SEARCH FOR THE BEGINNING OF CHAOS IN THE LOW-ENERGY REGION OF WELL DEFORMED EVEN–EVEN NUCLEI

Raymond K. Sheline

Chemistry and Physics Departments, Florida State University
Tallahassee, FL 32306, USA

Petr Alexa

Department of Physics, Technical University of Ostrava
17. listopadu 15, 708 33 Ostrava, Czech Republic

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A critical evaluation of all the data on the level structures of ^{178}Hf, ^{168}Er, ^{162}Dy and ^{164}Dy with special emphasis on the (n,γ) studies with average resonance capture (ARC) indicates the presence of a total of 4 states which are anomalous and may be chaotic (1 in ^{178}Hf below 1800 keV, 1 in ^{162}Dy below 2000 keV, and 2 in ^{164}Dy below 2000 keV) imbedded among the normal quadrupole deformed rotational states.

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1. Introduction

It has been recognized recently, that in the intermediate energy region where chaos is expected to hold full sway, there are kernels of imbedded regularity where the normal quantum numbers apply, and regular energy sequences can be found. Examples of this are: 1. superdeformed bands [1], 2. high spin isomers [2], and 3. ΔK forbiddenness in neutron capture resonances [3].

The "other side of the coin" of these phenomena would be the observation of states in the low energy regions where chaos has already begun but just for a particular state imbedded in a region of regular quadrupole deformation. Presumably, the rest of the states would all be members of well understood configurations with working quantum numbers implying that chaos has not yet begun. It would be particularly interesting to find such isolated chaotic states and to understand their early appearance.

One of the best places to look for such isolated chaotic states would be in the thoroughly studied low energy region of well deformed nuclei, where levels can be understood in terms of Nilsson configurations and collective vibrations and rotational bands.

In addition, one would need to have very carefully studied neutron capture gamma ray experiments in the highest possible resolution with average capture resonance spectroscopy (ARC) so that one could say that all states up to a certain energy between certain spin states have been observed. It would also be valuable if the neutron capturing state had fairly high spin so that one could look for possible missing rotational states both above and below in energy. Failure to find these rotational states would mean the state in question is anomalous and probably chaotic. Examples of nuclei with high spin in the capture state are ^{178}Hf and ^{168}Er where, assuming s-wave capture, the neutron capturing states are 3^- and 4^-, and 3^+ and 4^+, respectively. Primary gamma-ray transitions should then populate all states in the low energy spectra between $2^\pm$ and $5^\pm$. Two other nuclei which are not as well studied are ^{162}Dy and ^{164}Dy. Here the neutron capturing states, assuming s-wave capture, are 2^+ and 3^+, and 2^- and 3^- respectively, and the primary gamma-ray transitions should populate all low energy states between $1^\pm$ and $4^\pm$. It should be noted that in the cases of ^{168}Er and ^{178}Hf the low energy states with spins 2 and 5 and the same parity as the capturing states will be more weakly populated. A similar statement applies to the 1 and 4 states for the ^{162}Dy and ^{164}Dy nuclei.

We now consider the nuclei ^{178}Hf, ^{168}Er, ^{162}Dy and ^{164}Dy in this order.

2. The nucleus ^{178}Hf

Studies of ^{178}Hf levels following decays of ^{178}Lu and ^{178}Ta, and from reaction studies such as (n,γ), (α,xn), *etc.*, have been summarized in *Nuclear Data Sheets* [4]. Discussion of the microscopic structure of levels was given by Fogelberg and Bäcklin [5]. Hague *et al.* [6] used ^{177}Hf(n,γ) experiments to measure gamma rays with curved-crystal spectrometers, as well as conversion electrons and average resonance capture (ARC) spectra. This work established a large number of multipolarities and very precise transition energies. The result was a level scheme with a much more detailed set of spin-parity values. This study is particularly valuable in the present research because Hague *et al.* [6] are able to see all states between $J = 2$ and 5 up to 1.8 MeV. This aids us in showing the lack of band members connecting probable chaotic states.

States in ^{178}Hf which do not fit into rotational bands are presented in Table I. The first state which is not part of the band structure is the $J^\pi = 2^+$ state at 1561.533 keV. Since no lower lying, lower spin state is observed

TABLE I

States in ^{178}Hf below 2 MeV that do not fit into rotational structures.

State E [keV]	J^π	Comments
1561.533	2^+	$^{(i)}$
1808.272	2^+	$^{(ii)}$
1891.306	2^+	$^{(ii)}$
1942.009	$1^+, 2^+, 3^+$	$^{(ii)}$
1986.452	$1^+, 2^+, 3^+$	$^{(ii)}$
1997.460	3^+	$^{(ii)}$

$^{(i)}$ Interesting case.
$^{(ii)}$ Beyond the 1800 keV limit in which all spins between 2 and 5 are expected to be observed.

unassigned, we assume that this is the band head of a 2^+ band and look for other band members. Using the $K = 2^+$, γ-band at 1174.630 keV in ^{178}Hf as a model, we would expect to find 3^+, 4^+, and 5^+ states at ~ 1645, ~ 1771 and ~ 1910 keV. However, the next unassigned state is not observed until 1808.272 keV with $J^\pi = (2)^+$. Therefore, we can say with considerable confidence that the 2^+ state at 1561.533 keV is probably an example of a chaotic state in the sea of well ordered low energy states of ^{178}Hf. It is indeed fortunate that this 1561.533 keV state falls well below the 1800 keV limit where Hague *et al.* [6] see all the states with $J = 2, 3, 4$ and 5.

The next unassigned state at 1808.272 keV is already above the 1800 keV limit, which makes it difficult to make conclusive statements about its nature. It should also be noted that Hague *et al.* [6] believed the 2^+ state at 1808 keV to be the first member of a $K = 2^+$ band, the other members of which have been shown [7] very conclusively to be members of a $K = 3^+$ band. Therefore, the 1808 keV state has no band members and may also be chaotic in nature. However, we are limited to a single state in ^{178}Hf below 1800 keV which satisfies all our criteria and therefore is probably chaotic in nature. We note, however, that the $K = 0^+$ band at 1772.19 keV with 2^+ and 4^+ members has an erratic rotational structure and may be showing signs of severe mixing and the initial stages of chaotic behavior.

3. The nucleus ^{168}Er

Another example of an extremely well studied deformed even–even nucleus is ^{168}Er [8]. In all, 33 rotational bands [8–10] up to an energy of 3992 keV have been observed. Table II lists the states below 2 MeV in ^{168}Er which do not fit into rotational bands. In a remarkably similar manner to that in ^{178}Hf, the first candidates for probable chaotic states are observed at

1266.07 and 1768.17 keV. Furthermore, all states with spins between 2 and 5 are seen up to an energy of 2.0 MeV [9]. However, unfortunately the spins of these two states in ^{168}Er are much more indefinite than those in ^{178}Hf. More specifically, the spins of the 1266.07 keV state can only be limited to 1^+, 2^+, 5^+ or 6^+ whereas the 1768.17 keV level cannot be assigned spins. Because of these uncertainties in spin we cannot include these states in ^{168}Er among a list of probable chaotic states to be further investigated.

TABLE II

States in ^{168}Er below 2 MeV that do not fit into rotational structures.

State E [keV]	J^π	Comments
1266.07	1^+, 2^+, 5^+, 6^+	$^{(i)}$
1768.17	—	$^{(ii)}$

$^{(i)}$ Higher rotational band members for the 5^+ or 6^+ spin possibilities are beyond the range of ARC.
$^{(ii)}$ Data results from unresolved photon peak which makes its spin impossible to determine.

4. The nucleus ^{162}Dy

^{162}Dy is also reasonably well studied [8, 11] deformed even–even nucleus with 16 rotational bands up to an energy of 3838 keV. Table III presents the 3 levels below 2 MeV which do not fit into rotational bands. Only one of these levels at 1895.207 keV with spin 2^+ is a good candidate as a probable chaotic state. Table III gives the details on why the other 2 states do not uniquely qualify as probable chaotic states.

TABLE III

States in ^{162}Dy below 2 MeV that do not have rotational structure built on them.

State E [keV]	J^π	Comments
1895.207	2^+	$^{(i)}$
1951.385	$0^+ - 4^+$, $1^- - 6^-$	$^{(ii)}$
1982.107	2^+	$^{(iii)}$

$^{(i)}$ Interesting case.
$^{(ii)}$ Unresolved doublet.
$^{(iii)}$ Getting too close to 2 MeV limit to be certain of seeing rotational structure.

5. The nucleus ^{164}Dy

^{164}Dy has also been studied with a variety of experimental methods [8, 12]. In all, 9 rotational bands with energies up to 4212 keV have been observed. Table IV lists 9 levels up to 2 MeV which do not fit into rotational bands. Three levels at 1797.4 $(2)^+$, 1840.7 $1^{(\pm)}$ and 1933.0 $(2,3)^+$ are possible candidates for probable chaotic states. Unfortunately, the spin assignments for these three states are not definite.

TABLE IV

States in ^{164}Dy below 2 MeV with reasonably well known spin and parity that do not fit into rotational structures.

State E [keV]	J^π	Comments
1607.7	(4^+)	*(i)*
1736.4	$(1, 2^+)$	*(ii)*
1797.4	$(2)^+$	*(iii)*
1840.7	$1^{(\pm)}$	*(iii)*
1920.6	$(2, 3)^+$	*(iv)*
1933.0	$(2, 3)^+$	*(v)*
1949.0	$(3, 4)^-$	*(i) (ii) (vi)*
1978.9	$(2)^+$	*(v)*
1998.2	(4^+)	*(i) (iv) (vi)*

(i) Expected spin range of higher rotational band members outside the range of ARC.
(ii) Not observed in ARC. Therefore the data are questionable.
(iii) Interesting case.
(iv) 1920.6 (3^+) and 1998.2 (4^+) keV states could be rotational band members.
(v) The 1933.0 keV state observed to have spins 2^+ or 3^+ in the capture gamma ray spectroscopy depopulates to 6 states implying spins 3^- or 4^+. Because these spins do not agree with the spins $(2, 3)^+$, this state is excluded from consideration.
(vi) Too close to 2 MeV limit to be certain of seeing rotational band members.

6. Experimental summary

Table V summarizes all the states found in ^{178}Hf (below 1800 keV), ^{162}Dy and ^{164}Dy (below 2000 keV) which are not associated with rotational bands and, therefore, are probably chaotic. This does not mean that some of the other states listed in Tables I–IV may not also be chaotic. It implies that we cannot prove that the other states in Tables I–IV are probably chaotic with the present experimental information.

It is interesting to note that all of the probable chaotic states in Table V may have positive parity. Furthermore, a majority of them have $J^\pi = 2^+$.

A second criterion for a very low energy state being chaotic is that, since it must be a highly mixed state, gamma rays populating and depopulating it should only react to selection rules involving multipolarities and not to other selection rules such as K quantum number [1, 2].

All of the states in Table V are populated by gamma rays, following average resonance capture, all of the states have observed gamma rays depopulating them. The number of gamma rays depopulating these states equals 5 in the case of ^{178}Hf [4], 8 in the case of ^{162}Dy [11], 2 from the 1797.4 keV state of ^{164}Dy, and 2 from the 1840.7 keV state of ^{164}Dy [12]. In all cases we assume pure M1 multipolarity when the spins permit a mixture of M1 and E2 since we know nothing about the mixing.

TABLE V

States in ^{178}Hf (below 1800 keV), ^{162}Dy and ^{164}Dy (below 2000 keV) which do not fit into rotational bands and are probably chaotic.

State E [keV]	J^π	Nucleus
1561.533	2^+	^{178}Hf
1895.207	2^+	^{162}Dy
1797.4	$(2)^+$	^{164}Dy
1840.7	$1(^\pm)$	^{164}Dy

In ^{164}Dy all gammas depopulating 1797.4 + 1840.7 keV states populate the ground state band. For the 1797.4 keV state, the relative reduced transition probabilities [13,14] to the 2^+ and 4^+ members of the ground state band are 20(8) and 4.0(1.5), respectively. For the 1840.7 keV state, the relative reduced transition probabilities to the 0^+ and 2^+ members of the ground state band are 16.0 and 16.8, respectively [12]. The ratio of these relative reduced E1 or M1 transition probabilities 16.8/16.0 = 1.05 violates the Alaga rules that give

$$\frac{(I_i K_i \lambda K_f - K_i | I_f K_f)^2}{(I_i K_i \lambda K_f - K_i | I_f{}' K_f)^2} = \frac{(1\ 1\ 1\ \ -1|2\ 0)^2}{(1\ 1\ 1\ \ -1|0\ 0)^2} = 0.5\ . \tag{1}$$

From the 1895.207 keV state of ^{162}Dy, two gammas populate the 0^+ and 2^+ members of the ground state band, three the 2^+, 3^+ and 4^+ members of the gamma band, and three the 1^- (1637.190 keV), 2^- (1148.2266 keV) and 3^- (1357.923 keV) states of octupole vibrational bands. The relative reduced transition probabilities to the 0^+ member of the ground state band is 0.17(8), to the 2^+, 3^+ and 4^+ members of the gamma band, are 19.2(1.9), 18(4) and 17(6), respectively, and to the 1^-, 2^- and 3^- states, are 1688, 45(45) and 67(7), respectively [11].

From the 1561.533 keV state of ^{178}Hf, three gammas populate the 0^+, 2^+ and 4^+ members of the ground state band and two the 2^+ and 3^+ members of the gamma band. The relative reduced transition probabilities to 0^+, 2^+ and 4^+ members of the ground state band are 3.12(11), 10.9(3) and 32(1), respectively, and to the 2^+ and 3^+ members of the gamma band, are 22(7) and 45(3), respectively [4]. The ratio of the relative reduced E2 transition probabilities to 4^+ and 0^+ members of the ground state band $32/3.12 = 10.3(5)$ violates the Alaga rules that give

$$\frac{(2\ 2\ 2\ \ -2|4\ 0)^2}{(2\ 2\ 2\ \ -2|0\ 0)^2} = 0.0714\ . \tag{2}$$

Except for the direct gamma rays following neutron capture resonances, none of the states in Table V is populated by gamma rays from higher lying states, except in the case of the 1561.533 keV state in ^{178}Hf. In the case of ^{178}Hf, 4 gamma rays from 4 higher energy states are observed. However, there is no way from the existing data to get absolute or relative transition probabilities.

Unfortunately, this very modest amount of data on the gamma population and depopulation of the states in Table V, allows us to draw no further conclusions about the chaos of these states than that obtained from the lack of associated rotational structures described earlier in this article.

7. Theoretical summary

From the theoretical point of view in the standard approach the 2^+ and $1^\pm$ states in the region of our interest can be interpreted as weak one-phonon vibrational 2^+ states (corresponding to the second pole of the RPA equation), two-phonon 2^+ states, 1^+ collective state or 2^+ or $1^\pm$ two-quasiparticle states. The quasiparticle-phonon-model (QPM) is capable to describe low-lying collective 1^+ and 2^+ states [15,16]. In ^{162}Dy Soloviev *et al.* [16] predict the second 2^+ state at 2.1 MeV and identify it with the experimental 2^+ state at 1999 keV. In ^{164}Dy Soloviev *et al.* [16] predict the second 2^+ state at 1.7 MeV and identify it with the experimental 2^+ state at 1797.4 keV (our candidate for the probable chaotic state). The first 1^+ state is predicted at 2.0 MeV with the reduced transition probability to the ground state $B(\mathrm{M1}) = 0.08\ \mu_{\mathrm{N}}^2$, and identified with the experimental 1^+ state at 1840.7 keV (our candidate for the probable chaotic state). In ^{178}Hf QPM predicts the second 2^+ state at 2.11 MeV [17].

Our QPM calculations (quadrupole, hexadecapole deformation and pairing gaps taken from [18], Nilsson parameters κ and μ taken from [19], multipole–multipole interaction strengths fitted to experimental energies of quadrupole and octupole vibrational states or taken from systematics) give the second weak one-phonon quadrupole vibrational 2^+ states with $B(\mathrm{E2})$ to the ground state $\sim 10^{-4} \div 0.2$ W.u. at 1.86 MeV (^{162}Dy), 1.83 MeV (^{164}Dy) and 2.05 MeV (^{178}Hf, with a dominant two-quasiparticle configuration $\pi 5/2\,[402] \otimes \pi 1/2\,[411]$). Calculated energies of the lowest two-quasiparticle 2^+ and $1^\pm$ states are summarized in Table VI.

TABLE VI

Calculated energies of the lowest 2^+ and $1^\pm$ two-quasiparticle states in ^{178}Hf, ^{162}Dy and ^{164}Dy.

Energy [MeV]	J^π	Nucleus	Configuration
2.05	2^+	^{178}Hf	π 5/2 [402] $\otimes\pi$ 1/2 [411]
1.78	2^+	^{162}Dy	ν 5/2 [523] $\otimes\nu$ 1/2 [521]
1.72	2^+	^{164}Dy	ν 5/2 [523] $\otimes\nu$ 1/2 [521]
1.84	1^+	^{164}Dy	ν 7/2 [633] $\otimes\nu$ 5/2 [642]
1.84	2^+	^{164}Dy	ν 5/2 [512] $\otimes\nu$ 1/2 [521]
1.98	1^-	^{164}Dy	ν 7/2 [633] $\otimes\nu$ 5/2 [523]

Apart from missing rotational bands, in ^{162}Dy and ^{164}Dy there are predicted 2^+ and $1^\pm$ states close to our candidates for probable chaotic states. In contrast to these nuclei, in ^{178}Hf the predicted 2^+ state lie above 2 MeV and the position of the experimentally observed 2^+ state at 1561.533 keV is then very anomalous. If it were chaotic, it should be a highly mixed state comprising a lot of two-quasiparticle configurations. The ratio R of the relative reduced E2 transition probabilities to the 4^+ and 0^+ members of the ground state band can be obtained from

$$R = \left| \frac{\sum_i a_i (2\ K_i\ 2\ \ -K_i\ 4\ 0) \langle 0 | e_i \hat{O}(E2, -K_i) | K_i \rangle}{\sum_i a_i (2\ K_i\ 2\ \ -K_i\ 0\ 0) \langle 0 | e_i \hat{O}(E2, -K_i) | K_i \rangle} \right|^2 , \tag{3}$$

where a_i are random wave-function components of the two-quasiparticle configurations of the 2^+ state with K quantum number K_i, e_i is the effective charge ($0.991e$ for protons, $0.002e$ for neutrons [19]) and $\hat{O}(E2, -K_i)$ is the operator of the E2 transition. Taking into account all neutron and proton two-quasiparticle states up to 10 MeV (782 neutron states and 506 proton states) with random wave-function components generated by RANLUX [20] we get $R = (16{\pm}7)$ from 10 calculated values or $R = (47{\pm}37)$ from 100 calculated values that is consistent with the experimental value $R = (10.3 \pm 0.5)$.

It is clear that this is not sufficient to draw conclusions about the structure of the 2^+ state but it indicates that among the 4 proposed anomalous or probable chaotic states the 2^+ state at 1561.533 keV in ^{178}Hf is the most promising candidate.

In conclusion we wish to emphasize that we have not proven that the 4 states discussed are chaotic. We obviously would need a very large number of states to prove the classical Bohigas–Giannoni–Schmit conjecture [21] that these states belong to quantum systems where spectral fluctuations obey Random Matrix Theory and, therefore, correspond to chaotic states. This is the reason we say these states are "probably chaotic". Our goal in this paper is much simpler. We look in the low energy region of well deformed nuclei for states which do not fit into the well demonstrated rotational structures and find 4 states which do not fit into the existing rotational bands and have no rotational structure themselves. These levels are called "probably chaotic" and form an interesting counterpoint to the higher energy regions where there are isolated spectral regions which are not chaotic in regions which otherwise are. It is a challenge for future experimental and theoretical studies to investigate the structure of the 4 discussed states and to prove or disprove our hypothesis about the nature of these anomalous states.

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