Vol. 39 (2008) ACTA PHYSICA POLONICA B No 6

NUCLEAR MATTER WITH THREE-BODY FORCES FROM SELF-CONSISTENT SPECTRAL CALCULATIONS*

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(Received January 21, 2008; revised version received March 5, 2008)

We calculate the equation of state of nuclear matter in the self-consistent T-matrix scheme including three-body nuclear interactions. We study the effect of the three-body force on the self-energies and spectral functions of nucleons in medium.

PACS numbers: 21.65.+f, 24.10.Cn, 25.30.–c

The extrapolation of the energy per particle in dense nuclear systems from finite nuclei to infinite nuclear matter and to neutron matter in neutron stars involves theoretical estimates. Most of the approaches are using free nucleon–nucleon potentials when treating systems with many particles. Obviously a number of approximations must be done to obtain a result. Properties of dense nuclear matter have been estimated with a variety of different schemes: Brueckner–Hartree–Fock (BHF) calculations [1–4], variational calculations [5–9] and self-consistent T-matrix calculations [10–14]. The central issue of these studies is the calculation of the binding energy of cold symmetric and neutron rich nuclear matter. It has been realized that a realistic description of the nuclear matter at saturation density and beyond cannot neglect the presence of three-body forces between nucleons. Variational and BHF calculations that take the three-body forces into account reproduce the empirical saturation point density and the binding energy in symmetric nuclear matter [5, 6, 15–18]. The spectral T-matrix method using

* Supported by the Polish Ministry of Science and Higher Education under grant N202 034 32/0918.

in medium dressed nucleon propagators and the self-consistent T-matrix is well suited for the calculation of single-particle properties of nucleons in the medium [10], the effective scattering [19] and pairing correlations [20, 21]. One important feature of the scheme is the automatic fulfillment of thermodynamic consistency relations [22]. The equation of state of symmetric and neutron matter in the T-matrix approach have been evaluated as well [11–13, 23], however only two-body interactions have been included up to now. The resulting equation of state is similar to the one obtained by other methods with the same interactions. In this letter we present first results for the self-consistent T-matrix approximation with phenomenological three-body interactions taken into account. After reproducing the properties of the symmetric nuclear matter around the empirical saturation point we calculate the modified in-medium properties of dressed nucleons at several densities.

The in-medium self-consistent T-matrix [24–26] is defined by the two-body potential V as

$$T = V + VGT \tag{1}$$

which denotes the resummation of the two-nucleon ladder diagrams. The in-medium nucleon propagator

$$G = \frac{1}{\omega - p^2/2m - \Sigma} \tag{2}$$

is dressed by the self-consistently calculated self-energy

$$i\Sigma = \mathrm{Tr}\,[TG]\,. \tag{3}$$

The numerical calculations are performed in the real-time formalism for the finite-temperature Green's functions [10]. The spectral function for dressed nucleons is obtained from the retarded self-energy (with energy and momentum arguments explicitly written)

$$A(p,\omega) = \frac{-2\mathrm{Im}\Sigma(p,\omega)}{(\omega - p^2/2m - \mathrm{Re}\Sigma(p,\omega))^2 + \mathrm{Im}\Sigma(p,\omega)^2}\ . \tag{4}$$

At each density ρ the above set of equations is iterated until convergence and the Fermi energy μ is adjusted to fulfill the constraint

$$\rho = \int_{-\infty}^{\mu} \frac{d\omega}{2\pi} \int \frac{d^3p}{(2\pi)^3} A(p,\omega)\,. \tag{5}$$

The binding energy can be calculated from the Galitskii–Koltun's sum rule

$$\frac{E}{N} = \frac{1}{2\rho} \int \frac{d\omega}{2\pi} \int \frac{d^3p}{(2\pi)^3} (\omega + p^2/2m) A(p,\omega)\,, \tag{6}$$

a formula that works for Hamiltonians with two-body interactions. In the general case one should calculate the diagrams corresponding to the expectation value of the Hamiltonian $\langle H \rangle$ [27].

Several parameterizations of the nuclear two and three-body interactions are used in nuclear matter and finite nuclei calculations. Since the short range behavior of the nuclear force is not precisely known, differences in parameterization of the nucleon–nucleon interaction at high momenta can be compensated by differences in the associated three-body term [28, 29]. Explicit calculations using relativistic BHF formalism require a different (if any) three-body force. Still other three-body interactions are needed when using renormalized effective two-body potentials [30]. In the following we use a simple phenomenological way of taking the three-body interaction into account [31] and for the two-body potential we take the CD–Bonn interaction [32]. The three body term motivated by the two-pion exchange process has the form of an additional density dependent two-body interaction

$$V(r) = I^c T^2(r) \left(e^{-\gamma_1 \rho} - 1\right) , \tag{7}$$

where

$$T(r) = \left(1 + \frac{3}{\eta r} + \frac{3}{\eta^2 r^2}\right) \frac{e^{-\eta r}}{\eta r} \left(1 - e^{-cr^2}\right)^2 , \tag{8}$$

$\eta = 0.7$ fm^{-1}, $c = 2$ fm^{-2}, $I^c = -5.7$ MeV, $\gamma_1 = 0.15$ fm^3. The density dependent part is short range and repulsive and leads to a stiffening of the equation of state. We perform iterative self-consistent calculation of the dressed propagators and the in-medium T matrix with such density dependent interactions at several densities between $\rho = 0.6\rho_0$ and $\rho = 3\rho_0$, where the saturation density is $\rho_0 = 0.17$ fm^{-3}. Additionally a phenomenological attractive mean field energy is taken in the form [31]

$$\frac{E_{\rm TNA}(\rho)}{N} = 3\gamma_2 \rho^2 e^{-\gamma_3 \rho} , \tag{9}$$

where $\gamma_2 = -260$ MeV fm^6 and $\gamma_3 = 11$ fm^3. With the density dependent two-body interaction (7) the Galitskii–Koltun's sum rule gives the same binding energy as the expectation value of the Hamiltonian. For the chosen interaction the difference shows up only in the term $E_{\rm TNA}$, that should be added to the energy but the corresponding shift in the Fermi energy is $\delta(\rho E_{\rm TNA}(\rho)/N)/\delta\rho$.

The binding energy in symmetric nuclear matter is shown in Fig. 1 and compared to results of variational and BHF calculations with two-body and three-body forces [6, 17]. We observe that the inclusion of density dependent forces suffices to reproduce the empirical saturation point of nuclear matter, although our equation of state is slightly stiffer with the parameters of the short range density dependent interaction taken directly from [31]. As expected, in order to get realistic binding energies at higher densities three-body forces must be included in the calculation. The equation of state fixes the strength of the additional three body terms.

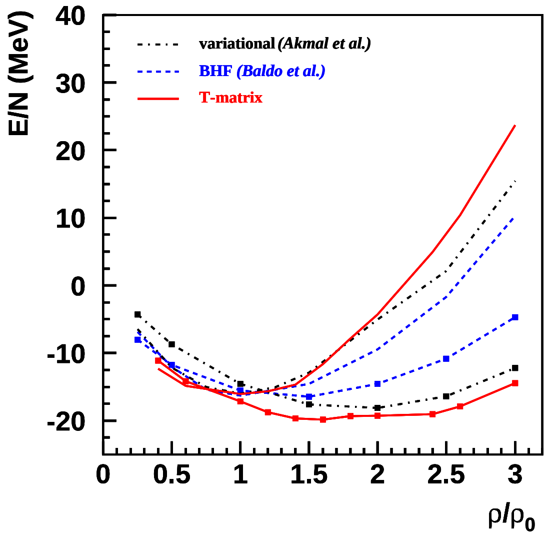

Fig. 1. The equation of state for symmetric nuclear matter. The solid line represents the result of the T-matrix calculation, the dashed line the BHF result [17] and the dashed–dotted line the variational result [6]. The lines with symbols denote the results of calculations including only the two-body force.

Within the T-matrix scheme we explore the consequences of the additional interaction terms on the single-particle properties. In Fig. 2 is shown the spectral function $A(p,\omega)$ at ρ_0. We see that the modification of the force changes the spectral function. This indicates that besides the equation of state the spectral function is sensitive to the chosen nuclear interaction. The binding energy is not sensitive enough to constraint the short range part of the nuclear interaction. The nuclear spectral function when compared to experimental results [33] gives insight into the nuclear two and three-body interactions in nuclei. In a future publication we shall present the results of an investigation of the sensitivity of the proton spectral functions on the density and isospin dependence of the three-body force. Within the parametrization of the three-body terms taken in this letter, the spectral functions shows a stronger quasi-particle peak, which is a consequence of a reduced scattering at the Fermi surface. It is interesting to analyze the real-part of the self-energy $\mathrm{Re}\Sigma(p,\omega_p)$ at the quasi-particle pole ($\omega_p = p^2/2m + \mathrm{Re}\Sigma(p,\omega_p)$). It is the in medium potential felt by the nucleon. The self-energy is given by

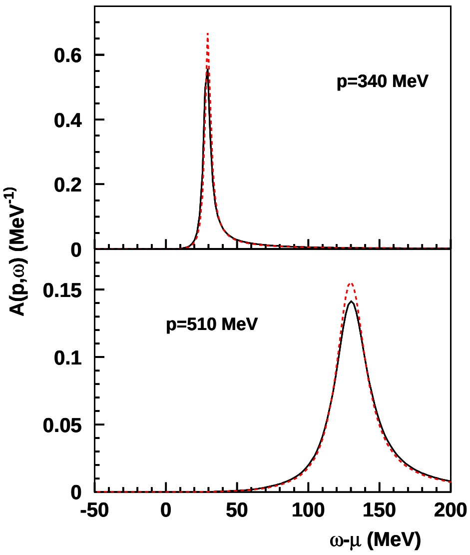

Fig. 2. The spectral function for dressed nucleons at saturation density. The dashed line is the result with the three-body force, the solid line represents the result of the calculation the with two-body interactions only.

dispersion relation

$$\begin{aligned} \mathrm{Re}\Sigma(p,\omega) &= \Sigma_{\mathrm{HF}}(p) + \int \frac{d\omega'}{\pi}\frac{\mathrm{Im}\Sigma(p,\omega')}{\omega'-\omega} \\ &= \Sigma_{\mathrm{HF}}(p) + \Sigma_{\mathrm{disp}}(p,\omega)\,. \end{aligned} \tag{10}$$

It is a sum of a dispersive self-energy Σ_{disp} and the mean-field self-energy $\Sigma_{\mathrm{HF}}(p)$. Different parameterizations of the nuclear interactions yield different Hartree–Fock and dispersive self-energies, hard-core potentials give less attractive mean-field. However the total self-energy $\mathrm{Re}\Sigma(p,\omega)$ is similar [11][1]. At saturation density the dispersive part is not modified much by the three-body forces, only the Hartree–Fock energy is lowered (Fig. 3). On the other hand at $\rho = 3\rho_0$, where the scattering is more important, the dispersive part of the potential is lowered when including three-body forces, but the mean-field is similar. Depending on the density, the shift of the single-particle energy caused by the three-body forces manifests itself in a different way. In all cases the total self-energy $\mathrm{Re}\Sigma(p,\omega_p)$ is more attractive when three-body forces are taken into account.

[1] Interactions yielding similar binding energies must give similar single-particle energies by thermodynamic consistency.

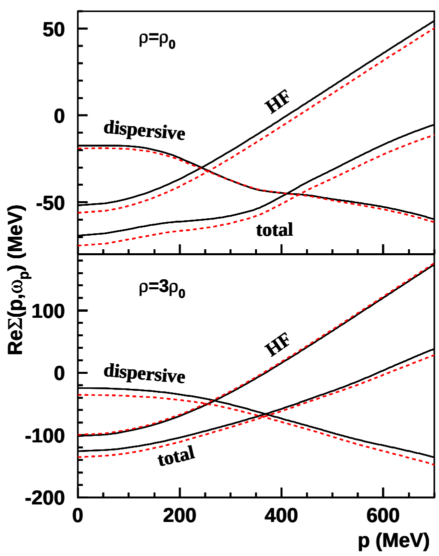

Fig. 3. The real part of the self-energy at the quasiparticle pole $\mathrm{Re}\Sigma(p,\omega_p)$, at the saturation density ρ_0 (upper panel) and at $3\rho_0$ (lower panel). We show the Hartree–Fock contribution, the dispersive part, and the sum (Eq. (10)). The dashed lines are the results including three-body forces and the solid lines represent the energies obtained with two-body forces only.

We calculate for the first time the properties of nuclear matter in the self-consistent T-matrix approximation with a phenomenological density dependent three-body term in the nucleon–nucleon interaction. The additional terms in the interaction and in the mean-field energy allow to reproduce the empirical saturation point of symmetric nuclear matter. We find a slightly stiffer equation of state at densities above $2\rho_0$ than other approaches (BHF and variational). We calculate also the spectral function and the in-medium nuclear self-energy. The spectral function shows a stronger quasi-particle peak and the potential for the nucleons is more attractive when three-body forces are included. The equation of state and the nucleon potential are not directly sensitive to the details of the short range part of the nuclear interaction. On the other hand the proton spectral function at high energies could be a probe of short range nuclear correlations. For the interaction studied in this paper we find less scattering when density dependent three body terms are included.

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