Vol. 39 (2008) *ACTA PHYSICA POLONICA B* No 6

REISSNER–NORDSTRÖM BLACK HOLE THERMODYNAMICS IN NONCOMMUTATIVE SPACES

Kourosh Nozari†, Behnaz Fazlpour

Department of Physics, Faculty of Basic Science, University of Mazandaran
P.O. Box 47416-1467, Babolsar, Iran

(Received February 18, 2008)

This paper considers the effects of space noncommutativity on the thermodynamics of a Reissner–Nordström black hole. In the first step, we extend the ordinary formalism of Bekenstein–Hawking to the case of charged black holes in commutative space. In the second step we investigate the effect of space noncommutativity on the thermodynamics of charged black holes. Finally we compare thermodynamics of charged black holes in commutative space with thermodynamics of Schwarzschild black hole in noncommutative space. In this comparison we explore some conceptual relation between charge and space noncommutativity.

PACS numbers: 02.40.Gh, 04.70.–s, 04.70.Dy

1. Introduction

Generally, black holes can be characterized by three (and only three) quantities: mass (M), electric charge (Q) and angular momentum ($\vec{J}$) (for a review see [1,2,3]). A charged black hole is a black hole that possesses electric charge. Since the electromagnetic repulsion in compressing an electrically charged mass is dramatically greater than the gravitational attraction (by about 40 orders of magnitude), it is not expected that black holes with a significant electric charge will be formed in nature. When the black hole is electrically charged, the Schwarzschild solution is no longer valid. In this case the Reissner–Nordström geometry describes the geometry of empty space surrounding a charged black hole.

If the charge of black hole is less than its mass (measured in geometric units $G = c = 1$), then the geometry contains two horizons, an outer horizon and an inner horizon. Between the two horizons space is much like a waterfall, falling faster than the speed of light, carrying everything with itself.

† knozari@umz.ac.ir

It should be stressed that fundamental charged particles such as electrons and quarks are not black holes: their charge is much greater than their respective masses, and they do not contain horizons.

The issue of black hole thermodynamics and its quantum gravitational correction has been studied extensively [4,–7]. Since this problem is a key attribute of quantum gravity proposal, investigation of its various aspects will shed light on the perspectives of ultimate quantum gravity scenario. It has been revealed that quantum corrections to the Bekenstein–Hawking formalism of black hole thermodynamics can be performed in several alternative approaches such as noncommutative geometry [6,8], the generalized uncertainty principle (GUP) [7,9,10] and modified dispersion relations [4,5,11,12]. The goal of the present paper is to proceed one more step in this direction. We consider the case of charged black holes. We firstly give an overview to the original formalism of Bekenstein–Hawking for charged black holes in commutative space. Then we consider the effects of space noncommutativity on the thermodynamical quantities of charged black holes. We compare Reissner–Nordström black hole thermodynamics in commutative space with thermodynamics of Schwarzschild black hole in noncommutative space. In this manner we are forced to conclude that space noncommutativity has something to do with charge. In other words, it seems that space noncommutativity and charge have the same effects on thermodynamics of a Schwarzschild black hole.

2. Charged black holes

By the Schwinger effect in the presence of a charged black hole, there are pair-creation of charged particles [13]. When we consider the quantum effects, a charged matter fluid will surround the singularity and then black hole charge will be screened. Therefore, we will have an electric field which modifies the geometry of the black hole.

Consider the Reissner–Nordström geometry, describing a static electrically charged black hole with the following line-element

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) , \tag{1}$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} . \tag{2}$$

This expression has been written in geometric units, where the speed of light and Newton's gravitational constant are set equal to unity, $c = G = 1$. This metric has two possible horizon which can be found as follows

$$r = M \pm \sqrt{M^2 - Q^2} . \tag{3}$$

These two values are corresponding to the outer and inner horizons. Therefore, when a black hole becomes charged, the event horizon shrinks, and another one appears, near the singularity. The more charged the black hole is, the closer the two horizons are. As more and more electric charge is thrown into the black hole, the inner event horizon starts to get larger, while the outer horizon starts to shrink. The maximum possible charge on the black hole is when the two horizons come together and merge. If one tried to force in more charge, both event horizons would disappear, leaving a naked singularity. Since in the limit of $Q = 0$ we should recover the Schwarzschild radius $r_{\rm s} = 2M$, we consider the plus sign in (3) and the corresponding radius will be radius of the outer horizon.
If we set $M = r_{\rm s}/2$ in equation (3), we find

$$r = \frac{r_{\rm s}}{2} \pm \sqrt{\frac{{r_{\rm s}}^2}{4} - Q^2} = \frac{1}{2}\Big(r_{\rm s} \pm \sqrt{{r_{\rm s}}^2 - 4Q^2}\Big)\,. \tag{4}$$

In what follows we consider only the outer radius which is corresponding to the radius of the Schwarzschild black hole. In this manner, equation (4) can be rewritten as follows

$$r = r_{\rm s} - \frac{Q^2}{r_{\rm s}} - \frac{Q^4}{{r_{\rm s}}^3}\,, \tag{5}$$

where we have considered only the three first terms of the right hand side since $Q < M$. When $Q = 0$, we get $r = r_{\rm s}$ which is corresponding to the event horizon radius of Schwarzschild black hole.

After a brief overview of the Reissner–Nordström black holes, we apply the original Bekenstein–Hawking formalism to this type of black holes.

3. Thermodynamics of a charged black hole

The Hawking temperature of the Schwarzschild black hole is given by

$$T = \frac{1}{8\pi M}\,. \tag{6}$$

Since $r_{\rm s} = 2M$, this relation can be written formally as $T = M/(2\pi r_{\rm s}^2)$. We generalize this relation to the case of Reissner–Nordström black hole using relation (5) for r instead of $r_{\rm s}$. We find

$$T \simeq \frac{M}{2\pi}\left(r_{\rm s} - \frac{Q^2}{r_{\rm s}} - \frac{Q^4}{{r_{\rm s}}^3}\right)^{-2}\,, \tag{7}$$

which leads to the following relation (note that $Q < M$)

$$T \simeq \frac{1}{8\pi M}\left(1 + \frac{Q^2}{2M^2} + \frac{5}{16}\frac{Q^4}{M^4}\right)\,. \tag{8}$$

Now we calculate entropy of the charged black hole. In the standard Bekenstein argument, the relation between energy and position uncertainty of a particle in the vicinity of black hole event horizon is given by $E \geq 1/\delta x$ [4]. We suppose $\delta x \sim r$ and, therefore, we find the following generalization

$$E \geq \frac{1}{\left(r_{\rm s} - \frac{Q^2}{r_{\rm s}} - \frac{Q^4}{{r_{\rm s}}^3}\right)}\,. \tag{9}$$

This relation implicitly shows the necessary modification of the standard dispersion relations. These modified dispersion relations have appeared in scenarios such as loop quantum gravity where Lorentz invariance violation has been encountered [4,14]. Now let us consider a quantum particle that starts out in the vicinity of an event horizon and then ultimately absorbed by black hole. For a black hole absorbing such a particle the minimal increase in the horizon area can be expressed as $(\Delta A)_{\rm min} \geq 4(\ln 2)E\delta x$ [4]. In this situation, the increase of the event horizon area can be given as follows

$$\Delta A \geq 4(\ln 2)\,\frac{1}{\left(1 - \frac{Q^2}{{r_{\rm s}}^2} - \frac{Q^4}{{r_{\rm s}}^4}\right)}\,, \tag{10}$$

where $\ln 2$ is the calibration factor. This statement leads us to the following relation

$$\frac{dS}{dA} \approx \frac{\Delta S_{\rm min}}{\Delta A_{\rm min}} \simeq \frac{\ln 2}{4(\ln 2)\,\frac{1}{\left(1-\frac{Q^2}{{r_{\rm s}}^2}-\frac{Q^4}{{r_{\rm s}}^4}\right)}}\,. \tag{11}$$

Therefore, we can write

$$\frac{dS}{dA} \simeq \frac{1}{4}\left[1 - \frac{Q^2}{{r_{\rm s}}^2} - \frac{Q^4}{{r_{\rm s}}^4}\right]. \tag{12}$$

Now we should calculate dA. Since $A = 4\pi r^2$, we find

$$A = A_{\rm s} - 8\pi Q^2 - \frac{(4\pi)^2 Q^4}{A_{\rm s}} + 2\frac{(4\pi)^3 Q^6}{{A_{\rm s}}^2}\,, \tag{13}$$

and therefore

$$dA = \left[1 + \left(\frac{4\pi}{A_{\rm s}}\right)^2 Q^4 - 4\left(\frac{4\pi}{A_{\rm s}}\right)^3 Q^6\right] dA_{\rm s}\,, \tag{14}$$

where $A_{\rm s} = 4\pi r_{\rm s}^2$. Integration of (12) leads to the following result

$$S \simeq \frac{A_{\rm s}}{4} - \pi Q^2 \ln\frac{A_{\rm s}}{4} + \frac{1}{3}\left(\pi Q^2\right)^2\left(\frac{4}{A_{\rm s}}\right) + \frac{5}{2}\left(\pi Q^2\right)^3\left(\frac{4}{A_{\rm s}}\right)^2 + \mathcal{O}\left(\left(\frac{4}{A_{\rm s}}\right)^3\right)\ldots\,, \tag{15}$$

If we calculate corrections of all orders, we will arrive at the following compact and generalized form for entropy of Reissner–Nordström black holes in commutative spaces

$$S = \frac{A_{\rm s}}{4} - \pi Q^2 \ln \frac{A_{\rm s}}{4} + \sum_{n=1}^{\infty} c_n \left(\frac{4}{A_{\rm s}} \right)^n + \mathcal{C} \,, \tag{16}$$

where $\mathcal{C}$ is a constant and expressions c_n are quantum gravity model dependent coefficients. A similar expression for entropy can be obtained in other alternative approaches such as string theory and loop quantum gravity. In the case of $Q = 0$ this equation yields the standard Bekenstein entropy, $S = A_{\rm s}/4$.

This is a generalization of Bekenstein–Hawking formalism to the case of charged black holes in commutative space. In what follows, we consider the effects of space noncommutativity on the Bekenstein–Hawking formalism of charged black holes.

4. The effect of space noncommutativity

A noncommutative space can be realized by the coordinate operators satisfying [15,16,17]

$$[\hat{x}_i, \hat{x}_j] = i\theta_{ij} \,, \qquad i, j = 1, 2, 3 \,, \tag{17}$$

where $\hat{x}_i$'s are the coordinate operators and θ_{ij} is a real, antisymmetric and constant tensor, which determines the fundamental cell discretization of space much in the same way as the Planck constant $\hbar$ discretizes the phase space. It has the dimension of $(length)^2$. Canonical commutation relations in noncommutative spaces read (with $\hbar = 1$)

$$[\hat{x}_i, \hat{x}_j] = i\theta_{ij} \,, \qquad [\hat{x}_i, \hat{p}_j] = i\delta_{ij} \,, \qquad [\hat{p}_i, \hat{p}_j] = 0 \,. \tag{18}$$

There is a new coordinate system with the following definitions

$$x_i = \hat{x}_i + \tfrac{1}{2}\, \theta_{ij} \hat{p}_j \,, \qquad p_i = \hat{p}_i \,. \tag{19}$$

With these new variables, the values of x_i and p_i satisfy the usual (commutative) commutation algebra

$$[x_i, x_j] = 0 \,, \qquad [x_i, p_j] = i\delta_{ij} \,, \qquad [p_i, p_j] = 0 \,. \tag{20}$$

In what follows we develop the arguments of the preceding section to the case where space noncommutativity is present. For a noncommutative charged black hole, we have

$$f(r) = 1 - \frac{2M}{\sqrt{\hat{r}\hat{r}}} + \frac{Q^2}{\hat{r}^2} \,, \tag{21}$$

where $\hat{r}$ satisfies conditions (19). The horizon of the noncommutative metric as usual satisfies the condition $\hat{g}_{00} = 0$, which leads to[18,19]

$$1 - \frac{2M}{\sqrt{\hat{r}\hat{r}}} + \frac{Q^2}{\hat{r}^2} = 0 \,. \tag{22}$$

By a coordinate transformation from $\hat{x}_i$ to x_i and then using relation (19), one can show that horizon of the noncommutative charged black hole satisfies the following approximate conditionuation

$$1 - \frac{2M}{\sqrt{\left(x_i - \frac{\theta_{ij}p_j}{2}\right)\left(x_i - \frac{\theta_{ik}p_k}{2}\right)}} + \frac{Q^2}{\left(x_i - \frac{\theta_{ij}p_j}{2}\right)\left(x_i - \frac{\theta_{ik}p_k}{2}\right)} = 0 \,. \tag{23}$$

This leads us to the following relation

$$\begin{aligned} &1 - \frac{2M}{r}\left(1 + \frac{x_i\theta_{ij}p_j}{2r^2} - \frac{\theta_{ij}\theta_{ik}p_jp_k}{8r^2} + \frac{3}{8}\frac{(x_i\theta_{ij}p_j)^2}{r^4}\right) \\ &+ \frac{Q^2}{r^2}\left(1 + \frac{x_i\theta_{ij}p_j}{r^2} - \frac{\theta_{ij}\theta_{ik}p_jp_k}{4r^2} + \frac{(x_i\theta_{ij}p_j)^2}{r^4}\right) + \mathcal{O}(\theta^3) + \ldots = 0 \,, \end{aligned} \tag{24}$$

where $\theta_{ij} = \frac{1}{2}\epsilon_{ijk}\theta_k$. Using the identity $\epsilon_{ijr}\epsilon_{iks} = \delta_{jk}\delta_{rs} - \delta_{js}\delta_{rk}$, one can rewrite (24) as follows

$$\begin{aligned} &1 - \frac{2M}{r}\left[1 + \frac{\vec{L}.\vec{\theta}}{4r^2} - \frac{\left(p^2\theta^2 - (\vec{p}.\vec{\theta})^2\right)}{32r^2} + \frac{3(\vec{L}.\vec{\theta})^2}{32r^4}\right] \\ &+ \frac{Q^2}{r^2}\left[1 + \frac{\vec{L}.\vec{\theta}}{2r^2} - \frac{\left(p^2\theta^2 - (\vec{p}.\vec{\theta})^2\right)}{16r^2} + \frac{(\vec{L}.\vec{\theta})^2}{4r^4}\right] + \mathcal{O}(\theta^3) + \ldots = 0 \,, \end{aligned} \tag{25}$$

where $L_k = \epsilon_{ijk}x_ip_j$, $p^2 = \vec{p}.\vec{p}$ and $\theta^2 = \vec{\theta}.\vec{\theta}$. If we set $\theta_3 = \theta$ and assuming that all remaining components of θ vanish (which can be done by a rotation or a re-definition of the coordinates), then $\vec{L}.\vec{\theta} = L_z\theta$ and $\vec{p}.\vec{\theta} = p_z\theta$. Since Reissner–Nordström black hole is non-rotating, we set $\vec{L} = 0$ and, therefore, $L_z = 0$. In this situation equation (25) can be written as

$$r^4 - 2Mr^3 + \frac{M\left(p^2 - p_z^2\right)\theta^2}{16}r + Q^2r^2 - \frac{Q^2\left(p^2 - p_z^2\right)\theta^2}{16} + \mathcal{O}(\theta^3) + \ldots = 0 \,. \tag{26}$$

From this equation one can see that space noncommutativity has no effect on Reissner–Nordström space-time in the first order approximation. Since

$p^2 = p_x^2 + p_y^2 + p_z^2$, one can write $(p^2 - p_z^2)\theta^2 = (p_x^2 + p_y^2)\theta^2$ and, therefore, (26) can be rewritten as follows

$$r^4 - 2Mr^3 + Q^2 r^2 + \frac{M\left(p_x^2 + p_y^2\right)\theta^2}{16} r - \frac{Q^2\left(p_x^2 + p_y^2\right)\theta^2}{16} = 0\,. \tag{27}$$

With the following definitions

$$\begin{aligned} a &\equiv -2M = -r_{\rm s}\,, \qquad\qquad b \equiv Q^2\,, \\ c &\equiv \frac{M\left(p_x^2 + p_y^2\right)\theta^2}{16}\,, \qquad d \equiv -\frac{Q^2\left(p_x^2 + p_y^2\right)\theta^2}{16}\,. \end{aligned} \tag{28}$$

We can write $c = M\alpha = (r_{\rm s}/2)\alpha$ and $d = -Q^2\alpha$, where

$$\alpha = \frac{\left(p_x^2 + p_y^2\right)\theta^2}{16}\,. \tag{29}$$

Note that α is so defined that contains the effects of space noncommutativity. Equation (27) has four roots but when $Q = 0$ and space-time is commutative we should recover $\hat{r}_{\rm s} = r_{\rm s}$. Therefore, only one root is acceptable which is given by

$$\hat{r}_{\rm s} = -a + \frac{b}{a} - \frac{c}{a^2} + \frac{2b^2}{3a^3} - \frac{b}{a^3}(A+B) - \frac{4cb}{3a^4} + \frac{2c}{a^4}(A+B)\,, \tag{30}$$

where

$$\begin{aligned} A = \frac{2^{\frac{1}{3}}}{3}(b^2 - 3ac + 12d)\Big(& 2b^3 - 9abc + 27c^2 + 27a^2 d - 72bd \\ &+ \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2 d - 72bd)^2}\Big)^{-\frac{1}{3}} \end{aligned} \tag{31}$$

and

$$\begin{aligned} B = \frac{1}{3 \times 2^{\frac{1}{3}}}\Big(& 2b^3 - 9abc + 27c^2 + 27a^2 d - 72bd \\ &+ \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2 d - 72bd)^2}\Big)^{\frac{1}{3}}\,. \end{aligned} \tag{32}$$

Substitution of the values of a, b, c and d from (28) leads to the following expression for radius of event horizon in noncommutative space

$$\hat{r}_{\rm s} = r_{\rm s} - \frac{Q^2}{r_{\rm s}} - \frac{\alpha}{2r_{\rm s}} - \frac{2Q^4}{3r_{\rm s}^3} + \frac{Q^2}{r_{\rm s}^3}(A+B) - \frac{2\alpha Q^2}{3r_{\rm s}^3} + \frac{\alpha}{r_{\rm s}^3}(A+B)\,, \tag{33}$$

where now A and B have the following explicit forms

$$A=\frac{2^{\frac{1}{3}}}{3}\left(2Q^4-24\alpha Q^2+3\alpha r_{\rm s}^2\right)\Bigg(16Q^6-180\alpha Q^2r_{\rm s}^2+54\alpha^2r_{\rm s}^2+576\alpha Q^4$$
$$-\sqrt{-4(4Q^4-48\alpha Q^2+6\alpha r_{\rm s}^2)^3+(-16Q^6+180\alpha Q^2r_{\rm s}^2-54\alpha^2r_{\rm s}^2-576\alpha Q^4)^2}\Bigg)^{-\frac{1}{3}} \quad (34)$$

and

$$B=\frac{1}{6\times 2^{\frac{1}{3}}}\Bigg(16Q^6-180\alpha Q^2r_{\rm s}^2+54\alpha^2r_{\rm s}^2+576\alpha Q^4$$
$$-\sqrt{-4(4Q^4-48\alpha Q^2+6\alpha r_{\rm s}^2)^3+(-16Q^6+180\alpha Q^2r_{\rm s}^2-54\alpha^2r_{\rm s}^2-576\alpha Q^4)^2}\Bigg)^{\frac{1}{3}} \quad (35)$$

If we simplify expressions of A and B, we obtain for $A+B$

$$A+B=\sum_{n,m,i}\eta\frac{Q^n\alpha^m}{r_{\rm s}^i}\,,\qquad n,m=0,1,2,\ldots\quad\text{and}\qquad i=0,2,4,\ldots\,, \quad (36)$$

where η is a numerical coefficient. Note that only even powers of $1/r_{\rm s}$ appear in this expansion and correspondingly, only odd powers of $1/r_{\rm s}$ will appear in (33).

To obtain charged black hole thermodynamics in noncommutative space we proceed in the line of previous section. For simplicity of calculations, we consider only three first terms of expansions. For black hole event horizon we have

$$\hat{r}_{\rm s}=r_{\rm s}-\frac{(2Q^2+\alpha)}{2r_{\rm s}}-\frac{2(Q^2+\alpha)Q^2}{3r_{\rm s}^3}\,. \quad (37)$$

Using equations (6) and (37), we obtain the following generalized statement for temperature of charged black holes in noncommutative spaces

$$T=\frac{M}{2\pi}\left(r_{\rm s}-\frac{(2Q^2+\alpha)}{2r_{\rm s}}-\frac{2(Q^2+\alpha)Q^2}{3r_{\rm s}^3}\right)^{-2}, \quad (38)$$

which leads to the following relation

$$T=\frac{1}{8\pi M}\left(1+\frac{(2Q^2+\alpha)}{4M^2}+\frac{\frac{13}{3}(Q^2+\alpha)Q^2+\frac{3}{4}\alpha^2}{16M^4}\right). \quad (39)$$

Now we calculate entropy of charged black hole in noncommutative space. As previous section with $\delta x=\hat{r}_{\rm s}$, we obtain the following generalization

$$E\geq\frac{1}{\hat{r}_{\rm s}}\,, \quad (40)$$

which after substitution of $\hat{r}_{\rm s}$ from (37) leads to the relation

$$E \geq \frac{1}{\left(r_{\rm s} - \frac{(2Q^2+\alpha)}{2r_{\rm s}} - \frac{2(Q^2+\alpha)Q^2}{3r_{\rm s}^3}\right)}\,. \tag{41}$$

Again, this relation implicitly shows the modification of standard dispersion relations in noncommutative spaces. In this manner, the increase of event horizon area in noncommutative space is given by

$$\Delta\hat{A}_{\rm s} \geq 4(\ln 2)\frac{1}{\left(1 - \frac{(2Q^2+\alpha)}{2r_{\rm s}^2} - \frac{2(Q^2+\alpha)Q^2}{3r_{\rm s}^4}\right)}\,. \tag{42}$$

which leads to the following relation

$$\frac{dS}{d\hat{A}_{\rm s}} \approx \frac{\Delta S_{(\min)}}{\Delta\hat{A}_{{\rm s}(\min)}} \simeq \frac{\ln 2}{4(\ln 2)\frac{1}{\left(1-\frac{(2Q^2+\alpha)}{2r_{\rm s}^2}-\frac{2(Q^2+\alpha)Q^2}{3r_{\rm s}^4}\right)}}\,. \tag{43}$$

Therefore, we can write

$$\frac{dS}{d\hat{A}_{\rm s}} \simeq \frac{1}{4}\left[1 - \frac{(2Q^2+\alpha)}{2r_{\rm s}^2} - \frac{2(Q^2+\alpha)Q^2}{3r_{\rm s}^4}\right]. \tag{44}$$

Now we should calculate dA. Since $\hat{A}_{\rm s} = 4\pi\hat{r}_{\rm s}^2$, we find

$$\begin{aligned}\hat{A}_{\rm s} \;=\; & A_{\rm s} - 4\pi(2Q^2+\alpha) + (4\pi)^2\frac{\left(-\frac{1}{3}(Q^2+\alpha)Q^2 + \frac{1}{4}\alpha^2\right)}{A_{\rm s}} \\ & + \frac{2}{3}(4\pi)^3\frac{Q^2\left(2Q^4+3\alpha Q^2+\alpha^2\right)}{A_{\rm s}^2}\end{aligned} \tag{45}$$

and therefore,

$$\begin{aligned}d\hat{A}_{\rm s} \;=\; & \Bigg[1 - \left(\frac{4\pi}{A_{\rm s}}\right)^2\left(-\frac{1}{3}(Q^2+\alpha)Q^2 + \frac{1}{4}\alpha^2\right) \\ & \quad - \frac{4}{3}\left(\frac{4\pi}{A_{\rm s}}\right)^3 Q^2\left(2Q^4+3\alpha Q^2+\alpha^2\right)\Bigg]dA_{\rm s}\,,\end{aligned} \tag{46}$$

where $A_{\rm s} = 4\pi r_{\rm s}^2$. Integration of (44) leads to the following relation for entropy of charged black holes in noncommutative spaces

$$\begin{aligned}S \simeq\; & \frac{A_{\rm s}}{4} - \pi\frac{(2Q^2+\alpha)}{2}\ln\frac{A_{\rm s}}{4} + \pi^2\left(\frac{1}{3}(Q^2+\alpha)Q^2 + \frac{1}{4}\alpha^2\right)\left(\frac{4}{A_{\rm s}}\right) \\ & + \frac{1}{4}\pi^3\left(6Q^6 + 9\alpha Q^4 + \frac{5}{2}\alpha^2 Q^2 - \frac{1}{4}\alpha^3\right)\left(\frac{4}{A_{\rm s}}\right)^2 + \mathcal{O}\left(\left(\frac{4}{A_{\rm s}}\right)^3\right) \ldots\,.\end{aligned} \tag{47}$$

Generally, this relation can be written as the following compact form

$$S \simeq \frac{A_{\rm s}}{4} - \pi\frac{(2Q^2+\alpha)}{2}\ln\frac{A_{\rm s}}{4} + \sum_{n=1}^{\infty} c_n\left(\frac{4}{A_{\rm s}}\right)^n + \mathcal{C}\,, \tag{48}$$

where $\mathcal{C}$ is a constant of integration. Such an event area dependence of entropy have been obtained in other alternative approaches such as string theory and loop quantum gravity (see for example [4,20]). Note that the logarithmic pre-factor is a model dependent quantity [21–24]. In the case where $Q = 0$ and $\alpha = 0$, this expression yields the standard Bekenstein entropy

$$S \simeq \frac{A_{\rm s}}{4}\,. \tag{49}$$

5. The relation between charge and space noncommutativity

As we have shown, thermodynamics of charged black holes in commutative space can be described with the following equations

$$T = \frac{1}{8\pi M}\left(1 + \frac{Q^2}{2M^2} + \frac{5}{16}\frac{Q^4}{M^4}\right) \tag{50}$$

and

$$S = \frac{A_{\rm s}}{4} - \pi Q^2 \ln\frac{A_{\rm s}}{4} + \sum_{n=1}^{\infty} c_n\left(\frac{4}{A_{\rm s}}\right)^n + \mathcal{C}\,. \tag{51}$$

On the other hand, based on a simple analysis much similar to approach presented in Section 4, one can show that temperature and entropy of a noncommutative space Schwarzschild black hole are given as follows [8]

$$T \approx \frac{1}{8\pi M}\left[1 + \frac{\alpha}{4M^2} - \frac{3\alpha^2}{8M^4}\right] \tag{52}$$

and

$$S = \frac{A_{\rm s}}{4} - \frac{\pi\alpha}{2}\ln\frac{A_{\rm s}}{4} + \sum_{n=1}^{\infty} c_n\left(\frac{4}{A_{\rm s}}\right)^n + \mathcal{C}\,. \tag{53}$$

Irrespective of numerical factors which are model dependent, comparison between equations (50) and (52) suggests that there is a similarity between the notion of space noncommutativity and the charge. The same result can be obtained in comparison of (51) and (53). Therefore, if we accept the universality of black holes thermodynamics, we can conclude that space noncommutativity has something to do with charge. In other words, at least in the spirit of black hole thermodynamics, charge and space noncommutativity have the same effects.

6. Summary and conclusion

In this paper we have developed formalism of Bekenstein–Hawking to the case of charged black holes without rotation. A general statement for entropy of charged black hole has been presented for this situation. Then we have investigated the effects of space noncommutativity on the thermodynamics of charged black holes. To formulate our proposal, we have considered the effect of space noncommutativity on the radius of event horizons (there is another point of view which considers the effect of space noncommutativity on the energy-momentum tensor on the right hand side of Einstein's equations [6]). In this manner, we have calculated approximate statements for temperature and entropy of charged black holes in noncommutative spaces. Our equations show a general mass or event horizon area dependence much similar to statements which have been obtained in other alternative approaches [4,5,20,25]. If we accept that quantum gravitational corrections of Bekenstein–Hawking formalism have enough generality (as it seems to be the case since several alternative approaches give the same mass or event horizon area dependence for temperature and entropy of black holes), then by comparing equations of charged black holes in commutative spaces with corresponding equations of Schwarzschild black holes in noncommutative spaces, we can conclude that charge and space noncommutativity have close relation. In other words at least their effects on black hole thermodynamics are the same. We are forced to conclude that charge can be considered as a source of space noncommutativity. This issue can be explained as follows: space noncommutativity comes back to the quantum nature of space-time at very short distances (string scale) where fractal nature of space-time leads to a minimal observable length scale and therefore the notion of space-time fuzziness. On the other hand, when charge is present, quantum mechanical properties will arise. So, since the origin of space noncommutativity goes back to quantum properties of space-time and these quantum properties can be attributed to charge, one can relate the notions of space noncommutativity and the charge.

REFERENCES

[1] A.V. Frolov *et al, Phys. Rev.* **D72**, 021501 (2005).
[2] P.K. Townsend, `gr-qc/9707012`.
[3] T. Padmanabhan, *Phys. Rep.* **406**, 49 (2005).
[4] G. Amelino-Camelia *et al.*, *Class. Quantum Grav.* **23**, 2585 (2006).
[5] K. Nozari, A.S. Sefiedgar, *Phys. Lett.* **B635**, 156 (2006).
[6] P. Nicolini *et al., Phys. Lett.* **B632**, 547 (2006).

[7] R.J. Adler *et al., Gen. Relativ. Gravitation* **33**, 2101 (2001).

[8] K. Nozari, B. Fazlpour, *Mod. Phys. Lett.* **A22**, 2917 (2007) [hep-th/0605109].

[9] K. Nozari, S.H. Mehdipour, *Mod. Phys. Lett.* **A20**, 2937 (2005).

[10] K. Nozari, S.H. Mehdipour, *Int. J. Mod. Phys.* **A21**, 4979 (2006) [gr-qc/0511110].

[11] G. Amelino-Camelia *et al., Int. J. Mod. Phys.* **D13**, 2337 (2004).

[12] G. Amelino-Camelia *et al., Phys. Rev.* **D70**, 107501 (2004).

[13] S. Iso, H. Umetsu, F. Wilczek, *Phys. Rev. Lett.* **96**, 151302 (2006).

[14] G. Amelino-Camelia *et al.*, gr-qc/0501053.

[15] R.J. Szabo, *Phys. Rep.* **378**, 207 (2003).

[16] A. Micu, M.M. Sheikh-Jabbari, *J. High Energy Phys.* **0101**, 025 (2001).

[17] N. Seiberg, E. Witten, *J. High Energy Phys.* **9909**, 032 (1999).

[18] F. Nasseri, *Gen. Relativ. Gravitation* **37**, 2223 (2005).

[19] M.R. Setare, *Int. J. Mod. Phys.* **A21**, 3007 (2006).

[20] A.J.M. Medved, E.C. Vagenas, *Phys. Rev.* **D70**, 124021 (2004).

[21] S. Hod, *Class. Quantum Grav.* **21**, L97 (2004).

[22] A.J.M. Medved, *Class. Quantum Grav.* **22**, 133 (2005).

[23] A.J.M. Medved, *Class. Quantum Grav.* **22**, 5195 (2005).

[24] K. Nozari, A.S. Sefiedgar, *Gen. Relativ. Gravitation* **39**, 501 (2007) [gr-qc/0606046].

[25] M.R. Setare, *Eur. Phys. J.* **C33**, 555 (2004) and references therein.