GIANT SUPPRESSION OF THE ACTIVATION RATE IN DYNAMICAL SYSTEMS EXHIBITING CHAOTIC TRANSITIONS*

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The phenomenon of giant suppression of activation, when two or more correlated noise signals act on the system, was found a few years ago in dynamical bistable or metastable systems. When the correlation between these noise signals is strong enough and the amplitudes of the noise are chosen correctly — the life time of the metastable state may be longer than in the case of the application of only a single noise even by many orders of magnitude. In this paper, we investigate similar phenomena in systems exhibiting several chaotic transitions: Pomeau–Manneville intermittency, boundary crisis and interior crisis induced intermittency. Our goal is to show that, in these systems the application of two noise components with the proper choice of the parameters in the case of intermittency can also lengthen the mean laminar phase length or, in the case of boundary crisis, lengthen the time the trajectory spends on the pre-crisis attractor. In systems with crisis induced intermittency, we introduce a new phenomenon we called quasi-deterministic giant suppression of activation in which the lengthening of the laminar phase lengths is caused not by the action of two correlated noise signals but by a single noise term which is correlated with one of the chaotic variables of the system.

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1. Introduction

In chaotic transitions a continuous change of one of the parameters of the system causes a discontinuous change in the properties of the chaotic attractor of the system [1]. The best known of these transitions are: the period-doubling bifurcation cascade, intermittency and crisis. In the case of

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the first kind of these chaotic transitions [2] the stable periodic orbit of the system undergoes successive period-doubling bifurcations. In the limit, the period becomes infinite and a chaotic attractor appears. This is a typical route to chaos for many dynamical systems. During intermittency irregularly long periods of the time occur in the time evolution of the system during which the statistical properties of the trajectory differ strongly from those in other periods. One example is type I intermittency [3] [4] — one of three kinds of intermittency originally introduced by Pomeau and Mannevile [3]. Other examples include on–off intermittency [5] or in–out intermittency [6]. Generally speaking, in these kinds of intermittency the system spends a significant amount of time very close to an invariant subspace, the dimension of which is smaller than the dimension of the whole attractor. The last category of the above mentioned chaotic transitions, the crises [7] [8], are present in such systems in which the chaotic attractors — depending on the type of intermittency — suddenly grow, vanish or merge with another chaotic attractor when the control parameter of the system is changed continuously. The first of these chaotic transitions is known as the interior crisis, the second — the boundary crisis and the last one — the attractor merging crisis. Crisis-induced intermittency is associated with the interior crisis.

In both cases *i.e.* in Pomeau–Mannevile intermittency and in crises, to describe the behavior of the system in which they are present, various statistical measures are used. The mean laminar phase length or the laminar phase length distribution are used in the case of a Pomeau–Mannevile intermittency [4]. The mean life time of the trajectory on the pre-crisis attractor and distribution of this time are used in the case of the boundary crisis. These statistics are characteristic for the specific type of chaotic transition and allow us to identify such a transition in real physical systems, when the form of equations describing the system is not known.

In real systems, noise is usually present. Due to the noise the above mentioned statistics measured in real systems differ from these found in simple models. In the case of additive white noise, the effect on the laminar phase length distribution and the mean laminar phase length was investigated and described by Hirsch [4]. The main conclusions were that in the presence of noise there are more shorter laminar phases than in the absence of noise and, consequently, the mean laminar phase length is also shorter. Usually, one expects noise to introduce disorder into the dynamics of the system.

In the theory of dynamical systems, however, there are cases in which adding noise makes system more "ordered". The giant suppression of activation (GSA) [9], present in bistable or metastable systems is a good example of such an effect. In such systems, adding two correlated Gaussian noise terms may cause the life time of the metastable states to be many orders of magnitude longer than in the presence of only a single noise component [10].

To obtain giant suppression of activation, the amplitudes of the noise components must be chosen properly.

A similar phenomenon was found [11] in a nonchaotic system — the logistic equation, which is a continuous time version of logistic map. The effect of two correlated noises, one additive and one parametric, was investigated. For the correlation coefficient close to one, the behavior of system became very regular. With a periodic signal acting on system, the phenomenon of stochastic resonance [12] was observed, characterized by the maximum of signal-to-noise ratio [11]. These results allows us to expect that similar phenomena of order in the dynamics of system induced by two correlated noise terms may be observed also in systems with chaotic transitions.

We show that for systems exhibiting several kinds of chaotic transitions, application of two correlated noise signals can also make the dynamics of the system more ordered *i.e.* the mean length of laminar phase may increase or the mean life time of the trajectory on the pre-crisis attractor may become larger than due to a single white noise only. The effect may be important for the analysis of dynamical systems. Also, in experiments, we may expect to observe properties similar to those obtained for the noise-free case — in spite of the presence of noise. In fact, our results show that the effect is obtained also when many noise components act on the system, even with very large amplitudes, provided they are correlated to each other.

The paper is organized as follows: in Sec. 2, we analyze the suppression of activation in systems with type I intermittency and give a numerical example of such a phenomena as well as a qualitative explanation of it. In Sec. 3, we show how two correlated noise components may cause a giant suppression of activation in a system with boundary crisis — a numerical example is also given. In Sec. 4, we introduce the quasi-deterministic giant suppression of activation (QDGSA), which occurs in crisis-induced intermittency. The increase in the mean laminar phase is then caused not by two correlated noise components but by only a single noise term, which is, however, correlated with one of the variables of the system. Discussion and conclusions are given in Sec. 5.

2. Giant suppression of activation in type I intermittency

2.1. General description of the phenomenon

Consider a dynamical system, in which the route to chaos via type I intermittency is obtained. In such a system, at a critical value of the control parameter a_{c}, a saddle-node bifurcation occurs. When $a < a_{\mathrm{c}}$ we obtain type I intermittency while for $a > a_{\mathrm{c}}$ two fixed points, one stable (a node) and one unstable (a saddle) are present.

Assume that the control parameter is such that the fixed point is stable. Such a situation resembles the case of a potential with a metastable state. In the absence of noise, the system reaches the stable fixed point and stays there. When the additive noise is present — the system may escape from the fixed point. Such a behavior may be observed in a mechanical system with a third-order potential for moderate amplitudes of the noise. In the neighborhood of the fixed point the dynamical system may be approximated by such a third-order potential (see *e.g.* [4]). However, far away from this neighborhood, this approximation is no longer valid — the higher order terms in the potential cause a return to the vicinity of the node. Thus, we may treat such a phenomenon as a kind of intermittency. This intermittency-like phenomenon is sometimes called noise-induced intermittency [13] and it is characterized by an exponential distribution of the laminar phase lengths. Adding another noise term to the system, *e.g.* a multiplicative one, may increase the mean laminar phase length if this second noise component is correlated to the first one. This is very similar to the GSA described in [9] — the only difference being that our system always returns to the metastable state.

Next, assume that the system is in the intermittency region. In this region, we do not have any stable fixed points, so we expect that the laminar phases (*i.e.* the periods of time, when the trajectory passes close to bifurcation point) will not be arbitrarily long even when the correlation between the two noise components is very strong. However, type I intermittency may be treated as a limiting case of metastability. *E.g.* Hirsch *et. al.* [4] have introduced a simple model of a particle in a third order potential to explain many properties of the type I intermittent system. So, we can expect that there exists such a combination of noise amplitudes that the behavior of the system may become very similar to the purely deterministic case *i.e.* one without the noise present.

For a one-dimensional dynamical system with discrete time (a map) we can write the equation of evolution in the form:

$$x_{n+1} = f\left(x_n\right) + \sigma_1 g_1\left(x_n\right)\xi_1 + \sigma_2 g_2\left(x_n\right)\xi_2\,, \tag{1}$$

where ξ_1 and ξ_2 are random variables with a normal distribution, a zero mean value and variance equal to one. If *e.g.* $g_2 = \text{const.}$, the noise ξ_2 becomes additive noise. But, in this subsection, we consider the general form of Eq. (1), with two multiplicative noise terms.

In type I intermittency, the system spends a considerable amount of time in the vicinity of the fixed point x^* that would be created by the saddle-node bifurcation. Inserting x^* into Eq. (1) we obtain:

$$x_{n+1} = f\left(x^*\right) + \sigma_1 g_1\left(x^*\right)\xi_1 + \sigma_2 g_2\left(x^*\right)\xi_2\ . \tag{2}$$

If $\xi_1 = \xi_2 = \xi$, we can rewrite it in the form:

$$x_{n+1} = f\left(x^*\right) + \left(\sigma_1 g_1\left(x^*\right) + \sigma_2 g_2\left(x^*\right)\right)\xi\,. \tag{3}$$

One can see that if we choose the values of σ_1 and σ_2 so that:

$$\frac{\sigma_1}{\sigma_2} = -\frac{g_2\left(x^*\right)}{g_1\left(x^*\right)} \tag{4}$$

the effect of the noise terms at the point x^* will cancel and the effective noise amplitude in the entire intermittency channel will be very small. So, we expect that in this case the laminar phase length distribution and the position of the right peak of this distribution to be similar to that for the deterministic case without noise. How to measure this similarity will be shown later.

Now, let us generalize the above case and consider ξ_1 and ξ_2 which are linearly dependent and so correlated. When the correlation coefficient is very close to but not equal to one, we can write

$$\xi_2 = \rho\xi_1 + \sqrt{1-\rho^2}\xi_3\,, \tag{5}$$

where ξ_3 is a Gaussian noise such that $\langle\xi_1\xi_3\rangle = 0$ and ρ is close to 1. The noise term in (2) can be now rewritten as

$$\xi\left(x^*\right) = [\sigma_1 g_1\left(x^*\right) + \rho\sigma_2 g_2\left(x^*\right)]\xi_1 + \sqrt{1-\rho^2}\sigma_2 g_2\left(x^*\right)\xi_3\,. \tag{6}$$

The random variable $\xi\left(x^*\right)$ is also Gaussian and its variance as a function of x^* can be expressed as

$$\sigma^2\left(x^*\right) = [\sigma_1 g_1\left(x^*\right) + \rho\sigma_2 g_2\left(x^*\right)]^2 + \left(1-\rho^2\right)\sigma_2^2 g_2^2\left(x^*\right)\,. \tag{7}$$

This variance is never equal to zero as long as g_1 or g_2 are not exactly equal to zero, but if the value of one of the noise terms, *e.g.* σ_2, is considered fixed, the function σ^2 has a minimum when

$$\sigma_1 = -\rho\sigma_2\frac{g_2\left(x^*\right)}{g_1\left(x^*\right)}\,. \tag{8}$$

At such an optimal choice of noise amplitudes the system behaves similarly as in the purely deterministic case.

2.2. An example — the logistic map

The logistic map is given by equation [1]

$$x_{n+1} = ax_n (1 - x_n) . \tag{9}$$

It is well known, that when the value of a is slightly smaller than $a_{\rm c} = 1 + \sqrt{8} \cong 3.828427125\ldots$, the logistic map exhibits type I intermittency [4]. At $a = a_{\rm c}$ the saddle-node bifurcation occurs and three stable points appear: $x_1^* = 0.156\ldots$, $x_2^* = 0.514\ldots$ and $x_3^* = 0.956\ldots$. Let us focus on the first one.

On Fig. 1(a) the laminar phase length distribution (LPLD) is shown. It has a bimodal form — typical for this type of intermittency. Fig. 1(b) depicts LPLD in the presence of additive noise. It can be seen that the right peak shifts to shorter laminar phase lengths and a long tail in the distribution appears. This effect was obtained by Hirsch [4].

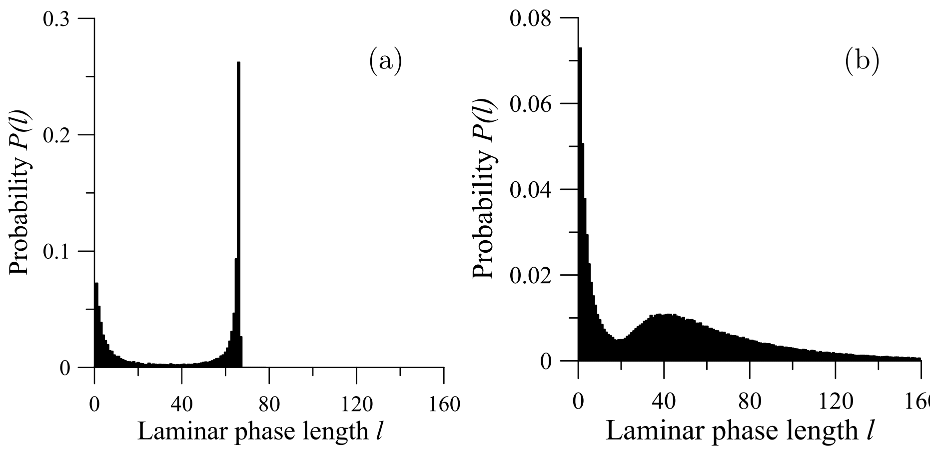

Fig. 1. The laminar phase distribution in the case of type I intermittency. (a) the noiseless case, (b) the case with additive Gaussian noise.

To measure the effect of noise on the LPLD, we introduce two quantities characterizing the plot. One of them is the right peak position (RPP) — the coordinate of right maximum of the LPLD. The second one is the length of the tail of the distribution (DTL) that is the distance between RPP and the coordinate for which the LPLD attains ten per cent of the height of its right maximum. The meaning of these two measures is shown in Fig. 2.

In the stationary case, DTL is always equal to zero, as obtained numerically and explained theoretically in [4]. The value of RPP depends on the difference of the control parameter and its critical value $a - a_{\rm c}$. For our choice of the control parameter $a = 3.8284$, in the absence of noise, RPP was 62. In the presence of additive noise, RPP decreases and DTL increases.

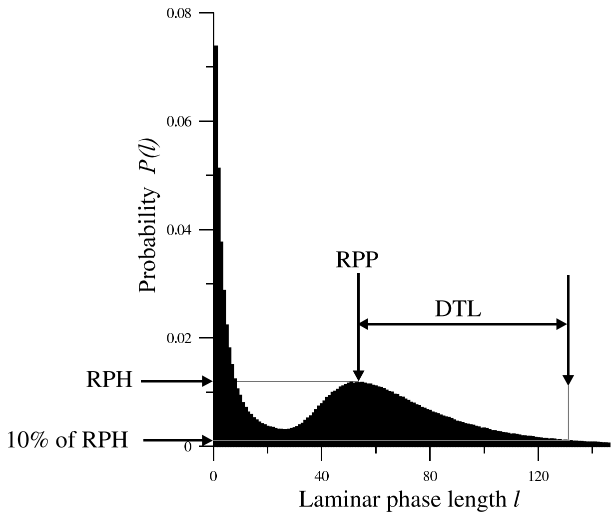

Fig. 2. Definition of the right peak position (RPP) and the distribution tail length (DTL), as described in text.

Let us now consider the third-iterate of the logistic map in the presence of two noise terms

$$x_{n+1} = f\left(x_n\right) + \sigma_1 \xi_1 - \sigma_2 x_n \xi_2 \,. \tag{10}$$

where the $f\left(x\right)$ is given by:

$$f\left(x\right) \;=\; f_1\left(f_1\left(f_1\left(x\right)\right)\right) \,, \tag{11}$$

$$f_1\left(x\right) \;=\; ax\left(1-x\right) \tag{12}$$

and the noise terms are correlated:

$$\langle \xi_1 \xi_2 \rangle = \rho \,. \tag{13}$$

RPP and DTL as functions of σ_2 (σ_1 was fixed at 0.0005) were obtained from histograms of about 10^6 laminar phases. Fig. 3 depicts these characteristics for three different values of ρ. We can easily identify the maximum of RPP and the minimum of DTL, most clearly visible for $\rho = 0.95$. The plot of DTL for the optimal value of σ_2 is shown in Fig. 4. It can be seen that it is very similar to the noiseless case depicted in Fig. 1(a).

The magnitude of the noise terms applied to the system may not be arbitrarily large as it may cause the system to leave the basin of attraction of the chaotic attractor (the interval $[0,1]$) and diverge to minus infinity. However, the phenomena studied in this paper occurred at noise magnitudes much

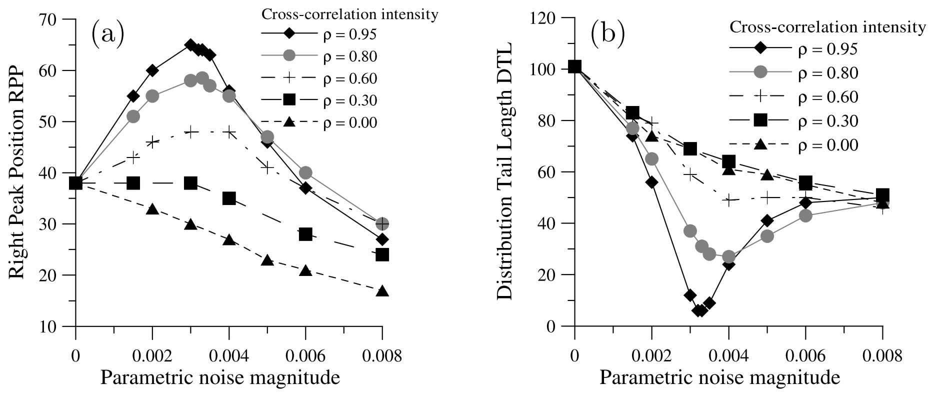

Fig. 3. RPP (a) and DTL (b) as functions of σ_2, when $\sigma_1 = 0.0005$ for five different values of the correlation coefficient. Note that the DTL curves coincide for the two lowest values of ρ.

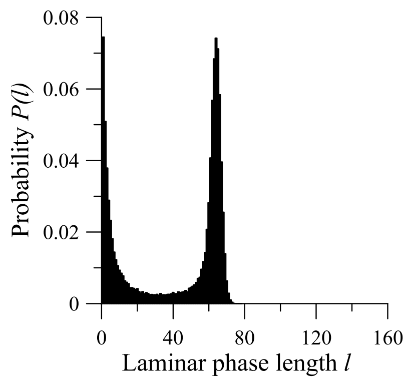

Fig. 4. The laminar phase distribution for the logistic map with two noise terms (Eq. (10)) for $\sigma_1 = 0.0005$, $\sigma_2 = 0.0033$ and $\rho = 0.95$. Note that the distribution is very similar to the noiseless case in Fig. 1(a).

less than those at which the divergence occurs. Note that the maximum of RPP and the minimum of DTL decrease in magnitude rather quickly with noise amplitude so that for larger noise amplitudes the GSA is difficult to observe.

3. Giant suppression of activation in systems with boundary crisis

3.1. General description

In a system with a boundary crisis, the attractor is destroyed as an effect of its collision with a hyperbolic saddle residing on the border of the basin of attraction. However, if the trajectory starts from a point on the attractor, it can stay on it for the duration of a shorter or a longer transient, before leaving the attractor definitely. It has been shown [7] that the residence time (RT) of the trajectory on the destroyed attractor has an exponential distribution and, for $a > a_{\rm c}$, the mean value of this time depends on the control parameter as:

$$\langle\tau\rangle = A\,(a - a_{\rm c})^{-\gamma}\ , \tag{14}$$

where $a_{\rm c}$ is the critical value of control parameter at which the crisis occurs and A and γ are positive constants, depending on the dynamics of system. For example, the logistic map undergoes the boundary crisis at $a = a_{\rm c} = 4.0$. For a larger, the attractor is no longer stable on the interval (0,1) — almost all the initial conditions diverge to minus infinity. But the transient, during which the trajectory still moves on the attractor — is clearly seen, especially for a close to $a_{\rm c}$. It has also been shown that the value of the exponent γ for the logistic map is equal to 0.5 [1, 7].

The main principle of this phenomenon is similar to that described in the previous section. If the control parameter is slightly less than $a_{\rm c}$, the pre-crisis chaotic attractor is stable. If we add some noise to the system — additive or multiplicative — the system may leave the basin of attraction. It has been shown [8] that, in this case, the distribution of RT looks very similar to the distribution for a completely deterministic boundary crisis: it is also exponential. If we now add a second noise term to the system, correlated with the first one, we can choose its amplitude so as to minimize the effective noise acting at the point of tangency of the attractor with the stable manifold responsible for the crisis. This effect increases the mean RT. Note that an arbitrarily long mean residence time is not possible. Even if the effective noise at the point of the tangency will be equal to zero — the effective noise in the neighborhood of this point will have a non-zero amplitude and the system will leave the basin of attraction through this neighborhood.

3.2. An example — the logistic map

Consider the logistic equation with noise:

$$x_{n+1} = (a + \sigma_1\xi_1)\,x_n\,(1 - x_n) - \sigma_2\xi_2 x_n^2\,(1 - x_n)^2\ , \tag{15}$$

where ξ_1 and ξ_2 — two correlated white noise terms. We assume that a is equal to 3.98, that is slightly less than $a_{\rm c} = 4.0$. This guarantees that, in the absence of the noise, the pre-crisis chaotic attractor is absolutely stable. The somewhat complicated form of the second noise term may require justification. We wanted to concentrate on the escape of the trajectory from the basin of attraction at $x = 0.5$. However, when x is close to 0 or 1 it is also possible that in the presence of noise the system may escape. To avoid the escape of the trajectory at these points, we use a noise term that converges to 0 when x is close to 0 or 1.

The tangency of the attractor and the stable manifold responsible for the crisis occurs in this case at $x_n = 1.0$, *i.e.* $x_{n-1} = 0.5$. Due to the action of the noise, at that point the probability for the system to escape from the attractor in the next iteration is the greatest. In fact, there are two other points very noise-sensitive: $x = 0$ and $x = 1$, but the noise terms in (15) have zero magnitudes at these points. That allows us to take only the point 0.5 into account. If we insert the value $x = 0.5$ into the expression for the effective noise

$$\xi_{\text{eff}} = \sigma_1 x (1-x) \xi_1 - \sigma_2 x^2 (1-x)^2 \xi_2 \tag{16}$$

we obtain for the correlation coefficient close to one (in the case of a linear correlation of two noise terms with identical distributions this means simply $\xi_1 \cong \xi_2$) the condition

$$\xi_{\text{eff}} (x = 0.5) \cong 0 \Leftrightarrow \sigma_2 = 4\sigma_1 \,. \tag{17}$$

The mean RT as a function of σ_2 (σ_1 is fixed at 0.05) is depicted in Fig. 5. RT was calculated as an arithmetic average of fifty thousand simulations. It can be seen that a maximum of this time is clearly visible, especially so for larger values of the correlation coefficient.

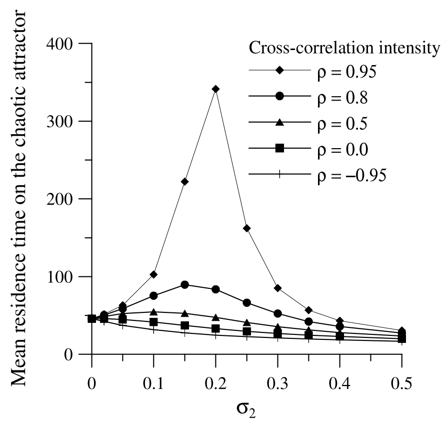

Fig. 5. The mean residence time of the trajectory on the destroyed attractor as a function of σ_2 (σ_1 is fixed at 0.05) for five different values of the correlation coefficient ρ. The maximum near $\sigma_2 = 0.2$ is clearly seen for $\rho = 0.95$ or $\rho = 0.80$.

4. Quasi-deterministic giant suppression of activation in systems with crisis

We may expect, that a similar phenomenon to that described in the previous section may be obtained for a system with an interior crisis. In fact, the two kinds of crisis are very similar chaotic transitions and are caused by a very similar collision between the chaotic attractor and an unstable periodic orbit (UPO) or its manifold. The only difference is that, in the case of interior crisis, this UPO lies in the interior of the basin of attraction and not on the boundary — so the attractor does not vanish but suddenly increases its size. And our expectations are, in fact, true — in such systems noise also decreases the critical value of the control parameter at which the crisis and crisis-induced intermittency occur [1]. We checked that two correlated noise terms acting on the system may cause the giant suppression of activation, a phenomenon in which the mean laminar phase length has a maximum as a function of the amplitude of one of the noise terms. The results of this investigation are presented on Fig. 6. The system considered was the Ikeda [14] system:

$$x_{n+1} = a + bx_n \cos\left(\kappa - \frac{\eta}{1 + x_n^2 + y_n^2}\right) - by_n \sin\left(\kappa - \frac{\eta}{1 + x_n^2 + y_n^2}\right) , \quad (18)$$

$$y_{n+1} = bx_n \sin\left(\kappa - \frac{\eta}{1 + x_n^2 + y_n^2}\right) + by_n \cos\left(\kappa - \frac{\eta}{1 + x_n^2 + y_n^2}\right) , \quad (19)$$

where a, b, κ and η are real parameters. The interior crisis in the Ikeda system [1] occurs when we set $a = 0.85$, $b = 0.9$, $\kappa = 0.4$ and the value of η, which will be our control parameter, is set to $\eta = \eta_c \cong 7.26884894\ldots$. For greater values of η the attractor suddenly increases in size.

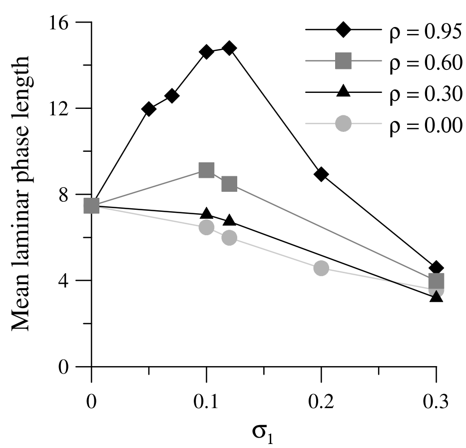

Fig. 6. The mean laminar phase length for the Ikeda system with five correlated noise terms, as a function of σ_1 for four values of ρ. The maximum indicating the GSA is clearly seen.

Let the value of the parameter η be equal to 7.25. Two noise terms were added to the first equation of this system, which then takes the form:

$$\begin{aligned} x_{n+1} \;=\; & a + bx_n \cos\left(\kappa - \frac{\eta}{1+x_n^2+y_n^2}\right) - by_n \sin\left(\kappa - \frac{\eta}{1+x_n^2+y_n^2}\right) \\ & + \sigma_1\xi_1 x - \sigma_2\xi_2 \,. \end{aligned} \tag{20}$$

We assume, that $\sigma_2 = 0.3$. In Fig. 6 it is clearly seen that if the correlation coefficient is equal to 0.95, for σ_1 equal to about 0.12 the mean laminar phase length as a function of σ_1 has a maximum.

One may expect that, because of the similarity of behavior of the stochastic and chaotic variables it may be possible to obtain a similar effect when only one noise term will be added provided the noise will correlated with one of the variables of the system. In the next part of this section we consider such a situation.

Let us introduce an additive noise term into our system. If a variable v of our system has a zero time average and its mean square value is equal to one, we may express the value of the noise in the $n+1^{st}$ iteration as:

$$\xi_{n+1} = \rho v_n + \sqrt{1-\rho^2}\zeta_n \,, \tag{21}$$

where ζ is a white noise independent of v. If none of the variables of the system have the above mentioned properties, we may renormalize one of variables of the system, say y, and define v as:

$$v = \frac{y - \langle y\rangle}{\sqrt{\left\langle (y-\langle y\rangle)^2\right\rangle}} \,. \tag{22}$$

We should emphasize, that in (22) $\langle y\rangle$ and $\left\langle (y-\langle y\rangle)^2\right\rangle$ are real and calculated as time averages along the trajectory. The variable v defined by (22) is automatically normalized, *i.e.* has a zero mean value and a unit mean square value.

In such an approach, we may treat the system as driven by two correlated noise terms: the first one is the gaussian noise ζ and the second one would be the chaotic variable v. In fact, the chaotic trajectories of some systems are so "random-like" that they are often used for random number generation [15]. However, when analyzing the effect of noise given by (21) on the system, we should take into account two new aspects which were absent when the system was driven by two stochastic noise terms.

One of these are the properties of the probability distribution of the noise. The noise given by (21) is not Gaussian because the probability distribution of v (which is just the natural measure distribution) is in general not Gaussian. The other problem, even more serious than the previous one, is that the crisis modifies the amplitude of the noise given by (21). Computing the variance of this noise and as

$$\langle \xi^2 \rangle = \rho^2 \langle v^2 \rangle + \left(1-\rho^2\right) \langle \zeta^2 \rangle = \rho^2 \langle v^2 \rangle + 1 - \rho^2 \,, \tag{23}$$

where we have taken into account that the noise ζ has a unit mean square value. More complicated is the computation of the value of $\langle v^2 \rangle$. The value of v is, of course, normalized, as can be seen from (22). That should allow us to write $\langle v^2 \rangle = 1$. However, that is true only for the pre-crisis attractor, *i.e.* when the noise is absent. When we turn on the noise, the interior crisis occurs and the variance of v is in general different from one because the trajectory visits the points of phase space which have not been visited before the crisis. To cope with this problem, we assume that the laminar phases are long and the excursions onto the expanded attractor short enough not to change the variance of v very much. Thus, we assume that the variance $\langle \xi^2 \rangle \cong 1$ but we should remember that in the case of short laminar phases this approximation is no longer true.

The pre-crisis attractor may be treated as an equivalent of the metastable state in typical mechanical systems. The slowing down of the escape from the "metastable-like" state caused by the noise coupled with the dynamics of the system can be called quasi-deterministic giant suppression of activation (QDGSA), in difference to the fully stochastic giant suppression of activation [9], where two random noise terms are needed to cause the phenomenon.

As an example, we analyze again the Ikeda system [14], which has two variables x and y with the same values of the parameters, as at the beginning of this section.

First consider $\eta < 7.26$, *e.g.* $\eta = 7.24$. We may then compute the statistical properties of *e.g.* the variable y. We obtain:

$$\langle y \rangle \cong 0.1764\ldots \,, \tag{24}$$

$$\langle y^2 \rangle \cong 0.06708\ldots \,. \tag{25}$$

The distribution of y is not Gaussian, that can be easily checked, but it is clear that not only these two moments but any moments of higher orders exist: y is bounded as the attractor of the noiseless Ikeda system has a finite size. Thus, our normalized variable v will be given by:

$$v = \frac{y - 0.1764}{0.259} \,. \tag{26}$$

And the noise, according to (21) will be given by:

$$\xi_n = \rho \frac{y_n - 0.1764}{0.259} + \sqrt{1-\rho^2}\zeta_n \,. \tag{27}$$

Similarly, one may consider the variable x instead of y.

Let us set the value of $\eta = 7.32$ which means that the attractor size has increased due to the crisis. This situation is analogous to the loss of stability of the metastable state due to the effect of noise. Now, we add to the equation for x the term $-\sigma\xi_n$, where the ξ_n is given by (27) and investigate the mean laminar phase length as a function of σ. The result (for four values of ρ) is depicted in Fig. 7; note the logarithmic scale. It can be seen that for a large correlation parameter value ($\rho = 0.95$) a well visible maximum occurs indicating GSA. This maximum decreases with decreasing correlation magnitude to disappear at about $\rho = 0.6$. What is characteristic is that the maximal value of the mean laminar phase length is now even a hundred or more times greater than in the absence of noise. In this case then we have a truly "giant" suppression of activation.

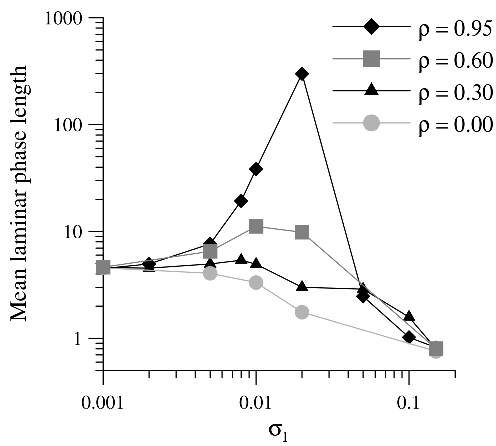

Fig. 7. The mean laminar phase length for the Ikeda system with a noise term dependent on one of the variables of the system, as a function of σ for four values of ρ in logarithmic scale. The maximum of this length, the signature of quasi-deterministic giant suppression of activation, is clearly seen for $\rho = 0.95$.

5. Conclusion

In this paper, we showed that a phenomenon analogous to giant suppression of activation, originally found in mechanical systems — are present also in several kinds of chaotic transitions. In the case of type I intermittency, we used the mechanical analogy for a system near the point of saddle-node bifurcation to a particle in a third-order potential field (this analogy was first proposed by Hirsch [4]). The similarity to the metastable state is clear. We also showed that in the case of other chaotic transitions a similar analogy can be found, *e.g.* the chaotic attractor just before the boundary crisis can be also treated as a specific kind of a metastable state although it is more complex than a fixed point. One can expect, that also several other phenomena found in metastable mechanical system are present in the chaotic systems described above.

We described also a new kind of giant suppression of activation, which is obtained by applying to a system near to a chaotic transition a single noise term correlated with one of the system variables. We called this phenomenon quasi-deterministic giant suppression of activation and showed that the phenomenon is several orders of magnitude stronger than the standard GSA. One may expect that if the distribution of applied noise term (ζ in (21)) is other than the Gaussian, the effect may even be stronger and it may be possible to obtain arbitrarily long laminar phases.

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