128 and 1147 keV the earlier assignments with $l = 2$ transitions (Figs. 5a, 5b) have been confirmed. The level at 706 keV previously assigned as (3/2, 5/2) exhibits an $l = 2$ character in the present study. The level at 959 keV was previously unknown, and the (d,t) angular distribution is equally well reproduced by $l = 2$ or 3 DWBA calculations (but the $l = 3$ is preferred). This level is also quite strongly populated in the (d,p) reaction indicating an $l = 3$ transition on the basis of the R value (1).

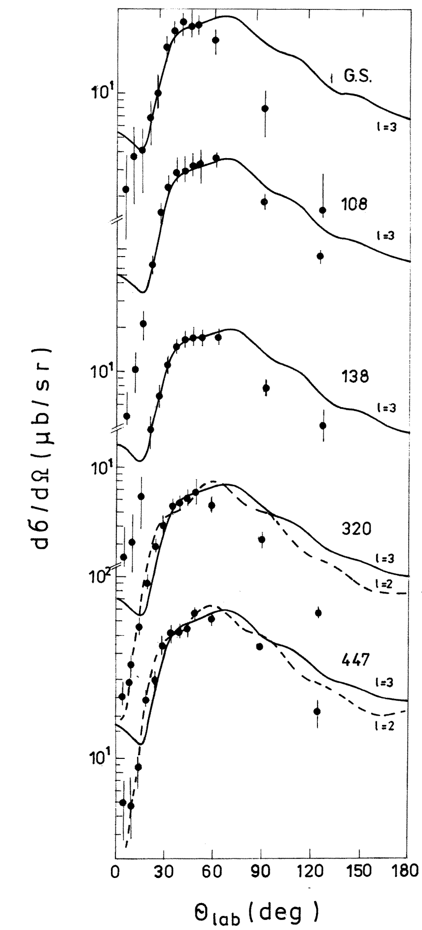

Fig. 6.

- Five levels populated with $l = 3$ transitions (Fig. 6) confirm the previous assignments.
- The levels populated by high l-value (4, 5, 6) transitions (Figs. 7 and 8) are well reproduced in the present paper. A new assignment with an $l = 5$ transition can be attributed to the level at 860 keV excitation.

3. Discussion

The ^{150}Nd nucleus can be regarded as a core of $N = 82$ neutrons building a closed shell with eight valence neutrons in the range $N = 82$ to 126. These eight valence neutrons are distributed among the $3p_{1/2,3/2}$; $2f_{5/2,7/2}$; $1h_{9/2}$

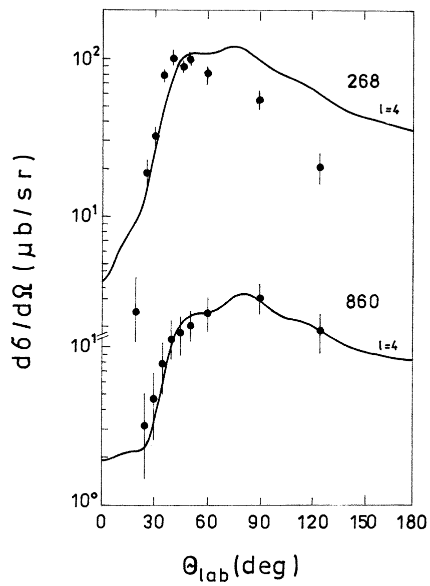

Fig. 7.

and $1i_{13/2}$ orbits which form the $N = 82$ to 126 shell and most of them should correspond to low lying particle states in the ^{149}Nd nucleus. By summing the (d,t) spectroscopic factors, $\sum S_{lj}^{(-)}$, obtained for all the states originating in the $N = 82$ to 126 shell, one should get the average number of particles in the target outside the $N = 82$ core. Theoretically this number should be eight for the ^{150}Nd nucleus. In Table I the spectroscopic strengths, $S_{lj}^{(-)}$, are listed for the states where the assignments are fairly certain. The spectroscopic strength, $\sum S_{lj}^{(-)}$, summed over all states originating in the $N = 82$ to 126 shell is 7.5, which is close to the expected value when a fairly large uncertainty in the spectroscopic strength is taken into account. The present spectroscopic factors $S_{jl}^{(-)}$ generally are in good agreement with previous determinations presented in Ref. [7].

The levels observed in the (d,t) reaction at higher excitation energies correspond to the removal of neutrons from the closed $N = 82$ core, mostly from the $3s_{1/2}$, $3d_{3/2}$, $1h_{11/2}$ and $1g_{7/2}$ neutron orbitals. Since these neutron orbitals are essentially completely filled, they should yield large spectroscopic factors in the pick-up reaction.

We also observed some levels with a large (d,p) spectroscopic strength $(2J + 1)\ S_{lj}^{(-)}$, indicating that these levels come mostly from the empty $N = 82$–126 neutron shell (indicating the single particle character of the level).

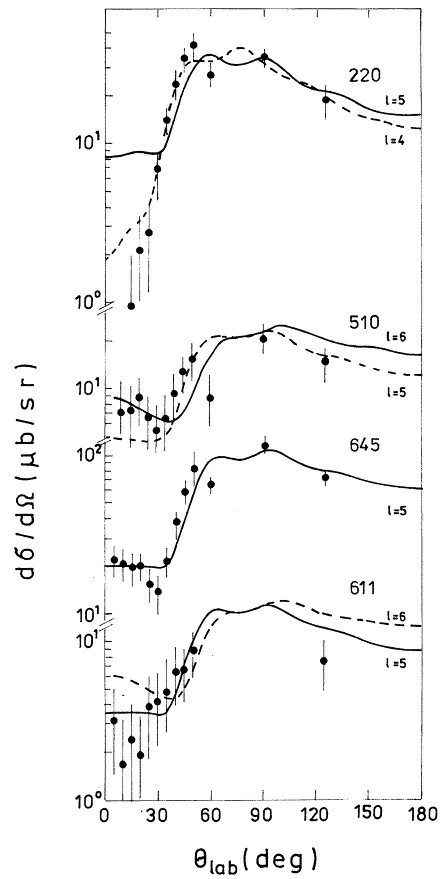

Fig. 8.

The previously published experimental data also indicate that the Nilsson model can be applicable in the interpretation of the level structure of the ^{149}Nd nucleus [2, 6, 7, 9–11]. In the paper of Burke *et al.* [2] the cross sections observed for the population of the ^{149}Nd levels were discussed in terms of the Nilsson model. In this paper arguments that the $5/2^-$ [523] or $5/2^-$ [512] Nilsson orbitals are good candidates for the ground state and the first excited state at 108 keV excitation energy are presented. An excellent example that the Nilsson model is applicable to the $N = 89$ nuclei [6] is the rotational band associated with the $11/2^-$ [505] Nilsson orbital. For the neodymium $N = 87$ and 89 isotopes this band can be described using a rotational parameter $\hbar^2/2J$ which is not significantly different from the ones used to described well deformed nuclei. In the paper of Pinston *et al.* [9] some success was obtained in describing the positive parity levels of the

^{149}Nd nucleus using the Nilsson model with the inclusion of the Coriolis coupling effect and $\Delta N = 2$ interactions for the $1/2^+$ [660]; $3/2^+$ [651]; $1/2^+$ [400] and $3/2^+$ [402] Nilsson orbitals. Also Pinston *et al.* [9] concluded that negative parity states cannot be grouped in any obvious rotational band and it seems to be very hard to explain them in the framework of the Nilsson model. The papers of Haugen *et al.* [7] and Katajanheimo *et al.* [11] support the conclusions of Pinston *et al.* [9] but with a few discrepancies. According to the paper of Katajanheimo *et al.* [11] the $7/2^-$ level at 108.5 keV excitation energy seems to be the first rotational member of the $5/2^-$ [523] ground state band. Also Pinston *et al.* [9] suggest that the ground state can be identified as the $5/2^-$ [523] or $5/2^-$ [512] Nilsson orbital but with some restrictions. Reference [10] suggests that the ground state rotational band can be constructed on the $5/2^-$ (GS), $7/2^-$ 108.5 keV and $9/2^-$ 220.7 keV levels.

Taking into account the arguments of previous papers [2–11] we also tried to interpret some low lying levels of the ^{149}Nd nucleus populated in the present (d,t) and (d,p) studies in the framework of the Nilsson model.

If one performs a one step process for single nucleon transfer reactions (pick-up $(-)$ or stripping $(+)$) on a deformed even–even target nucleus, the cross section, according Elbek and Tjom [22], for the level of spin J in a rotational band is given by the formula:

$$d\sigma/d\Omega^{(\pm)} = 2N^{(\pm)}C_{jl}^2\sigma^{(\pm)}(\theta)f^2(U,V)\,, \tag{4}$$

where and $N^{(\pm)}$ are $\sigma^{(\pm)}\theta$ defined as in formulae (2) and (3), the C_{jl}^2 values are the expansion coefficients of the Nilsson state from which the nucleon was picked-up (or transferred) in terms of spherical shell model states described by j and l ($j = J$ for a zero spin target); $f^2(U,V)$ is the occupation amplitude U or V for stripping or pick-up reactions, respectively. The expansion coefficients C_{jl}^2 have been tabulated by Chi [23] for different values of the deformation parameter β. According to formula (4) the resulting distribution of intensities of the rotational states is characteristic of the Nilsson band and often suffices for its identification. The intensity patterns are frequently called "fingerprints" and are commonly used for identifying Nilsson bands in studies of well deformed nuclei by one nucleon transfer reactions [22]. The ^{149}Nd nucleus is not a good case as a deformed nucleus for application of the Nilsson model but it is nevertheless the method we used to describe the negative parity ground state band and also one of the positive parity band. The only states which would be expected on the basis of the Nilsson model to have appreciable $l = 3$ strength in this region are the $5/2^-$ [523] and $5/2^-$ [512] Nilsson orbitals. The first of these appears as the ground state, according the Elbek and Tjøm [22] systematics, of the Gd, Dy, Er and Yb isotopes with $N = 95$ and 97 and is therefore also possibly expected to be found near or above the Fermi surface in the $N = 89$ nuclei. The second Nilsson orbital appears as a ground state for Er and Yb with $N = 103$ and for $N = 89$ is expected to be found quite far above the Fermi surface. Table II represents the so called "fingerprint" pattern for these two Nilsson orbitals in comparison with experiments for (d,t) and (d,p) reactions for the ground state rotational band. These theoretical values were calculated using expression (4) with normalisation factors $N^{(\pm)}$ equal to 1.5 and 3.0 for the (d,p) and (d,t) reactions, respectively. The DWBA single-particle transfer cross sections $\sigma_l^{(\pm)}(\theta)$ were calculated at $\theta = 90°$ for a (d,p) reaction Q-value of $+3$ MeV and for a (d,t) reaction Q-value of -2 MeV. These Q-values are used as reference values to which all experimental cross sections were reduced before comparison with the theoretical cross sections. The occupation amplitudes U^2 and V^2 were used as 1. The C_{jl}^2 were taken from the Chi [23] compilation for a deformation parameter of $\beta = 0.3$. The spins of the ground state (GS) $5/2^-$, first excited state at 108.5 keV $7/2^-$ and fourth excited state at 220.7 keV $9/2^-$ are well established by previous work and the present (d,t) and (d,p) reaction studies indicate $l = 3$ and $l = 5$ transitions, respectively. These states may be regarded as members of the GS rotational band and comparisons with theoretical calculations are presented in Table II. From Table II we see that the "fingerprint" pattern for

TABLE II

(d,t) and (d,p) population of the $5/2^-$ [523] and $5/2^-$ [512] Nilsson orbitals "fingerprint pattern".

	$(d,t)(d\sigma/d\Omega)(\mu\text{b/sr})$ $\theta=90°, Q=-2$ MeV, $V^2=1$				$(d,p)(d\sigma/d\Omega)(\mu\text{b/sr})$ $\theta=90°, Q=+3$ MeV, $U^2=1$			
Nilsson orbital/spin	5/2	7/2	9/2	11/2	5/2	7/2	9/2	11/2
$5/2^-$ [523] theory	41	47	76	5	32	37	59	4
$5/2^-$ [512] theory	5	450	14	6	4	356	11	4
Exp.	8	141	35	—	9	461	85	—

the $5/2^-$ [512] Nilsson orbital reproduces reasonably well the experimental results. The $5/2^-$ [512] band is characterised by a strong $7/2^-$ member, which reflects experiment where this level is strongly excited by both (d,t) and (d,p) reactions. Also, the inertial parameters obtained are between 14–15 keV, a reasonable value for a well deformed nucleus. In Table III similar calculations are presented for the $1/2^+$ [400] Nilsson orbital. The theoretical calculations are compared to the cross sections of positive parity states with spins and excitation energies of 985 keV $1/2^+$, 1045 keV $3/2^+$ and 1129 keV $5/2^+$. The observed small (d,p) cross section for the $1/2^+$ state is due to the fact that the $1/2+$ [400] are hole states with a V^2 value close to unity and U^2 close to zero. The observed $1/2^+$ at 985 keV excitation energy also represents a fraction of the full theoretical intensity due to the $\Delta N = 2$ coupling with the $1/2^+$ [660] Nilsson orbital [22].

TABLE III

(d,t) and (d,p) population of the $1/2^+$ [400] orbital.

	$(d,t)(d\sigma/d\Omega)(\mu\text{b/sr})$ $\theta=90°, Q=-2$ MeV, $V^2=1$					$(d,p)(d\sigma/d\Omega)(\mu\text{b/sr})$ $\theta=90°, Q=+3$ MeV, $U^2=1$				
Nilsson orbital/spin	1/2	3/2	5/2	7/2	9/2	1/2	3/2	5/2	7/2	9/2
$1/2^+$ [400] theory	865	157	81	4	1	541	132	68	3	1
Exp.	580	55	85	—	—	18	—	—	—	—

4. Conclusions

Measurements of the (d,p) and (d,t) reactions have proved to be useful in determining the angular momentum l (for the case $l = 0$ also spin) and parities of the populated levels. A number of previous assignments have been confirmed. It has also been possible to assign new levels (see Table I). It has been shown in this work that it was possible to describe in the framework of the Nilsson model the negative parity ground state band $5/2^-$ [512] and one of the positive parity bands, $1/2^+$ [400] (see Tables II and III).

This work is the result of a collaboration between the Niels Bohr Institute, Tandem Accelerator Laboratory, Risø and the Institute for Nuclear Studies, Warsaw. The author is grateful to the staff of the NBI for making available unpublished data. Furthermore, I like to thank Dr N. Keeley for a critical reading of the manuscript and constructive suggestions.

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