PARTICLE CREATION IN OSCILLATING CAVITIES WITH CUBIC AND CYLINDRICAL GEOMETRY

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In the present paper we study the creation of massless scalar particles from the quantum vacuum due to the dynamical Casimir effect by oscillating cavities with cubic and cylindrical geometry. To the first order of the amplitude we derive the expressions for the number of the created particles.

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1. Introduction

The Casimir effect is regarded as one of the most striking manifestation of vacuum fluctuations in quantum field theory. The presence of reflecting boundaries alters the zero-point modes of a quantized field, and results in the shifts in the vacuum expectation values of quantities quadratic in the field, such as the energy density and stresses. In particular, vacuum forces arise acting on constraining boundaries. The particular features of these forces depend on the nature of the quantum field, the type of spacetime manifold and its dimensionality, the boundary geometries and the specific boundary conditions imposed on the field. Since the original work by Casimir in 1948 [1] many theoretical and experimental works have been done on this problem (see, *e.g.*, [2–5] and references therein). The Casimir effect can be viewed as a polarization of vacuum by boundary conditions. A new phenomenon, a quantum creation of particles (the dynamical Casimir effect) occurs when the geometry of the system varies in time. In two dimensional spacetime and for conformally invariant fields the problem with dynamical boundaries can be mapped to the corresponding static problem and hence allows a complete study [2, 6]. In higher dimensions the problem is much more complicated and is solved for some simple geometries. The vacuum stress induced by uniform acceleration of a perfectly reflecting plane is considered in [7]. The corresponding problem for a sphere expanding in the four-dimensional spacetime with constant acceleration is investigated by Frolov and Serebriany [8, 9] in the perfectly reflecting case and by Frolov and Singh [10] for semi-transparent boundaries. Particle creation from the quantum scalar vacuum by expanding or contracting spherical shell with Dirichlet boundary conditions is considered in [11]. In another paper the case is considered when the sphere radius performs oscillation with a small amplitude and the expression are derived for the number of created particles to the first order of the perturbation theory [12]. A new application of the dynamical Casimir effect has recently appeared in connection with the suggestion by Schwinger [13] that the photon production associated with changes in the quantum electrodynamic vacuum state arising from a collapsing dielectric bubble could be relevant for sono-luminescence (the phenomenon of light emission by a sound-driven gas bubble in a fluid [14]). The possibility of particle production due to space-time curvature has been discussed by Schrodinger [15], while other early work is due to DeWitt [16] and Imamura [17]. The first thorough treatment of particle production by an external gravitational filed was given by Parker [18, 19].

In the present paper we study the creation of massless scalar particles from the quantum vacuum due to the dynamical Casimir effect by oscillating cavities with cubic and cylindrical geometry.

2. Quantum scalar field inside a cubic with time-dependent walls

Consider a massless scalar field satisfying Dirichlet boundary condition on the surface of a cubic with time-dependent walls

$$\left(\Delta-\frac{\partial^2}{\partial t^2}\right)u_{\boldsymbol{k}}\left(\boldsymbol{x},t\right)=0\,,\qquad u_{\boldsymbol{k}}|_{\text{boundaries}}=0\,. \tag{1}$$

we expand the corresponding eigenfunctions for the interior region in a series with respect to the instantaneous basis:

$$u_{\boldsymbol{k}}\left(\boldsymbol{x},t\right)=\sum_{\boldsymbol{p}}Q_{\boldsymbol{p}}^{(\boldsymbol{k})}\left(t\right)\varphi_{\boldsymbol{p}}\left(\boldsymbol{x},t\right), \tag{2}$$

where

$$\varphi_{\boldsymbol{p}}\left(\boldsymbol{x},t\right)=\sqrt{\frac{8}{L_x\left(t\right)L_y(t)L_z\left(t\right)}}\sin\left(\frac{p_x\pi x}{L_x\left(t\right)}\right)\sin\left(\frac{p_y\pi y}{L_y\left(t\right)}\right)\sin\left(\frac{p_z\pi z}{L_z\left(t\right)}\right). \tag{3}$$

Substituting (3) into field equation (1) we arrive at an infinite set of coupled differential equations

$$\sum_{\boldsymbol{p}} \left\{ \varphi_{\boldsymbol{p}} \left[\ddot{Q}_{\boldsymbol{p}}^{(k)} + \omega_{\boldsymbol{p}}^2 Q_{\boldsymbol{p}} \right] + 2\dot{Q}_{\boldsymbol{p}}^{(k)} \dot{\varphi}_{\boldsymbol{p}} + Q_{\boldsymbol{p}}^{(k)} \ddot{\varphi}_{\boldsymbol{p}} \right\} = 0 \,, \tag{4}$$

where overdot stands for the time derivative. Let us multiply this equation by $\varphi_{\boldsymbol{j}}^{\star}$ and integrate over the region inside a cubic at a given moment t. Using the orthonormality relation

$$\int \varphi_{\boldsymbol{p}} \varphi_{\boldsymbol{j}}^{\star} d^3x = \delta_{\boldsymbol{p}j} \tag{5}$$

this yields

$$\ddot{Q}_{\boldsymbol{p}}^{(k)} + \omega_{\boldsymbol{p}}^2 Q_{\boldsymbol{p}}^{(k)} = -2 \sum_{\boldsymbol{j}} \dot{Q}_{\boldsymbol{j}}^{(k)} \int \dot{\varphi}_{\boldsymbol{j}} \varphi_{\boldsymbol{p}}^{\star} d^3x - \sum_{\boldsymbol{j}} Q_{\boldsymbol{j}}^{(k)} \int \ddot{\varphi}_{\boldsymbol{j}} \varphi_{\boldsymbol{p}}^{\star} d^3x \,, \tag{6}$$

where we have introduced notations

$$g_{\boldsymbol{jp}} = -\int \dot{\varphi}_{\boldsymbol{j}} \varphi_{\boldsymbol{p}}^{\star} d^3x \,, \qquad g_{\boldsymbol{jp}}^{(1)} = -\int \ddot{\varphi}_{\boldsymbol{j}} \varphi_{\boldsymbol{p}}^{\star} d^3x \tag{7}$$

with integrations are over the inside region of the cavity. By making use of the completeness condition for eigenfunction (3) we obtain following relation between these coefficients

$$g_{\boldsymbol{j}p}^{(1)} = \frac{\partial}{\partial t} g_{\boldsymbol{j}p} + \sum_{\boldsymbol{s}} g_{\boldsymbol{js}} g_{\boldsymbol{ps}} \,. \tag{8}$$

Then we can write

$$\begin{aligned} g_{\boldsymbol{jp}} &= -\int \left(\frac{\partial \varphi_{\boldsymbol{j}}}{\partial L_x} \dot{L}_x + \frac{\partial \varphi_{\boldsymbol{j}}}{\partial L_y} \dot{L}_y + \frac{\partial \varphi_{\boldsymbol{j}}}{\partial L_z} \dot{L}_z \right) \varphi_{\boldsymbol{p}}^{\star} d^3x \\ &= \lambda_1 g_{1\boldsymbol{pj}} + \lambda_2 g_{2\boldsymbol{pj}} + \lambda_3 g_{3\boldsymbol{pj}} \end{aligned} \tag{9}$$

where

$$\begin{aligned} g_{1\boldsymbol{p}j} &= L_x \int \frac{\partial \varphi_{\boldsymbol{j}}}{\partial L_x} \varphi_{\boldsymbol{p}}^{\star} d^3x \,, \qquad \lambda_1 = \frac{\dot{L}_x}{L_x} \,, \\ g_{2\boldsymbol{p}j} &= L_y \int \frac{\partial \varphi_{\boldsymbol{j}}}{\partial L_y} \varphi_{\boldsymbol{p}}^{\star} d^3x \,, \qquad \lambda_2 = \frac{\dot{L}_y}{L_y} \,, \\ g_{3\boldsymbol{p}j} &= L_z \int \frac{\partial \varphi_{\boldsymbol{j}}}{\partial L_z} \varphi_{\boldsymbol{p}}^{\star} d^3x \,, \qquad \lambda_3 = \frac{\dot{L}_z}{L_z} \,, \\ g_{i\boldsymbol{p}j} &= -g_{i\boldsymbol{pj}} \,. \end{aligned}$$

Thus from equation (6) we obtain

$$\ddot{Q}^{(k)}_{\boldsymbol{p}} + \omega^2_{\boldsymbol{p}} Q^{(\boldsymbol{k})}_{\boldsymbol{p}} = 2\left[\sum_{\boldsymbol{j}} \left(\lambda_1 g_{1\boldsymbol{pj}} + \lambda_2 g_{2\boldsymbol{pj}} + \lambda_3 g_{3\boldsymbol{pj}}\right) \dot{Q}^{(\boldsymbol{k})}_{\boldsymbol{j}}\right] + \sum_{\boldsymbol{j}} \left(\dot{\lambda}_1 g_{1\boldsymbol{pj}} + \dot{\lambda}_2 g_{2\boldsymbol{pj}} + \dot{\lambda}_3 g_{3\boldsymbol{pj}}\right) Q^{(\boldsymbol{k})}_{\boldsymbol{j}}\,, \tag{10}$$

where we have neglected the second term in relation (8). The field operator in the Heisenberg representation may be expanded in terms of the corresponding eigenfunctions

$$\phi\left(\boldsymbol{x},t\right) = \sum_{\boldsymbol{k}} a_{\boldsymbol{k}} u_{\boldsymbol{k}}(\boldsymbol{x},t) + a^{\dagger}_{\boldsymbol{k}} u^{*}_{\boldsymbol{k}}(\boldsymbol{x},t)\,. \tag{11}$$

We assume that all the sides of the cavity move in the time range $0 < t < T$ with the initial conditions

$$Q^{(\boldsymbol{k})}_{\boldsymbol{p}}\left(0\right) = \frac{1}{\sqrt{2\omega_{\boldsymbol{k}}}}\delta_{\boldsymbol{pk}}\,, \qquad \dot{Q}^{(\boldsymbol{k})}_{\boldsymbol{p}}\left(0\right) = -i\sqrt{\frac{\omega_{\boldsymbol{k}}}{2}}\delta_{\boldsymbol{pk}}\,. \tag{12}$$

$a^{\rm in}_{\boldsymbol{p}}$ and $a^{\dagger\,{\rm in}}_{\boldsymbol{p}}$ are creation and annihilation operators in the "in" region ($t > 0$). $a^{\rm out}_{\boldsymbol{p}}$ and $a^{\dagger\,{\rm out}}_{\boldsymbol{p}}$ are those of the "out" region ($t > T$), these two sets are related by the Bogoliubov transformation as follows

$$a^{\rm out}_{\boldsymbol{p}} = \sum_{\boldsymbol{k}} a^{\rm in}_{\boldsymbol{k}} \alpha_{\boldsymbol{kp}} + a^{\dagger\,{\rm in}}_{\boldsymbol{k}} \beta^{\star}_{\boldsymbol{kp}}\,. \tag{13}$$

Now we must obtain $\alpha_{\boldsymbol{kp}}$ and $\beta_{\boldsymbol{kp}}$. If the walls of the cavity for $t > T$ return to their initial positions the right side of the equation (10) vanishes thus its solution is

$$Q^{(\boldsymbol{k})}_{\boldsymbol{p}}\left(t > T\right) = A^{(\boldsymbol{k})}_{\boldsymbol{p}} e^{i\omega_{\boldsymbol{p}} t} + B^{(\boldsymbol{k})}_{\boldsymbol{p}} e^{-i\omega_{\boldsymbol{p}} t} \tag{14}$$

and we obtain [20]

$$\alpha_{\boldsymbol{kp}} = \sqrt{2\omega_{\boldsymbol{p}}}\, B^{(\boldsymbol{k})}_{\boldsymbol{p}}\,, \qquad \beta_{\boldsymbol{kp}} = \sqrt{2\omega_{\boldsymbol{p}}}\, A^{(\boldsymbol{k})}_{\boldsymbol{p}}\,. \tag{15}$$

The number of photons created in the mode $\boldsymbol{p}$ is the average value of $a^{\dagger{\rm out}}_{\boldsymbol{p}} a^{\rm out}_{\boldsymbol{p}}$ with respect to the initial vacuum state

$$\langle N_{\boldsymbol{p}}\rangle = \langle 0_{\rm in}|\, a^{\dagger out}_{\boldsymbol{p}} a^{\rm out}_{\boldsymbol{p}}\, |0_{\rm in}\rangle = \sum_{\boldsymbol{k}} 2\omega_{\boldsymbol{p}} \left|A^{(\boldsymbol{k})}_{\boldsymbol{p}}\right|^2. \tag{16}$$

We assume the motion of the sides to be as

$$\begin{aligned} L_x(t) &= L_x\left(1+\varepsilon\sin\left(\Omega t\right)\right) , \\ L_y(t) &= L_y\left(1+\varepsilon\sin\left(\Omega t\right)\right) , \\ L_z(t) &= L_z\left(1+\varepsilon\sin\left(\Omega t\right)\right) , \end{aligned} \tag{17}$$

where $\varepsilon \ll 1$. By substituting in equation (10) we obtain

$$\begin{aligned} \ddot{Q}_{\boldsymbol{p}}^{(\boldsymbol{k})} + \omega_{\boldsymbol{p}}^2 Q_{\boldsymbol{p}}^{(\boldsymbol{k})} &= 2\varepsilon\left[\left(\frac{\pi p_x}{L_x}\right)^2 + \left(\frac{\pi p_y}{L_y}\right)^2 + \left(\frac{\pi p_z}{L_z}\right)^2\right]\sin\left(\Omega t\right) Q_{\boldsymbol{p}}^{(\boldsymbol{k})} \\ &- \varepsilon\Omega^2 \sin\left(\Omega t\right)\sum_{\boldsymbol{j}} g_{\boldsymbol{pj}} Q_{\boldsymbol{j}}^{(\boldsymbol{k})} \\ &+ 2\varepsilon\Omega\cos\left(\Omega t\right)\sum_{\boldsymbol{j}} g_{\boldsymbol{pj}} \dot{Q}_{\boldsymbol{j}}^{(\boldsymbol{k})} , \end{aligned} \tag{18}$$

where we have used following relations

$$\lambda_1 = \frac{\dot{L}_x}{L_x} = \frac{L_x\varepsilon\Omega\cos\left(\Omega t\right)}{L_x\left(1+\varepsilon\sin\left(\Omega t\right)\right)} \approx \varepsilon\Omega\cos\left(\Omega t\right) , \qquad \dot{\lambda}_1 = -\varepsilon\Omega^2\sin\left(\Omega t\right) . \tag{19}$$

It is well known that a naive perturbative solution of these equations in powers of ε breaks down after a short amount of time, of order $(\varepsilon\Omega)^{-1}$. This happens for those particular values of the external frequency such that there is a resonant coupling with the eigenfrequencies of the static cavity. In this situation, to find a solution valid for longer times (of order $(\varepsilon^{-2}\Omega^{-1})$) we use the MSA technique [20, 21]. We introduce a second time scale $\tau = \varepsilon t$ and expand $Q_{\boldsymbol{p}}^{(k)}$ to first order in ε as follows

$$Q_{\boldsymbol{p}}^{(\boldsymbol{k})} = Q_{\boldsymbol{p}}^{(\boldsymbol{k})(0)}(t,\tau) + \varepsilon Q_{\boldsymbol{p}}^{(\boldsymbol{k})(1)}(t,\tau) . \tag{20}$$

The derivatives with respect to t are

$$\dot{Q}_{\boldsymbol{p}}^{(\boldsymbol{k})} = \partial_t Q_{\boldsymbol{p}}^{(\boldsymbol{k})(0)} + \varepsilon\left[\partial_\tau Q_{\boldsymbol{p}}^{(\boldsymbol{k})(0)} + \partial_t Q_{\boldsymbol{p}}^{(\boldsymbol{k})(1)}\right] , \tag{21}$$

$$\ddot{Q}_{\boldsymbol{p}}^{(\boldsymbol{k})} = \partial_t^2 Q_{\boldsymbol{p}}^{(\boldsymbol{k})(0)} + \varepsilon\left[2\partial_{t\tau} Q_{\boldsymbol{p}}^{(\boldsymbol{k})(0)} + \partial_t^2 Q_{\boldsymbol{p}}^{(\boldsymbol{k})(1)}\right] , \tag{22}$$

with the initial conditions

$$Q_{\boldsymbol{p}}^{(\boldsymbol{k})(0)}(0) = \frac{1}{\sqrt{2\omega_{\boldsymbol{p}}}}\,\delta_{\boldsymbol{pk}} , \qquad \dot{Q}_{\boldsymbol{p}}^{(k)(0)} = -i\sqrt{\frac{\omega_{\boldsymbol{p}}}{2}}\,\delta_{\boldsymbol{pk}} . \tag{23}$$

To zeroth order in ε we obtain

$$Q_{\boldsymbol{p}}^{(\boldsymbol{k})(0)}(t,\tau) = A_{\boldsymbol{p}}^{(\boldsymbol{k})}(\tau)\, e^{i\omega_{\boldsymbol{p}} t} + B_{\boldsymbol{p}}^{(\boldsymbol{k})}(\tau)\, e^{-i\omega_{\boldsymbol{p}} t}\,. \tag{24}$$

By the initial conditions we obtain

$$A_{\boldsymbol{p}}^{(\boldsymbol{k})}(\tau=0) = 0\,, \qquad B_{\boldsymbol{p}}^{(\boldsymbol{k})}(\tau=0) = \frac{1}{\sqrt{2\omega_{\boldsymbol{p}}}}\,\delta_{\boldsymbol{pk}}\,. \tag{25}$$

To first order in ε we have

$$\partial_t^2 Q_{\boldsymbol{p}}^{(\boldsymbol{k})(1)} + \omega_{\boldsymbol{p}}^2 Q_{\boldsymbol{p}}^{(\boldsymbol{k})(1)} = -2\partial_{\tau t}^2 Q_{\boldsymbol{p}}^{(\boldsymbol{k})(0)} + 2\left[\left(\frac{\pi k_x}{L_x}\right)^2 + \left(\frac{\pi k_y}{L_y}\right)^2 + \left(\frac{\pi k_z}{L_z}\right)^2\right]$$
$$\times \sin(\Omega t)\, Q_{\boldsymbol{p}}^{(\boldsymbol{k})(0)} - \Omega^2 \sin(\Omega t) \sum_{\boldsymbol{j}\neq\boldsymbol{p}} g_{\boldsymbol{pj}} Q_j^{(k)(0)} + 2\Omega\cos(\Omega t) \sum_{\boldsymbol{j}\neq\boldsymbol{p}} g_{\boldsymbol{pj}} \partial_t Q_j^{(k)(0)},$$

where

$$g_{\boldsymbol{pj}} = g_{1\boldsymbol{pj}} + g_{2\boldsymbol{pj}} + g_{3\boldsymbol{pj}}\,.$$

In MSA technique for preventing secularities we must set the coefficients of $e^{\pm i\omega_{\boldsymbol{p}} t}$ to zero. By doing this we obtain

$$\begin{aligned}
\frac{dA_{\boldsymbol{p}}^{(\boldsymbol{k})}}{d\tau} \;=\; & -\left[\frac{\pi^2 p_x^2}{2\omega_{\boldsymbol{p}}^2 L_x^2} + \frac{\pi^2 p_y^2}{2\omega_{\boldsymbol{p}}^2 L_y^2} + \frac{\pi^2 p_z^2}{2\omega_{\boldsymbol{p}}^2 L_z^2}\right] B_{\boldsymbol{p}}^{\boldsymbol{k}}\, \delta(2\omega_{\boldsymbol{p}} - \Omega) \\
& + \sum_{\boldsymbol{j}} \left(-\omega_{\boldsymbol{j}} + \frac{\Omega}{2}\right) \delta(-\omega_{\boldsymbol{p}} - \omega_{\boldsymbol{j}} + \Omega)\, \frac{\Omega}{2\omega_{\boldsymbol{p}}}\, g_{\boldsymbol{pj}} B_{\boldsymbol{j}}^{(\boldsymbol{k})} \\
& + \sum_{\boldsymbol{j}} \left[\left(\omega_{\boldsymbol{j}} + \frac{\Omega}{2}\right) \delta(\omega_{\boldsymbol{p}} - \omega_{\boldsymbol{j}} - \Omega) + \left(\omega_{\boldsymbol{j}} + \frac{\Omega}{2}\right) \delta(\omega_{\boldsymbol{p}} - \omega_{\boldsymbol{j}} + \Omega)\right] \\
& \times \frac{\Omega}{2\omega_{\boldsymbol{p}}}\, g_{\boldsymbol{pj}} A_{\boldsymbol{j}}^{(\boldsymbol{k})}\,,
\end{aligned} \tag{26}$$

$$\begin{aligned}
\frac{dB_{\boldsymbol{p}}^{(\boldsymbol{k})}}{d\tau} \;=\; & -\left[\frac{\pi^2 p_x^2}{2\omega_{\boldsymbol{p}} L_x^2} + \frac{\pi^2 p_y^2}{2\omega_{\boldsymbol{p}} L_y^2} + \frac{\pi^2 p_z^2}{2\omega_{\boldsymbol{p}} L_z^2}\right] A_{\boldsymbol{p}}^{(\boldsymbol{k})} \delta(2\omega_{\boldsymbol{p}} - \Omega) \\
& + \sum_{\boldsymbol{j}} \left(-\omega_{\boldsymbol{j}} + \frac{\Omega}{2}\right) \delta(-\omega_{\boldsymbol{p}} - \omega_{\boldsymbol{j}} + \Omega)\, \frac{\Omega}{2\omega_{\boldsymbol{p}}} g_{\boldsymbol{pj}} A_{\boldsymbol{j}}^{(\boldsymbol{k})} \\
& + \sum_{\boldsymbol{j}} \left[\left(\omega_{\boldsymbol{j}} + \frac{\Omega}{2}\right) \delta(\omega_{\boldsymbol{p}} - \omega_{\boldsymbol{j}} - \Omega) + \left(\omega_{\boldsymbol{j}} - \frac{\Omega}{2}\right) \delta(\omega_{\boldsymbol{p}} - \omega_{\boldsymbol{j}} + \Omega)\right] \\
& \times \frac{\Omega}{2\omega_{\boldsymbol{p}}} g_{\boldsymbol{p}j} B_{\boldsymbol{j}}^{(\boldsymbol{k})}\,.
\end{aligned} \tag{27}$$

If $\Omega = 2\omega_{\boldsymbol{p}}$ and if we assume $|\omega_{\boldsymbol{p}} \pm \omega_{\boldsymbol{j}}| \neq \Omega$, that is $\omega_{\boldsymbol{j}} - \omega_{\boldsymbol{p}} \neq 2\omega_{\boldsymbol{p}}$ which means that we do not have coupling between the mode $\boldsymbol{p}$ and the other modes the equations (26)and (27) reduce to

$$\frac{dA_{\boldsymbol{p}}^{(\boldsymbol{k})}}{d\tau} = \frac{-\omega_{\boldsymbol{p}}}{2} B_{\boldsymbol{p}}^{(\boldsymbol{k})}, \qquad \frac{dB_{\boldsymbol{p}}^{(\boldsymbol{k})}}{d\tau} = \frac{-\omega_{\boldsymbol{p}}}{2} A_{\boldsymbol{p}}^{(\boldsymbol{k})}. \tag{28}$$

The solutions to the above equations are

$$\begin{aligned} B_{\boldsymbol{p}}^{(\boldsymbol{k})} &= \frac{1}{\sqrt{2\omega_{\boldsymbol{p}}}} \delta_{\boldsymbol{pk}} \cosh\left(\frac{\omega_{\boldsymbol{p}}}{2}\tau\right), \\ A_{\boldsymbol{p}}^{(\boldsymbol{k})} &= -\frac{1}{\sqrt{2\omega_{\boldsymbol{p}}}} \delta_{\boldsymbol{pk}} \sinh\left(\frac{\omega_{\boldsymbol{p}}}{2}\tau\right). \end{aligned} \tag{29}$$

From equation (16) we obtain for the mean number of created photons

$$\langle N_{\boldsymbol{p}} \rangle = \sinh^2\left(\frac{\omega_{\boldsymbol{p}}}{2}\tau\right). \tag{30}$$

3. Quantum scalar field inside a cylinder with time-dependent surface

In the previous section we discussed particle creation in a cubical box with time dependent walls. In this section we consider a cylinder and discuss particle creation in this geometry. In this case we have

$$\varphi_{\boldsymbol{p}}(\boldsymbol{x}, t) = \sqrt{\frac{2}{L_z(t)}} \sin\left(\frac{p_z \pi z}{L_z(t)}\right) v_{\boldsymbol{p}_\perp}(\boldsymbol{x}_\perp) \tag{31}$$

where

$$v_{\boldsymbol{p}_\perp}(\boldsymbol{x}_\perp) = \frac{1}{\sqrt{\pi} R(t) J_n(y_{nm}) \sqrt{1 - n^2/y_{nm}}} J_n\left(\frac{y_{nm}\,\rho}{R(t)}\right) e^{\mathrm{i}n\phi}. \tag{32}$$

By substituting the above relations into equation (1) we obtain

$$\begin{aligned} \ddot{Q}_{\boldsymbol{p}}^{(\boldsymbol{k})} + \omega_{\boldsymbol{p}}^2 Q_{\boldsymbol{p}}^{(\boldsymbol{k})} &= 2\left[\sum_{\boldsymbol{j}} (\lambda_1 g_{1\boldsymbol{pj}} + \lambda_2 g_{2\boldsymbol{pj}}) \dot{Q}_{\boldsymbol{j}}^{(\boldsymbol{k})}\right] \\ &\quad + \sum_{\boldsymbol{j}} \left(\dot{\lambda}_1 g_{1\boldsymbol{pj}} + \dot{\lambda}_2 g_{2\boldsymbol{pj}}\right) Q_{\boldsymbol{j}}^{(\boldsymbol{k})}, \end{aligned} \tag{33}$$

where

$$\begin{aligned}
\omega_{\boldsymbol{p}} &= \omega_{nmp_z} = \sqrt{\left(\frac{y_{nm}}{R\,(t)}\right)^2 + \left(\frac{p_z\pi}{L_z\,(t)}\right)^2}\,, \\
\lambda_1 &= \frac{\dot{L}_z}{L_z}\,, \qquad g_{1\boldsymbol{pj}} = L_z \int d^3x \frac{\partial\varphi_{\boldsymbol{p}}}{\partial L_z}\,\varphi^{\star}_{\boldsymbol{j}} \\
\lambda_1 &= \frac{\dot{R}}{R}\,, \qquad g_{1\boldsymbol{pj}} = R \int d^3x \frac{\partial\varphi_{\boldsymbol{p}}}{\partial R}\,\varphi^{\star}_{\boldsymbol{j}}\,.
\end{aligned}$$

If we assume the time dependency of the radius and the length of the cylinder as follows

$$L_z\,(t) = L_z\,[1 + \varepsilon \sin{(\Omega t)}]\;, \qquad R\,(t) = R\,[1 + \varepsilon \sin{(\Omega t)}]\;. \tag{34}$$

from equation (33) we have

$$\begin{aligned}
\ddot{Q}^{(\boldsymbol{k})}_{\boldsymbol{p}} + \omega^2_{\boldsymbol{p}} Q^{(\boldsymbol{k})}_{\boldsymbol{p}} &= 2\varepsilon \left(\frac{y_{nm}}{R}\right)^2 \sin{(\Omega t)}\, Q^{(k)}_{p} + 2\varepsilon \left(\frac{p_z\pi}{L}\right)^2 \varepsilon \sin{(\Omega t)}\, Q^{(\boldsymbol{k})}_{\boldsymbol{p}} \\
&\quad - \varepsilon\Omega^2 \sin{(\Omega t)} \sum_{\boldsymbol{j}} (g_{1\boldsymbol{pj}} + g_{2\boldsymbol{pj}}) Q^{(\boldsymbol{k})}_{\boldsymbol{j}} \\
&\quad + 2\varepsilon\Omega \cos{(\Omega t)} \sum_{\boldsymbol{j}} (g_{1\boldsymbol{pj}} + g_{2\boldsymbol{pj}})\, \dot{Q}^{(\boldsymbol{k})}_{\boldsymbol{j}}\,. \qquad (35)
\end{aligned}$$

Similar to the previous section we use MSA technique to solve this equation. Thus we write

$$Q^{(\boldsymbol{k})}_{\boldsymbol{p}} = Q^{(\boldsymbol{k})(0)}_{\boldsymbol{p}}(t,\tau) + \varepsilon Q^{(\boldsymbol{k})(1)}_{\boldsymbol{p}}(t,\tau)\,. \tag{36}$$

As in the previous section to zeroth order in ε we obtain

$$Q^{(\boldsymbol{k})(0)}_{\boldsymbol{p}}(t,\tau) = A^{(k)}_{p}(\tau) e^{i\omega_{\boldsymbol{p}} t} + B^{(k)}_{p}(\tau) e^{-i\omega_{\boldsymbol{p}} t} \tag{37}$$

and to the first order in ε we obtain for $Q^{(\boldsymbol{k})(1)}_{\boldsymbol{p}}$ the following equation

$$\begin{aligned}
\partial^2_t Q^{(\boldsymbol{k})(1)}_{\boldsymbol{p}} + \omega^2_{\boldsymbol{p}} Q^{(\boldsymbol{k})(1)}_{\boldsymbol{p}} &= -2\partial^2_{\tau t} Q^{(\boldsymbol{k})(0)}_{\boldsymbol{p}} \\
&\quad + 2\left[(\frac{y_{nm}}{R})^2 + (\frac{p_z\pi}{L_z})^2\right] \sin(\Omega t) Q^{(\boldsymbol{k})(0)}_{\boldsymbol{p}} \\
&\quad - \Omega^2 \sin{(\Omega t)} \sum_{\boldsymbol{j}\neq\boldsymbol{p}} (g_{1\boldsymbol{pj}} + g_{2\boldsymbol{pj}}) Q^{(\boldsymbol{k})(0)}_{\boldsymbol{j}} \\
&\quad + 2\Omega \cos{(\Omega t)} \sum_{\boldsymbol{j}\neq\boldsymbol{p}} (g_{1\boldsymbol{pj}} + g_{2\boldsymbol{pj}})\, \partial_t Q^{(\boldsymbol{k})(0)}_{\boldsymbol{j}}\,, \qquad (38)
\end{aligned}$$

setting the coefficients of $e^{\pm i\omega_{\boldsymbol{p}}t}$ to zero we obtain

$$\frac{dA_{\boldsymbol{p}}^{(\boldsymbol{k})}}{d\tau} = -\left[\frac{\pi^2 p_z^2}{2\omega_{\boldsymbol{p}}L^2} + \frac{y_{nm}^2}{2\omega_{\boldsymbol{p}}R^2}\right] B_{\boldsymbol{p}}^{(\boldsymbol{k})}\delta\left(2\omega_{\boldsymbol{p}} - \Omega\right) + \text{other terms similar to the cubical case}\,, \tag{39}$$

and

$$\frac{dB_{\boldsymbol{p}}^{(\boldsymbol{k})}}{d\tau} = -\left[\frac{\pi^2 p_z^2}{2\omega_{\boldsymbol{p}}L^2} + \frac{y_{nm}^2}{2\omega_{\boldsymbol{p}}R^2}\right] A_{\boldsymbol{p}}^{(\boldsymbol{k})}\delta\left(2\omega_{\boldsymbol{p}} - \Omega\right) + \text{the same as the above equation}\,. \tag{40}$$

Again for the case of $\Omega = 2\omega_{\boldsymbol{p}}$ and $|\omega_{\boldsymbol{p}} \pm \omega_{\boldsymbol{j}}| \neq \Omega$ we obtain the same equations as in the previous section

$$\frac{dA_{\boldsymbol{p}}^{(\boldsymbol{k})}}{d\tau} = -\frac{\omega_{\boldsymbol{p}}}{2}B_{\boldsymbol{p}}^{(\boldsymbol{k})}\,, \qquad \frac{dB_{\boldsymbol{p}}^{(\boldsymbol{k})}}{d\tau} = -\frac{\omega_{\boldsymbol{p}}}{2}A_{\boldsymbol{p}}^{(\boldsymbol{k})}\,. \tag{41}$$

Thus for the mean number of created photons we obtain

$$\langle N_{\boldsymbol{p}}\rangle = \sinh^2\left(\frac{\omega_{\boldsymbol{p}}}{2}\tau\right)\,. \tag{42}$$

4. Conclusion

In this paper we discussed the particle creation from cubical and cylindrical geometries by considering Dirichlet boundary conditions. Following the previous works by one of authors [11, 12] and also the work by Dalvit *et al.* [22] here we consider all the walls of the cavity to be time dependent. By using MSA technique and assuming the resonance case we derived the mean number of photons created during the motion of the walls of the cavity. One of the difficulties in detection the particles created in the dynamical Casimir effect is the low number of particles created [23], therefore, considering all the walls of the cavity maybe useful from this point of view. We also predict interesting phenomena if we consider different oscillating frequencies for the walls of the cavity.

REFERENCES

[1] H.B.G. Casimir, *Proc. K. Ned. Akad. Wet.* **51**, 793 (1948).

[2] V.M. Mostepanenko, N.N. Trunov, *The Casimir Effect and Its Applications*, Clarendon, Oxford 1997.

[3] G. Plunien, B. Muller, W. Greiner, *Phys. Rep.* **134**, 87 (1986).

[4] *The Casimir Effect, 50 Years Later*, edited by M. Bordag, World Scientific, Singapore 1999.

[5] M. Bordag, U. Mohidden, V.M. Mostepanenko, *Phys. Rep.* **353**, 1 (2001).

[6] N.D. Birrell, P.C.W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, 1982.

[7] P. Candelas, D. Deutsch, *Proc. Roy. Soc.* **A354**, 79 (1977).

[8] V.P. Frolov, E.M. Serebriany, *J. Phys. A: Math. Gen.* **12**, 2415 (1979).

[9] V.P. Frolov, E.M. Serebriany, *J. Phys. A: Math. Gen.* **13**, 3205 (1980).

[10] V.P. Frolov, D. Singh, *Class. Quantum Grav.* **16**, 3693 (1999).

[11] M.R. Setare, A.A. Saharian, *Mod. Phys. Lett.* **A16**, 927, (2001).

[12] M.R. Setare, A.A. Saharian, *Mod. Phys. Lett.* **A16**, 1269, (2001).

[13] J. Schwinger, *Proc. Nat. Acad. Sci.* **90**, 985 2105 4505 7285 (1993); **91**, 6473 (1994).

[14] B.P. Barber, R.A. Hiller, R. Löfstedt, S.J. Putterman, *Phys. Rep.* **281**, 65 (1997).

[15] E. Schrodinger, *Physica (Utrecht)* **6**, 899, (1939).

[16] B.S. DeWitt, *Phys. Rev.* **90**, 357 (1953).

[17] T. Imamura, *Phys. Rev.* **118**, 1430 (1960).

[18] L. Parker, *Phys. Rev. Lett.* **21**, 562 (1968).

[19] L. Parker, *Phys. Rev.* **183**, 1057 (1969).

[20] M. Crocce, D.A.R. Dalvit, F.D. Mazzitelli, *Phys. Rev.* **A64**, 013808 (2001).

[21] C.M. Bender, S.A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, McGraw Hill, New York 1978.

[22] D.A.R. Dalvit, F.D. Mazzitelli, X. Orsi Milla'n, *J. Phys. A: Math. Gen.* **39**, 6261 (2006).

[23] M. Brown-Hayes, J.H. Brownell, D.A.R. Dalvit, W.J. Kim, A. Lambrecht, F.C. Lombardo, F.D. Mazzitelli, S.M. Middleman, V.V. Nesvizhevsky, R. Onofrio, S. Reynaud, *J. Phys. A: Math. Gen.* **39**, 6195 (2006).