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DISSIPATION IN THE VERY EARLY STAGE OF THE HYDRODYNAMIC EVOLUTION IN RELATIVISTIC HEAVY ION COLLISIONS

Piotr Bożek

Institute of Physics, Rzeszów University
Al. Rejtana 16 A, 35-959 Rzeszów, Poland
and
The H. Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences
Radzikowskiego 152, 31-342 Kraków, Poland
Piotr.Bozek@ifj.edu.pl

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We propose a modification of the hydrodynamic model of the dynamics in ultrarelativistic nuclear collisions. A modification of the energy-momentum tensor at the initial stage describes the lack of isotropization of the pressure. Subsequently, the pressure is relaxing towards the equilibrium isotropic form in the local comoving frame. Within the Bjorken scaling solution a bound is found on the decay time of the initial anisotropy of the energy-momentum tensor. For the strongest dissipative effect allowed, we find a relative entropy increase of about 30%, a significant hardening of the transverse momentum spectra, and no effect on the HBT radii.

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1. Introduction

Relativistic hydrodynamics is a common framework for the modeling of the expansion and the freeze-out of the hot and dense region formed in ultrarelativistic nuclear collisions [1]. Depending on the initial conditions and on the freeze-out temperature a substantial amount of collective flow can be built up during the hydrodynamic evolution. The physical picture of the freeze-out combines a thermal emission of particles from the local fluid element with the collective flow due to the movement of the fluid element. Choosing azimuthally asymmetric initial conditions for collisions at finite impact parameter, the hydrodynamic evolution generates elliptic asymmetry in the momentum distributions similar to the experimentally observed one. Perfect fluid dynamics (without shear viscosity) generates a strong elliptic flow, and a relatively moderate radial flow. The evolution has to be followed for a long time in order to reproduce the transverse momentum spectra and the elliptic flow for different particle species [1]. It means that the freeze-out temperature is low. Stronger asymmetry of the shape of the initial interaction zone is predicted by the Color-Glass-Condensate model [2] or is due to event-by-event eccentricity fluctuations [3]; naturally it leads to stronger elliptic flow. The strong elliptic flow is reduced if shear viscosity is important for the liquid formed in ultrarelativistic collisions at RHIC energies [4]. Shear viscosity reduces the work of the fluid in the longitudinal direction, which means a slower cooling rate, and consequently a larger pressure to drive the transverse radial flow. For the Hanbury–Brown–Twiss (HBT) radii the presence of viscous terms at the freeze-out could be important, but does not explain the observed HBT radii [5].

The actual value and the range of applicability of the ideal fluid hydrodynamics in the development of the dense medium in the collision is uncertain. Recent calculations indicate that the shear viscosity coefficient should be small [6], of the order of the conjectured lower bound [7]. The build up of the transverse collective flow is greatly facilitated by the shear viscosity but the effect is reduced by viscous corrections at the freeze-out; calculations indicate that a required amount of transverse flow appears.

Even if we assume that the bulk evolution of the hot and dense matter created in heavy ion collisions is well described by the ideal fluid hydrodynamics, dissipative effects may appear in the very early stage of the collision. The physical motivation for this picture is the fact that in the initial stage, for evolution times before 1 fm$/c$, one cannot expect a full local thermalization of the medium. This effect can be phenomenologically taken into account in the hydrodynamic evolution as a modification of the initial condition for the energy-momentum tensor. This idea is at the origin of an extreme scenario in the collision dynamics, where the expansion is two-dimensional, only in the transverse direction [8–10]. Assuming that the anisotropy of local momentum distribution does not equilibrate during the evolution of the fireball, a strong effect on the transverse expansion is seen. The transverse flow builds up faster [8, 9].

In the present work, we study a more realistic scenario where the anisotropy in the initial conditions dissipates with time and eventually the limit of the ideal fluid with an isotropic pressure is recovered. The dynamics of the equilibration of the local pressure is based on the second order dissipative relativistic hydrodynamics with shear viscosity [11]. In this paper we study of the effects of the initial anisotropy only, therefore we assume that the shear viscosity coefficient is zero and the energy-momentum tensor relaxes to the ideal fluid one. A new phenomenological parameter is introduced, the relaxation time of the pressure anisotropy. We study the modified hydrodynamic evolution with initial anisotropy for two different geometries of the flow: the Bjorken scaling solution with longitudinal expansion only, and the boost-invariant flow in the longitudinal direction with an azimuthally symmetric expansion in the transverse directions. The dissipation in the early stage of the evolution leads to an increase of the entropy of the system. This effect must be compensated by a suitable retuning of the initial conditions. As a result, a slight hardening of the transverse momentum spectra of emitted particles is found and almost no effect on HBT radii in central collisions is visible.

2. Early dissipation

Nonzero shear viscosity is believed to be the most important modification of ideal fluid hydrodynamics in ultrarelativistic collisions [4,12]. The energy-momentum tensor is modified by the shear tensor $\pi^{\mu\nu}$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - g^{\mu\nu} p + \pi^{\mu\nu} \,, \tag{1}$$

u^μ is the velocity of the fluid element, the energy density ϵ and the pressure p are related by the equation of state. Besides the equation of state and the hydrodynamic equations

$$\partial_\mu T^{\mu\nu} = 0 \tag{2}$$

one has a relaxation equation for the shear tensor [11]

$$\begin{aligned} \tau_\pi \Delta^\mu_\alpha \Delta^\nu_\beta u^\gamma \partial_\gamma \pi^{\alpha\beta} &= \eta \left\langle \nabla^\mu u^\nu \right\rangle - \pi^{\mu\nu} \\ &\quad -\frac{1}{2}\eta T \pi^{\mu\nu} \partial_\alpha \left(\frac{\tau_\pi u^\alpha}{\eta T} \right) + \tau_\pi \pi^{\alpha(\mu} \omega^{\nu)}_{\alpha} \,, \end{aligned} \tag{3}$$

$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$. The last term contains the vorticity of the fluid $\omega^{\mu\nu} = \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta - \partial_\beta u_\alpha\right)$, which is zero for the flows considered here. T is the local temperature and

$$\left\langle \nabla^\mu u^\nu \right\rangle = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu} \nabla_\alpha u^\alpha \,, \tag{4}$$

η is the shear viscosity coefficient, and τ_π is the relaxation time of the shear tensor. The term $\eta \left\langle \nabla^\mu u^\nu \right\rangle$ in Eq. (3) is the Navier–Stokes (first-order) viscous correction to the energy-momentum tensor. The viscosity and the relaxation time are related to the rates of equilibration processes in the plasma. The viscosity coefficient can be estimated to be $\eta \simeq 1.04s$ for a Boltzmann massless gas [13], $\eta \simeq 0.7$–$1.1s$ for ($N_{\rm f} = 0$) QCD [14], and is expected [7] to fulfill the bound $\eta \geq \frac{1}{4\pi}s$, where s is the entropy density. Estimates for the ratio of τ_π and η range from $\tau_\pi/\eta = \frac{6}{Ts}$ to $\tau_\pi/\eta \simeq \frac{0.2}{Ts}$.

In the early stage of the collision the flow is dominated by the flow in the longitudinal (z) direction. We assume a Bjorken flow with the four-velocity of the form $u^\mu = (t/\tau, 0, 0, z/\tau)$, $\tau = \sqrt{t^2 - z^2}$. For the Bjorken boost invariant scaling solution of hydrodynamic equations the stress tensor is

$$\pi^{\mu\nu} = \begin{pmatrix} -\sinh^2 y & 0 & 0 & -\sinh y \cosh y \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ -\sinh y \cosh y & 0 & 0 & -\cosh^2 y \end{pmatrix} \mathit{\Pi}\,, \tag{5}$$

where $\mathit{\Pi}$ is the solution of a dynamical equation [12]

$$\tau_\pi \frac{d\mathit{\Pi}(\tau)}{d\tau} = \frac{4}{3}\frac{\eta}{\tau} - \mathit{\Pi}(\tau) - \frac{\mathit{\Pi}(\tau)}{2}\left(\frac{\tau_\pi}{\tau} + \frac{T\eta}{\tau_\pi}\frac{d}{d\tau}\left(\frac{\tau_\pi}{T\eta}\right)\right) \tag{6}$$

and y is the rapidity of the fluid element. In the first-order dissipative hydrodynamics we have the steady-flow result

$$\mathit{\Pi}(\tau) = \frac{4\eta}{3\tau} \tag{7}$$

for the shear viscosity corrections to the energy-momentum tensor [4]. Solving the dynamical equation (6) requires the knowledge of the value of the viscous strength $\mathit{\Pi}(\tau_0)$ at the initial time. The role of the initial condition in the further evolution until the freeze-out depends on the relaxation time τ_π. For a typical choice of parameters τ_π and η, the initial value of $\mathit{\Pi}(\tau)$ relaxes fast and is not determinant for the further evolution [6, 15, 16]. The initial value for viscous corrections should fulfill the conditions of the applicability of hydrodynamics with viscosity $\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}} \ll p$. In practice, for the Bjorken flow, the condition $\frac{\mathit{\Pi}(\tau_0)}{p+\epsilon} \leq 1$ or $\mathit{\Pi}(\tau_0) \leq p$ is used. Another natural restriction is the condition that the relative viscous correction $\frac{\mathit{\Pi}}{p+\epsilon}$ decreases with time [16].

In this paper we study a different type of local deviation from equilibrium in the hydrodynamic evolution. The initial local momentum distributions in the transverse and longitudinal directions could be different, there is no reason to expect instantaneous isotropization of the momentum distributions. One could assume that this asymmetry remains until the freeze-out [8, 9]. Such a hydrodynamic evolution in the transverse direction only gives stronger transverse flow, faster expansion, small HBT radii and a realistic elliptic flow [8–10]. A fast build up of the transverse flow is compatible with experimental indications of a rapid break-up and hadronization of the fireball. The assumption that the local momentum distribution is of the form [8, 9]

$$f(p) \propto \delta(y-\eta) f_{\mathrm{thermal}}(p_{\mathrm{T}}), \tag{8}$$

where y and η are the rapidity and the space-time rapidity, whereas the thermal distribution f_{thermal} applies only for the transverse momenta, is very strong. One expects that during the evolution of the system the local momentum distribution is naturally driven towards an isotropic thermal distribution $f_{\mathrm{thermal}}(p)$ depending on the total momentum p of the particle in the local fluid element rest frame. An anisotropic momentum distribution is natural in the initial condition, reflecting the presence of the longitudinal direction in the collision. However, during the expansion of the system, the density drops. Local thermalization processes adjust the temperature, which decreases as well. Similar, but not necessarily the same, processes would also lead to the isotropization of the momentum distribution. Hydrodynamics requires local equilibrium, with only small deviations introduced in the form of viscous corrections. The range of deviations from equilibrium we consider, *i.e.* the relaxation from a system with two-dimensional equilibrium, which is far from the three-dimensional one, to the usual isotropic equilibrium, breaks the condition that the corrections are small. In fact in the initial state the correction to the longitudinal pressure is equal in magnitude and opposite in sign to the equilibrium pressure itself. It means that the generalization we propose must be understood as phenomenological description of the process of local equilibration.

The energy momentum tensor in the local rest fame is allowed to have an anisotropic pressure. It starts with two components of nonzero pressure in the transverse direction and relaxes to a full isotropic one. It is written as a sum of an ideal fluid energy momentum-tensor and a correction

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Pi/2 & 0 & 0 \\ 0 & 0 & \Pi/2 & 0 \\ 0 & 0 & 0 & -\Pi \end{pmatrix}. \tag{9}$$

Maximal asymmetry at the initial time means $\Pi(\tau_0) = p(\tau_0)$. For the subsequent evolution of the deviation from the equilibrium pressure we assume a relaxation equation similar to Eq. (6)

$$\Pi(\tau) = \Pi(\tau_0) \exp\left(-\frac{\tau-\tau_0}{\tau_\pi}\right), \tag{10}$$

which means that apart from the initial deviation from equilibrium shear viscosity effects are small. The simplest relaxation equation (10) is the most natural assumption for a phenomenological description of the local equilibration. Microscopic processes behind the isotropization of the pressure could be collisions or instabilities [17]. Without a reliable estimate of the relaxation timescale we use a phenomenological constant parameter τ_π .

For the boost-invariant flow the hydrodynamic equations reduce to the evolution of the energy density

$$\frac{d\epsilon(\tau)}{d\tau} = -\frac{\epsilon(\tau)+p(\tau)}{\tau} + \Pi(\tau_0)\exp\left(-\frac{\tau-\tau_0}{\tau_\pi}\right) . \tag{11}$$

The above equation is equivalent to an entropy production equation

$$\frac{d\left(s(\tau)\tau\right)}{d\tau} = \frac{\Pi(\tau)}{T} . \tag{12}$$

Deviations from the ideal energy-momentum tensor lead to a gradual entropy production [11, 18]. If we relate the entropy per unit rapidity to the particle multiplicity, a constraint on the dissipative effects in the hydrodynamic evolution appears [16]. If the ratio of the entropies at the end at the beginning of the hydrodynamic evolution was known, one could estimate its production in the dissipative hydrodynamics. A measure of the ratio of the final to initial entropy is given as the ratio of the particle multiplicity per unit rapidity as observed in heavy-ion collisions to the multiplicity predicted in models without a hydrodynamic collective stage. It is difficult to falsify or confirm directly the predictions of such models, since a strongly interacting collective phase in the dynamics, such as the hydrodynamic evolution, cannot be excluded. An argument in favor of such models is that they predict the centrality dependence of the multiplicity [19, 20]. These models explain through an initial state effect, the increased multiplicity of particles produced in heavy-ion collisions as compared to proton–proton collisions and its centrality dependence at the same time. Consequently, there is not much additional entropy production allowed during any supplementary hydrodynamic evolution. However, a different scenario is not excluded. Assuming that the initial entropy (multiplicity) per participating nucleon is the same as in nucleon–nucleon collision, the increased particle production in heavy-ion collision is due to the entropy production during the hydrodynamic evolution and the hadron cascade stages. Nontrivial centrality dependence of the multiplicity of produced particles can be explained within a core-mantle model [21]. The interaction region is composed of a dense core, where hydrodynamic evolution and entropy production takes place and an outer mantle, where after initial particle production not much rescattering (entropy production) takes place. Depending on the centrality, the ratio of the core and mantle volume changes, reproducing the centrality dependence of the multiplicity scaled by the number of participating nucleons [21]. The relative increase of the multiplicity in the core region is of 60%. This number represents an upper limit on allowed entropy production in the collective stage of the evolution of the fireball. For the most central collisions 95% of the particles are emitted from the dense core [21].

3. One-dimensional expansion and entropy production

For the one-dimensional Bjorken expansion and the relativistic gas equation of state $p = \epsilon/3$, the solution of the dissipative hydrodynamic equation (11) can be written in a scaling form

$$\begin{aligned}\epsilon(\tau) &= f(\tau/\tau_0, \tau_\pi/\tau_0) \\ &= \epsilon(\tau_0)\left[e^{\xi}\xi\left(u^{2/3}E_{2/3}(1/\xi) - uE_{2/3}(u/\xi)\right)\right. \\ &\quad \left. +(9+3\xi)u^{2/3} - 3e^{1/\xi}e^{u/\xi}\right] \Big/ (9u^2) \end{aligned} \tag{13}$$

with $u = \tau/\tau_0$, $\xi = \tau_\pi/\tau_0$, and $E_x(z) = \int_1^\infty \frac{e^{-z/t}}{t^x}dt$. Similar scaling forms can be written for the pressure or the entropy. At the initial time we have $\frac{dp(\tau)}{d\tau}|_{\tau_0} = -\frac{p(\tau_0)}{\tau_0}$ and $\frac{d\Pi(\tau)}{d\tau}|_{\tau_0} = -\frac{\Pi(\tau_0)}{\tau_\pi} = -\frac{p(\tau_0)}{\tau_\pi}$. From the requirement $\Pi(\tau) \leq p(\tau)$ we get $\tau_\pi \leq \tau_0$. Within our phenomenological generalization of hydrodynamics we obtain a bound on the relaxation time τ_π.

For the scaling solution the relative entropy production is a function of τ_π/τ_0 only

$$\lim_{\tau\to\infty}\frac{s(\tau)\tau}{s(\tau_0)\tau_0} = \lim_{\tau\to\infty}\frac{\tau}{\tau_0}\left(f\left(\frac{\tau}{\tau_0},\frac{\tau_\pi}{\tau_0}\right)\right)^{3/4}. \tag{14}$$

In Fig. 1 the entropy increase is plotted for several values of the initial starting times $\tau_0 = 0.2,\ 0.5, 1$ fm$/c$ and for $\tau_\pi/\tau_0 = 1$. Independently of the starting time of the hydrodynamic evolution the relative entropy production is the same 28.4%. The entropy production is limited to the time of the order of $4\tau_\pi$ after τ_0, whereas in the case of nonzero shear viscosity the dissipative processes are taking place through the whole evolution. (dashed and dotted lines in Fig. 1). The entropy production in the dissipative expansion depends on the ratio of the relaxation time τ_π and the initial time τ_0 (Fig. 2). In the following we take $\tau_\pi = \tau_0$ exhibiting the largest effects of the dissipative phase.

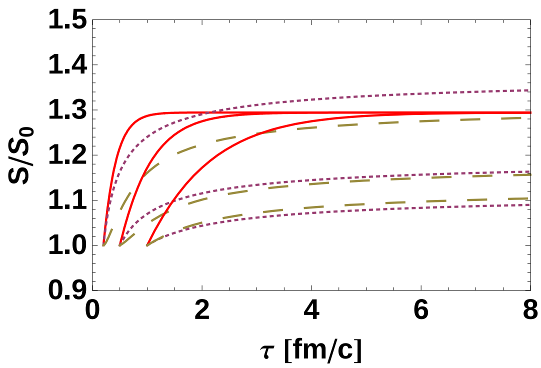

Fig. 1. Relative increase of the entropy from dissipative processes in the early stage of the collision for several initial times τ_0 of the evolution. The dotted line represents the entropy production from the Navier–Stokes shear viscosity tensor (7) with $\eta = 0.1\ s$, the dashed line represents the increase of the entropy obtained from the second order viscous hydrodynamic equation (6) with $\eta = 0.1\ s$, $\tau_\pi = 6\eta/Ts$, and $\Pi(\tau_0) = \frac{4\eta}{3\tau_0}$, and the solid represents the relative entropy production due to the stress tensor term of the form $\Pi(\tau) = p(\tau_0)\exp(-(\tau - \tau_0)/\tau_0)$ (12).

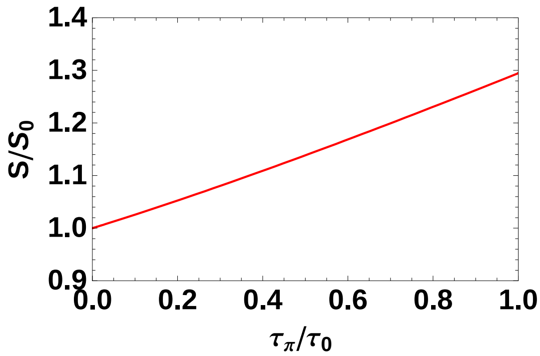

Fig. 2. Relative increase of the entropy from dissipative processes in the early stage of the collision as a function of the scaled relaxation time of the pressure anisotropy τ_π/τ_0.

4. Radial expansion

We consider a system with azimuthal symmetry and boost-invariance in the longitudinal direction. The initial conditions are given by no transverse flow and a Bjorken flow in the longitudinal direction. The initial entropy profile in the transverse plane (as a function of the transverse radius r) is given by the wounded nucleon distribution from the Glauber model for central collisions, with the nucleon–nucleon cross section of 42 mb [23]. In the local rest frame the energy-momentum tensor is given by (9). The correction to the pressure $\Pi(\tau, r)$ describing the pressure anisotropy in the local frame is again given by $\Pi(\tau, r) = p(\tau_0, r) \exp\left(-(\tau - \tau_0)/\tau_\pi\right)$. The energy momentum tensor is boosted by the transverse and longitudinal velocity of the fluid element, and the equation [24]

$$u^\gamma \partial_\gamma \epsilon = -(\epsilon + p) \nabla_\mu u^\mu + \tfrac{1}{2} \Pi^{\mu\nu} \left\langle \nabla_\mu u_\nu \right\rangle \tag{15}$$

and the radial component of

$$(\epsilon + p) u^\gamma \partial_\gamma u^\mu = \nabla^\mu p - \Delta^\mu_\nu \nabla_\alpha \Pi^{\nu\alpha} + \Pi^{\mu\nu} u^\gamma \partial_\gamma u_\nu \tag{16}$$

are solved numerically for the transverse velocity and the energy density. The equation of state used is a combination of the lattice results at large temperatures and a massive hadron gas equation of state at lower temperatures and was presented by Chojnacki and Florkowski (Fig. 3). The details of the parameterization are given in Ref. [22], the equation of state exhibits only a moderate softening around the critical temperature $T_\mathrm{c} = 170$ MeV. The dissipation happens early, in the plasma phase. We consider two different starting times $\tau_0 = 0.5$ and 1 fm/c and for each case both the ideal and dissipative hydrodynamic evolutions. The initial temperature at the center of the fireball is 300 MeV and 365 MeV for the initial time of 1 fm/c and 0.5 fm/c respectively, for the ideal hydrodynamics. For the dissipative evolution we scale the initial entropy, corresponding to the chosen equation of state by a factor 1/1.3 to accommodate for the entropy production in the hydrodynamics phase. Such a procedure leads to similar total multiplicities after the freeze-out in all the simulations.

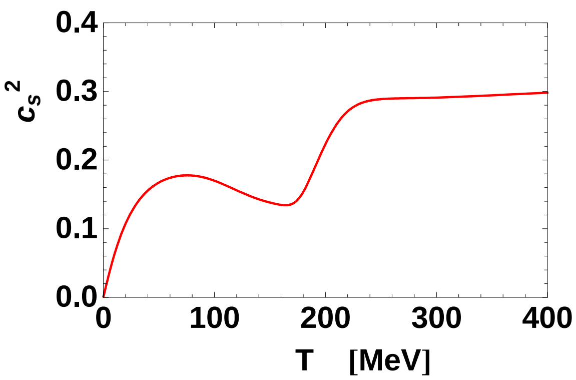

Fig. 3. Square of the velocity of sound as a function of the temperature for an equation of state interpolating between the hadron gas and the quark–gluon plasma [22].

The freeze-out hypersurfaces at the temperature $T_{\rm f}=160$ MeV in transverse-direction-time plane are shown in Fig. 4. The effect of the slower cooling in the longitudinal direction in the evolution with dissipation is compensated by the reduction of the initial temperature. As a result, the time to reach the freeze-out temperature is very similar for in the scenarios.

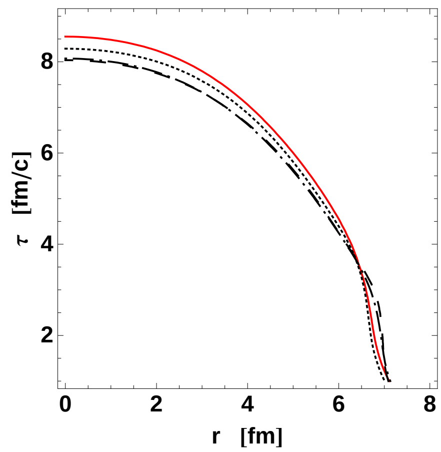

Fig. 4. Freeze-out hypersurfaces in the radial direction for $T_{\rm f}=165$ MeV. Solid and dashed-dotted line are for the ideal hydrodynamics starting at $\tau_0=1$ fm$/c$ and $\tau_0=0.5$ fm$/c$ respectively.The dotted and dashed lines are for the dissipative evolution corresponding to $\tau_0=1$ fm$/c$ and $\tau_0=0.5$ fm$/c$.

We calculate the transverse momentum spectra assuming boost invariance and Boltzmann distributions for both pions and protons. Corrections to the local equilibrium are significant only in the very early stage of the collision. For the dominant part of the freeze-out hypersurface dissipative corrections to the statistical distribution function are negligible. From the Cooper–Frye formula [25] one obtains in this case [26]

$$\begin{aligned}\frac{d^3N}{d^2p_\perp dy} &= \frac{1}{2\pi^2}\int\limits_0^{r_{\rm max}} rdr\tau(r)[m_\perp K_1(\gamma_\perp m_\perp/T)I_0(v_\perp\gamma_\perp p_\perp/T)\\ &\quad -\frac{d\tau(r)}{dr}p_\perp K_0(\gamma_\perp m_\perp/T)I_1(v_\perp\gamma_\perp p_\perp/T)]\,, \qquad (17)\end{aligned}$$

where $m_\perp=\sqrt{p_\perp^2+m^2}$, $v_\perp$ is the transverse velocity and $\gamma_\perp=1/\sqrt{1-v_\perp^2}$, $\tau(r)$ parameterizes the freeze-out hypersurface in the range of transverse radii $[0,\ r_{\rm max}]$, and K_i, I_i are the Bessel functions. At the temperature of 165 MeV most of the particles are resonances, which only latter decay into pions and protons. The decays modify mainly the low momentum part of the spectra. We take this effect and the nonzero baryon chemical potential $\mu \simeq 30$ MeV into account by multiplying the obtained direct pion and proton spectra (Eq. 17) by a factor 4 for pions and $5 \times \exp^{\mu/T}$ for protons [28]. The slope of the high momentum part of the spectra is not modified considerably by resonance decays.

The dissipative stage in the hydrodynamic evolution leads to an increased transverse pressure, this drives a stronger transverse flow, and gives flatter spectra (larger effective slopes) for the same freeze-out temperature. Such a stronger build-up of the transverse flow for viscous hydrodynamics has been observed [6]. Combining a small initial time for the hydrodynamic evolution with a dissipative increase of the transverse pressure one can qualitatively reproduce the observed effective slopes in transverse momentum spectra (dashed lines in Figs. 5 and 6). More detailed analysis should include resonance decays and effects of hadronic rescattering, combined with

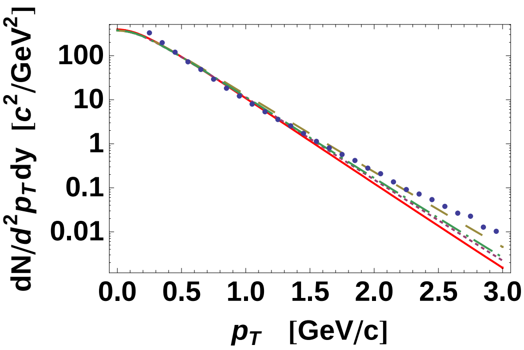

Fig. 5. π^+ spectra from hydrodynamic calculations (same lines as in figure 4). Data are from the PHENIX Collaboration [27] for most central events (0–5%).

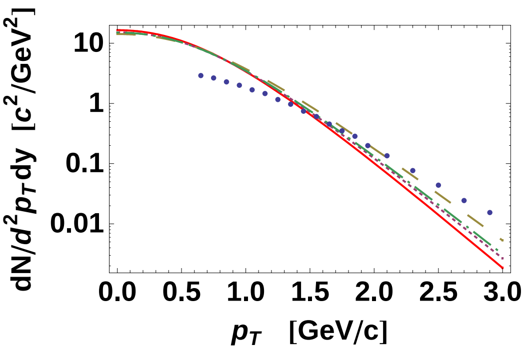

Fig. 6. Proton spectra from hydrodynamic calculations (same lines as in figure 4). Data are from the PHENIX Collaboration [27] for most central events (0–5%).

a study of the elliptic flow. The increase in the transverse flow from the early dissipation is similar for both initial starting times of the evolution; the integrated dissipative effects are comparable, as observed already for the relative entropy increase (Sec. 3).

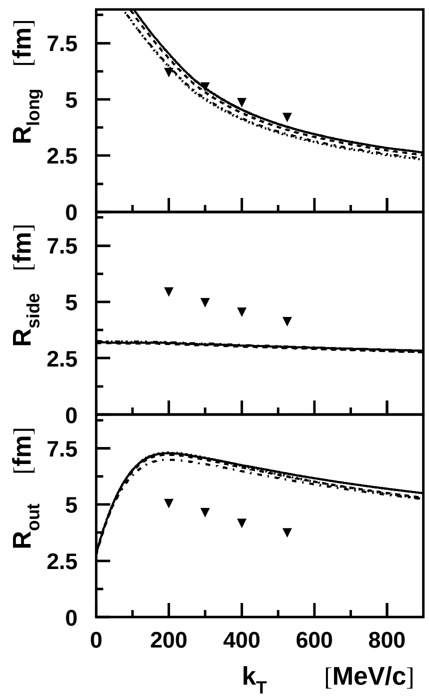

Fig. 7. HBT radii from hydrodynamic calculations as a function of the average particle momentum in the pair (same lines as in figure 4). Data are from the STAR Collaboration [29] for most central events (0–5%).

The emission of indistinguishable particle pairs from the fireball causes two-particle quantum correlations. In the Bertsch–Pratt formula for the two-particle correlation function $C(p_1, p_2)$, particle momenta p_1 and p_2 in the longitudinally comoving frame of the pair are parameterized by: the average momentum $k_\perp$ and the three components of the relative momentum of the pair, $q_{\rm long}$ along the longitudinal axis, $q_{\rm out}$ along $k_\perp$, and $q_{\rm side}$ in the third perpendicular direction [30, 31]. Using a Gaussian formula in the three directions $C(p_1,p_2) = 1 + \exp(-R^2_{\rm long}q^2_{\rm long} - R^2_{\rm out}q^2_{\rm out} - R^2_{\rm side}q^2_{\rm side})$, the HBT radii $R_{\rm long}$, $R_{\rm out}$, and $R_{\rm side}$ can be extracted from the width of the correlation function at mid-height. For that purpose, the two-particle correlation function on the freeze-out hypersurface is calculated for three kinematic configurations $p_{1,2} = k \pm q_{\rm long}/2$, $p_{1,2} = k \pm q_{\rm out}/2$, and $p_{1,2} = k \pm q_{\rm side}/2$. Explicit formulas are given in Ref. [26]. In Fig. 7 the HBT radii are plotted as a function of the average transverse momentum of the particles in the pair. The effect of the early dissipation on the HBT radii is negligible, all the calculations lead to essentially the same HBT radii. This is in contrast to a strong sensitivity of the HBT radii on the viscous effects *at the freeze-out* as noticed in Refs. [4, 5]. The HBT radii obtained in our hydrodynamic evolution cannot reproduce the observed ratio $R_{\rm out}/R_{\rm side}$. The agreement with the experiment could be improved following the recent work [32], combing modified initial conditions, including a more complete hadron spectrum at freeze-out and the decay of resonances. The effects of the early dissipation could be included in such realistic calculations, and would require a retuning of the initial temperature to accommodate the additional entropy production in the early phase. A different question is related to the effects of nonzero shear or bulk viscosities. Nonzero viscosities modify the latter stages of the evolution and lead to significant modifications of the final obseravbles [4–6, 33, 34]. This additional dissipation can be taken into account besides the early dissipation discussed in the present work.

5. Discussion

We propose a dissipative relaxation mechanism to describe the relaxation of the pressure tensor from a two-dimensional initial state to the isotropic three-dimensional form. The final effect of this early dissipative stage depends on the value of the relaxation time. Reduced work in the longitudinal direction and increased transverse pressure lead to a faster build-up of the transverse flow in the very early stage. A relative entropy increase of up to 30% is possible. Also a hardening of the transverse momentum spectra of particles at the freeze-out is noticeable. Since at the freeze-out the corrections to the ideal energy-momentum tensor disappear, we find very little effect on the HBT radii. After the system relaxes to the isotropic equilibrium state its evolution could be described by ideal fluid hydrodynamics with an initial transverse flow [35]. We find that the effect of the asymmetry of the pressure and its subsequent relaxation could be significant for relaxation times of the order of 1 fm/c. We analyze the isolated effect of the initial dissipation only. The initial dissipation described in this paper could be accompanied by a standard shear viscosity with a non-negligible viscosity coefficient η an a different relaxation time τ_π in the latter evolution.

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