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SUPPRESSION OF STATISTICAL BACKGROUND IN THE EVENT STRUCTURE OF AWAY-SIDE $\Delta\phi$ DISTRIBUTION

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An approach is proposed to analyze the azimuthal distribution of particles produced on the away side in heavy-ion collisions without background subtraction. Measures in terms of factorial moments are suggested that can suppress the statistical background, while giving clear distinction between one-jet and two-jet event structures on the away side. It is also possible to map the position and strength of the recoil jet to suitably chosen asymmetry moments.

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1. Introduction

Recently an important area of investigation in relativistic heavy-ion collisions is the azimuthal angular distribution of hadrons produced on the opposite side of particles triggered at high $p_{\rm T}$ [1–3]. That distribution reveals the properties of the medium effect on a hard recoil parton traversing the dense system formed by a central collision. Several possibilities have been suggested for what may happen to the hard parton and what signatures may be registered on the away side [4–8]. Our concern in this paper is not so much on the nature of the possible signatures, but on how best to extract them from a sea of noisy background that is inherent in every nuclear collision. The conventional approach is to subtract the background. That is accomplished by first summing over all events subject to specific kinematical cuts. It is meaningless and impossible to make background subtraction event by event. However, summing over all events is a process in analysis that often degrades the signal, since the recoil parton may be absorbed by the medium, or may emerge on the other side as a reduced minijet, or may generate a shock wave. To enhance the signature what is needed is a measure that filters out the statistical fluctuation in every event to the extent possible, and then let that measure be summed over all events. The aim of this paper is to propose such a measure and to demonstrate its effectiveness in the framework of a simple model that simulates events with jets in the midst of a large statistical background.

The measure that we propose is factorial moments. Before going into the details, let us first give some historical background. The factorial moments were first used in the study of intermittency, which quantifies the self-similar behavior of branching processes in multiparticle production [9, 10]. It has also been suggested for critical behavior in heavy-ion collisions [11]. Although intermittency has been found in elementary collisions, its study in nuclear collisions has been plagued by a number of difficulties, among which is the problem of statistical fluctuations in large systems despite the theoretical virtues of the factorial moments. There are several differences between those intermittency studies and our problem here. First, we do not analyze fluctuations in rapidity, and do not consider a wide range of bin size to search for fractal behavior. Second, we consider only a subset of rarer events selected by high p_{T} triggers, and examine the away-side $\Delta\phi$ distribution that consists of far lower multiplicities of particles. Third, in intermittency studies the dynamical fluctuation is convoluted with statistical fluctuation, whereas our jet signal on the away side is additive relative to the background. Having made these remarks to dissociate our work from whatever vestige there may be from the past, we proceed now to a description of our measure from the basics.

The outline of the remainder of this paper is as follows. Section 2 gives the definition of the appropriate of factorial moment measures of present interest. They are the normalized factorial moments (NFM). Throughout this work the term: "FM" approach will be used as a generic term to refer to analyses which make use of the NFMs. In that section we will introduce a simple model to illustrate the application of the FM approach in analyzing the away-side azimuthal distribution. In Section 3, we illustrate how those event-by-event fluctuations in the away-side spectrum may be evaluated in the FM approach. In Section 4, we discuss an approach which reveals the position and shape information of the 1j case using the FM method. A brief summary is included in Section 5.

2. Factorial moments of a simple model

2.1. Definition of FM measures

For notation convenience, we define $\varphi = \Delta\phi - \pi$, with the usual $\Delta\phi$ variable defined relative to the trigger momentum in the transverse plane. Consider an interval I around $\varphi = 0$, which, for definiteness, may be taken to be from -1.5 to $+1.5$, although the suitable range is an experimental decision. Let I be divided into M equal bins so that the bin size is $\delta = I/M$. In an event let n_j denote the number of particles in jth bin. Define the factorial moment (FM) by

$$f_q = \left\langle \frac{n!}{(n-q)!} \right\rangle = \frac{1}{M} \sum_{j=1}^{M} n_j(n_j-1)\cdots(n_j-q+1)\,, \tag{1}$$

and the normalized factorial moment (NFM) by

$$F_q = \frac{f_q}{f_1^q}\,, \tag{2}$$

for each event. It is important to recognize that Eq. (1) is used to determine the FM event by event, and the summation there may be regarded as the horizontal average.

Let us now consider the hypothetical case where the fluctuation of n_j from bin to bin can be described by a Poisson distribution as an ideal representation of the background

$$P_{\bar{n}}(n) = \frac{\bar{n}^n}{n!} e^{-\bar{n}}\,, \tag{3}$$

where n is the bin multiplicity that fluctuates and $\bar{n}$ is the average that depends on δ and total event multiplicity N. Then Eq. (1) can be written as

$$f_q = \sum_{n=q}^{N} \frac{n!}{(n-q)!} P_{\bar{n}}(n)\,. \tag{4}$$

If N is large compared to $\bar{n}$, it can be approximated by infinity in Eq. (4); it then follows from Eq. (3) that $f_q^{(\mathrm{stat})} = \bar{n}^q$, so we have $F_q^{(\mathrm{stat})} = 1$ for any δ in the ideal statistical case. When N is finite but large, the corresponding $F_q^{(\mathrm{stat})}$ should be slightly less than 1. Furthermore, it has been pointed out to us that F_q as defined in Eq. (2) is independent of detector efficiency [12].

In reality, $F_q^{(\mathrm{stat})}$ is not exactly 1. We expect F_q to fluctuate from event to event. Let us define in general the event-averaged NFM

$$\langle F_q \rangle(\delta) = \frac{1}{\mathcal{N}_{\mathrm{evt}}} \sum_{i=1}^{\mathcal{N}_{\mathrm{evt}}} F_q^{(i)}(\delta, N_i)\,, \tag{5}$$

where N_i is the multiplicity of the ith event and $\mathcal{N}_{\rm evt}$ is the total number of events. The central question is whether $\langle F_q \rangle$ stays approximately at 1 for pure background in the real data and becomes larger than 1 significantly enough, when jets exist, so that $\langle F_q \rangle$ can be used as an effective measure to distinguish the signal from the noise. We stress that the (vertical) average over all events in Eq. (5) is performed after the horizontal average is done in Eq. (1), not the other way around. Thus if the answer to the above central question is in the affirmative, then one may regard F_q as effectively suppressing the statistical background event-by-event. Since it is impossible to make background subtraction event-by-event, to suppress the statistical contribution to a suitably chosen measure seems to be the best that one can do, and it should be done at the level of each event.

2.2. A simple model of away-side distribution

To test the effectiveness of the measure suggested above, let us consider a simple model to simulate the φ distributions of events with jets in the presence of significant background. Let N_i particles be distributed randomly in the interval I with $-1.5 < \varphi < 1.5$. They will be regarded as the background of the ith event. For now, we limit ourselves to the simplest case of N_i being a constant, but later generalize N_i to be Gaussian distributed. For a 1j event, we add a cluster of particles of multiplicity, $m = 5$, bunched in a small interval in φ of extent $\epsilon = 0.04$ rad. The cluster is randomly located in the interval $-1 < \varphi < +1$. For a 2j event (to mimic a Mach cone structure) we add two clusters of 5 particles each, and place them symmetrically about $\varphi = 0$, uniformly distributed in the interval $|\varphi| = 0.6$–0.8. We label the three cases by bg, bg+1j and bg+2j, respectively, where bg denotes background only without jets.

In Fig. 1 we show the distributions of (a) F_2 and (b) F_3 for the three cases, for $N_i = 60, M = 30$ and $\mathcal{N}_{\rm evt} = 2000$. Although the distributions have significant overlap, their average values $\langle F_q \rangle$ are distinctly separated, as shown by the three lines for $q = 2, 3, 4$ in Fig. 2, for various bin numbers M. In (a) the average values $\langle F_q \rangle$ for the background are all less than, but close to, 1. In (b) for the case including 1j and in (c) for the bg+2j case, $\langle F_q \rangle$ are distinctly higher than 1, especially for $q > 2$. These results represent the first indication that the FM method is useful, and that the suppression of the statistical fluctuation is realized upon event averaging without explicit subtraction of the background. That is, with $\langle F_q \rangle$ being around 1 for background only, any significant enhancement of $\langle F_q \rangle$ above 1 would indicate the presence of non-statistical signal.

In a more realistic situation elliptic flow introduces a φ dependence to the background. We have accordingly modified the statistical background by a factor $1 + 2v_2\cos2\varphi$, and find that Fig. 2 remains essentially the same

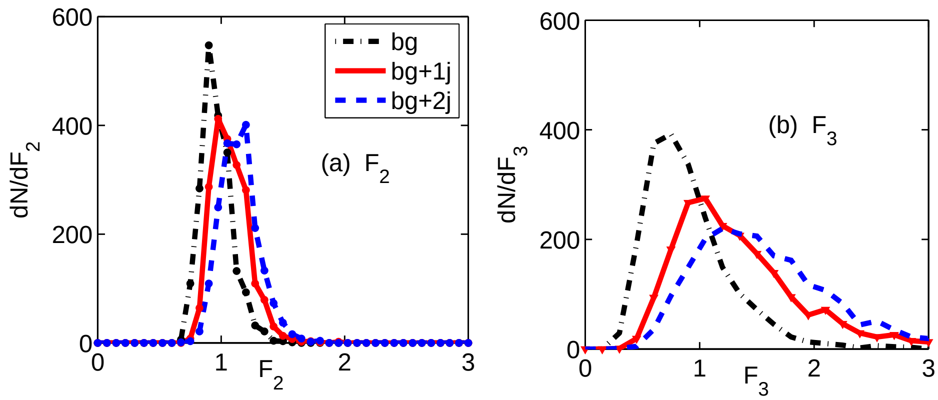

Fig. 1. Event-by-event distributions of F_q for (a) $q = 2$ and (b) $q = 3$, M=30. The case shown is for $dN/d\varphi|_{\rm bg} = 20$.

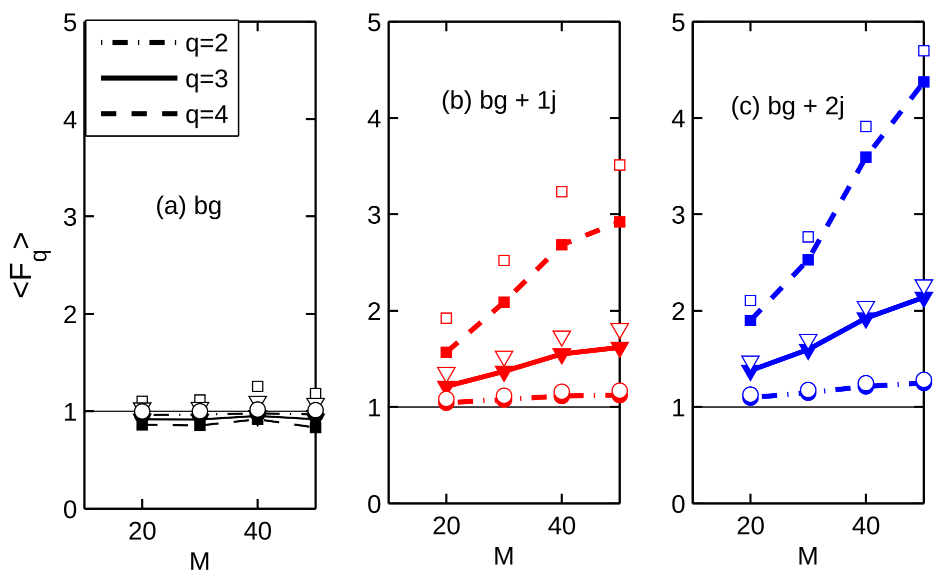

Fig. 2. Event-averaged $\langle F_q \rangle$ *versus* bin number M for (a) bg only, (b) bg+1j, and (c) bg+2j. Circles, triangles and squares are for $q = 2, 3, 4$, respectively. The solid points are for $v_2 = 0$, the open points for $v_2 = 0.1$. For $dN/d\varphi|_{\rm bg} < 50$, the 1j and 2j signals quantified by $\langle F_q \rangle$ can be distinguished from the background. The case shown is for $dN/d\varphi|_{\rm bg} = 20$.

for v_2 increasing up to 0.1. We do not show the modified F_q distributions in Fig. 1 because they overlap with the existing ones (without flow effect) so much as to cause substantial sacrifice in clarity. However, the modified $\langle F_q \rangle$

can be shown clearly by the open points in Fig. 2 for $v_2 = 0.1$. They are slightly higher than the corresponding curves for $v_2 = 0$, with the deviation most noticeable for the bg+1j case at $q = 4$. Still, the general features of the result remain the same.

In the test cases presented the background consists of 60 particles, distributed over an interval of $I = [-1.5, 1.5]$, which corresponds to $dN/d\varphi|_{\rm bg} \approx 20$. We use $dN/d\varphi$ to denote the event-averaged azimuthal distribution, as is conventionally done, although in our simulation here the same notation is used for the distribution in any event, for brevity's sake. In some experimental data the lower bound of $p_{\rm T}^{\rm assoc}$ has been set as low as 0.15 GeV/c [1], for which the background height is as high as $dN/d\varphi|_{\rm bg} \approx 200$. In such cases the signal of jets on the away side is at the 1% level; it is therefore in the presence of statistical bin-to-bin fluctuations that can be significantly larger, and our method cannot be expected to be effective. We have found that for $dN/d\varphi|_{\rm bg} < 50$, the 1j and 2j signals quantified by $\langle F_q \rangle$ can be distinguished from the background.

It should also be noted that our model for a jet is a cluster of 5 particles in a small interval ϵ. Obviously, if a cluster is widely spread out without a corresponding increase of particle multiplicity in the jet, then the effect of jet becomes indistinguishable from the background fluctuation and one cannot expect any method to be able to extract its properties. Since our purpose here is to illustrate a new method of analysis rather than to construct realistic jet characteristics, the model we use suffices toward that end.

3. Event-by-event fluctuations about $\varphi = 0$

A disadvantage in working with F_q is that one no longer sees visually the peaks in $\Delta\phi$ distribution associated with jets. Thus one gets a quantitative measure of the peaks at the expense of losing information about the locations of the peaks. However, the loss can be reduced if we elevate the level of horizontal analysis of the FM by being more specific about the spatial regions in φ. To that end let us define

$$f_q^{\pm} = \frac{1}{M_{\pm}} \sum_{j \in R_{\pm}} n_j(n_j - 1) \cdots (n_j - q + 1)\,, \tag{6}$$

where $R_{\pm}$ stands for the region where φ is $\gtrless 0$, and $M_{\pm} = M/2$ is the number of bins in $R_{\pm}$. Thus we have $f_q = (f_q^+ + f_q^-)/2$ for each event. We further define the NFM, as in Eq. (2), by

$$F_q^{\pm} = f_q^{\pm}/(f_1)^q\,, \tag{7}$$

which implies $F_q = (F_q^+ + F_q^-)/2$ for every event.

Since event averaging is likely to erase the distinction between F_q^+ and F_q^- if a jet fluctuates between being on the + and − sides, it is useful to consider the difference moments $D_q = |F_q^+ - F_q^-|$ and the sum $S_q = F_q^+ + F_q^-$. To amplify the effect of the fluctuations let us define the vertical average of the p-th moment of D_q by

$$\left\langle D_q^p \right\rangle (\delta) = \frac{1}{\mathcal{N}_{\mathrm{evt}}} \sum_{i=1}^{\mathcal{N}_{\mathrm{evt}}} D_q^{(i)p}(\delta, N_i)\,, \tag{8}$$

and similarly of S_q. The superscript p is a power of D_q. A plot of $\langle D_q^p \rangle (\delta)$ *versus* $\langle S_q^p \rangle (\delta)$ for the three cases of bg, 1j and 2j for various values of δ can reveal some characteristics of interest. In our simulation we place the two jets always on the opposite sides of $\varphi = 0$ in order to contrast the 2j from the 1j events. In Fig. 3 we show an array of such plots for $p, q = 2, 3, 4$. In all, the background points are clustered together, while the 1j and 2j points fan out from the origin, mostly in straight lines. Clearly, two jets on two sides of $\varphi = 0$ reduce D_q, resulting in lower $\langle D_q^p \rangle$ compared to the same from 1j. Similar plots of the experimental data would indicate first of all

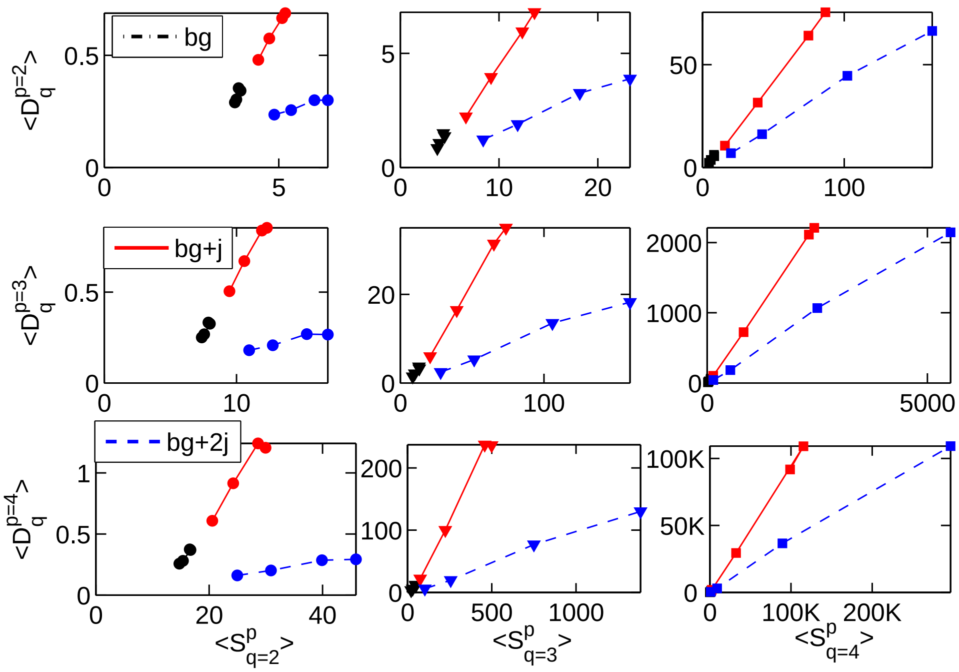

Fig. 3. $\left\langle D_q^p \right\rangle$ *versus* $\left\langle S_q^p \right\rangle$. The rows are for $p = 2, 3$ and 4, and the columns for $q = 2, 3$ and 4. Connected points are at $M = 20, 30, 40$, and 50.

whether the $\langle D_q^p \rangle$ and $\langle S_q^p \rangle$ points of the background cluster together at the lower-left corner, and then secondly whether the points arising from 1j and 2j are well separated from those from just the background.

The implication of Fig. 3 is that the bin size δ is not an essential variable, and that the normalized asymmetry

$$A_{qp} = \langle D_q^p \rangle / \langle S_q^p \rangle \tag{9}$$

can be a more succinct measure. To see that explicitly we show in Fig. 4 the values of A_{qp} *versus* δ^{-1} for the 1j and 2j points. For $q = 3$ and 4 the dependences on p and δ are not sensitive, and the differentiation between the 1j and 2j cases can easily be made, even for $q = 2$. It seems that this type of analysis renders more quantitative and distinctive results than 3-particle correlation.

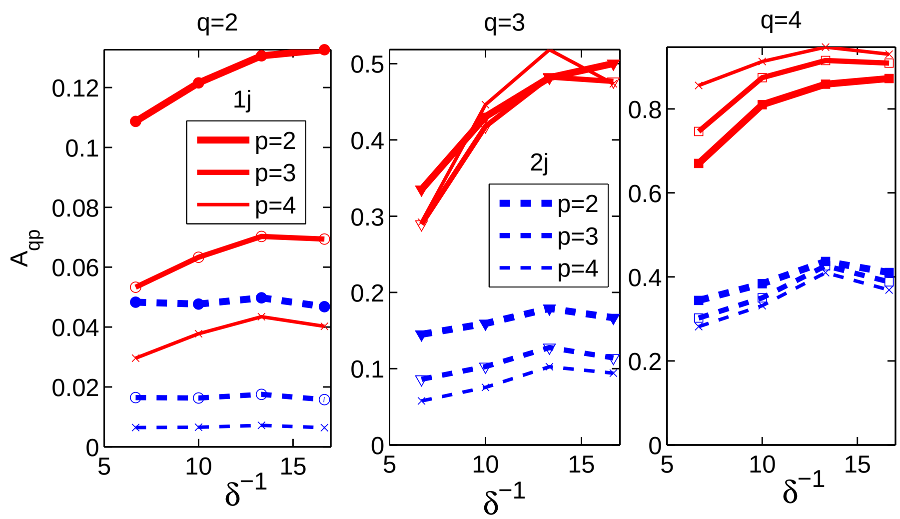

Fig. 4. A_{qp} *versus* δ^{-1} for (a) $q = 2$, (b) $q = 3$, and (c) $q = 4$. Thick, medium, thin lines are for $p = 2, 3, 4$; solid (dashed) lines for bg+1j (bg+2j).

We have generalized the value of N_i to be Gaussian distributed, and then also the jet multiplicity m. We have found no significant effect on the asymmetry measure A_{qp}. Since our aim here is only to suggest useful measures, and not their detail numbers, we will not detail the results of the Gaussian-distributed multiplicities here.

4. A measure reflecting φ dependence

Restricting our attention now to only the 1j case, we consider the question of how best to reveal the position and shape of a peak on the away side using FM. To that end we need to introduce a cut in φ. Define

$$f_q^{\gtrless}(\varphi_c) = \frac{1}{M_{\gtrless}} \sum_{j \in S_{\gtrless}} n_j(n_j-1)\cdots(n_j-q+1)\,, \tag{10}$$

where $S_{\gtrless}$ is the set of bins in the range $|\varphi|\gtrless\varphi_c$, and $M_{\gtrless}$ is the number of bins in $S_{\gtrless}$. The overall FM is then given by $f_q = r_< f_q^< + r_> f_q^>$, where $r_{\gtrless} = M_{\gtrless}/M$. The corresponding NFM is

$$F_q^{\gtrless}(\varphi_c) = f_q^{\gtrless}(\varphi_c)\,/(f_1)^q \ , \tag{11}$$

and the overall NFM is $F_q = r_< F_q^< + r_> F_q^>$. The relative magnitude of $F_q^>$ *versus* $F_q^<$, as φ_c is varied, provides a measure of the shape of the φ distribution of the jet location. Let us define

$$B_q = \frac{\left\langle F_q^< - F_q^> \right\rangle}{\left\langle F_q^< + F_q^> \right\rangle}\,. \tag{12}$$

To test its usefulness we consider three cases for which the event-averaged distributions above the background are shown in Fig. 5(a); the heights of the background, $dN/d\varphi|_{\rm bg}$, (not shown) are [i] 20, [j] 2, and [k] 0.2. The widths and locations of the peaks are: [i] (0.4, $\pm$0.8), [j] (0.3, $\pm$0.4), [k] (0.3, 0), respectively. The signal in case [i] is at the 1% level, and corresponds to very low $p_{\rm T}^{\rm assoc}$, whereas case [k] has a sharp peak above a low background that can arise from high $p_{\rm T}^{\rm trig}$ and $p_{\rm T}^{\rm assoc}$. In Fig. 5(b) $B_4(\varphi_c)$ is shown for the three cases, which are well separated. In all cases background alone gives B_q very nearly 0 because either $\left\langle F_q^< \right\rangle \cong \left\langle F_q^> \right\rangle$ for $\varphi_c > 0.1$ or their sum is much larger than their difference. Note that even in case [i] where the double peaks are small compared to background, we obtain without background subtraction nonzero $B_4(\varphi_c)$, albeit small. In case [j] with $dN/\varphi|_{\rm bg} = 2$ the average bin multiplicity $\bar{n}$ of the background is only 0.2, which is much less than $q \geq 1$. Nevertheless, $f_q^{\gtrless}$ defined in Eq. (10) does not vanish due to the fluctuations of the bin multiplicities, so $F_q^{\gtrless}$ exists, though very large. The event-averaged $\left\langle F_4^{\gtrless} \right\rangle$ are between 0 and 4, resulting in B_4 to be around 0.5. In case [k] the strong peak at $\varphi = 0$ results in $F_q^<$ being large compared to $F_q^>$ except when φ_c is small; thus B_4 is large, specially at large φ_c. Where it deviates from 1 is a measure of the narrowness of the peak. B_q for $q = 2, 3$ give similar results, so we do not show them for the sake of clarity in the figure.

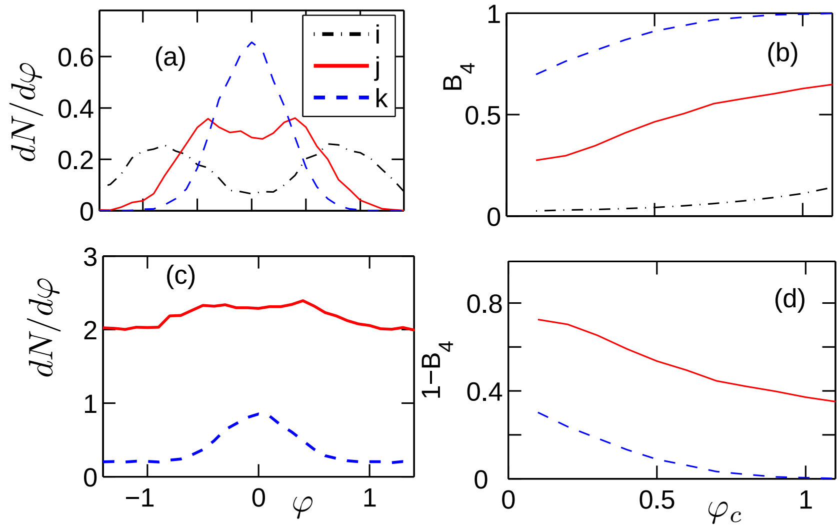

Fig. 5. (a) Event-averaged φ distribution for $\mathcal{N}_{\rm evt} = 5000$ with $M = 30$. The background contributions are not shown; their levels are [i] 20, [j] 2, [k] 0.2. The widths and locations of the peaks are: [i] (0.4, $\pm$0.8), [j] (0.3, $\pm$0.4), [k] (0.3, 0), respectively. (b) The corresponding $B_4(\varphi_c)$ distributions. (c) $dN/d\varphi$ for [j] and [k] with bg included. (d) $1 - B_4$ for the distributions in (c).

To achieve a better perspective of the mapping between $dN/d\varphi$ and B_4, we show in Fig. 5(c) the former for cases [j] and [k] with background included. The corresponding plot of $1 - B_4(\varphi_c)$, which is more intuitive, is given in (d). Here [j] exhibits the features of both the broad bump in (a) and the high background in (c), while the peak of [k] reflects the same in (a) and (c). These properties are remarkable due to the drastic difference in the nature of the measures displayed. This demonstrates that without background subtraction the FM analysis event-by-event results in clear and quantitative description of the $\Delta\phi$ characteristics.

We end our presentation of figures here with a general remark on the statistical errors. By doubling the total number of events used, we have found that curves shown in all the figures presented are stable with respect the variations of the total number of events used. In other words, the error bars due to statistics are not significant.

5. Summary

In this work we have considered the use of factorial moments to analyze the $\Delta\phi$ distribution of particles produced opposite a trigger. In practice, for a given set of experimental data, one needs to first determine the applicability of the FM approach by previewing $\langle F_q \rangle$'s, to check whether there are at least some of them which is significantly greater than unity. For FM-applicable data, the advantage of such a method of analysis is that no explicit background subtraction is necessary. The FM of each event is sensitive to the jet characteristics, while suppressing the effect of statistical fluctuation of the background; it is also insensitive to the smooth variation due to elliptic flow. The asymmetry moments A_{qp} can well separate 1j and 2j recoil scenarios, and B_q can give a quantitative description of the single-jet characteristics. The application of this method to the analysis of the RHIC data on jet correlation may provide a common framework to compare results from widely different experimental conditions and various subtraction schemes.

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