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EXPANSION OF THE UNIVERSE — MISTAKE OF EDWIN HUBBLE? COSMOLOGICAL REDSHIFT AND RELATED ELECTROMAGNETIC PHENOMENA IN STATIC LOBACHEVSKIAN (HYPERBOLIC) UNIVERSE

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As an alternative to the Big Bang (the standard model), we present a mathematical theory of cosmological redshift. We show that a fundamental formula of Lobachevskian (hyperbolic) geometry describes cosmological redshift and the Doppler effect as well. As presented here, the cosmological redshift preserves wavelength ratios (it shifts uniformly the whole electromagnetic spectrum), it is scale invariant, it is a monotonically increasing function of distance, and it is source independent. It agrees with all experimental data. The distortion introduced by imaging from hyperbolic into Euclidean space and the limitations of Special Relativity are discussed. Physical observations in Lobachevskian space are discussed and the new formula relating redshift and/or Doppler shift to aberration is given. An analysis is presented of an erroneous origin of Hubble's so called velocity distance law.

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1. Introduction

To read the paper no special skills are required, however a rudimentary knowledge of Lobachevskian geometry will surely help.

We start with a simple example. Consider a mapping of the sphere S^2 into the Euclidean plane E^2 given by:

$$\varphi \to x\,, \tag{1}$$

$$\theta \to a\tan\theta = y\,, \tag{2}$$

where φ, θ are intrinsic coordinates on S^2 and x, y are rectangular coordinates on E^2. The radius of the sphere is a.

In cartography, Eq. (1) and (2) are used to project the surface of the Earth onto a piece of the Euclidean plane. This way of imaging is known as a Mercator projection due to the Flemish cartographer Gerhard Mercator (1512–1594). Looking at this type of map we notice that a strip along the equator (small θ) is depicted quite correctly while the northern territories of Canada, Alaska, and Greenland are significantly deformed — see Fig. 2. We notice that the distortion of the mapping of S^2 into E^2 is introduced by the **tangent** function, tan(.), in Eq. (2).

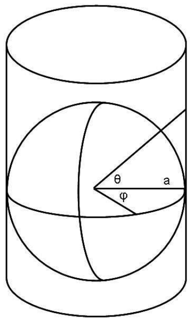

Fig. 1. The mapping from non Euclidean space S^2 to Euclidean space E^2.

Distortion of the intrinsic cosmological redshift (hyperbolic space induced shift in wavelength) and the distortion of the intrinsic Doppler effect (relative velocity induced shift in wavelength) arise when images (functions) in three-dimensional real Lobachevskian (hyperbolic) space L^3 are mapped onto images in three-dimensional Euclidean space E^3. The distortion in this case is due to the **hyperbolic tangent** function, tanh(.).

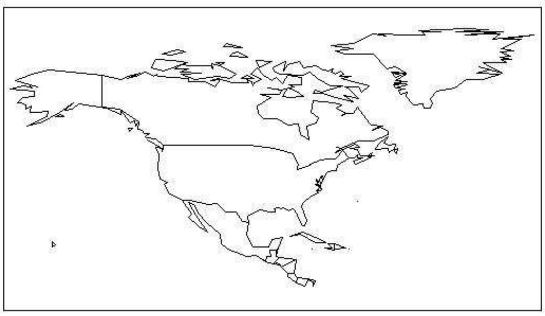

Fig. 2. Mercator image. Deformation of the Northern territories is clearly visible. Deformation is due to the tan(.) function which realizes a projection from non-Euclidean space S^2 into Euclidean space E^2. In this paper we will be concern ourselves with the projection of a non-Euclidean (Lobachevskian) space L^3 into Euclidean space E^3.

The cosmological redshift presented here, given by Eq. (5), has the following features:

- It is a physical realization of a geometrical theorem of an abstract Lobachevskian geometry.
- It uniformly shifts the whole electromagnetic spectrum, thus:
- It preserves wavelength ratios.
- It is scale invariant (it is expressed in dimensionless numbers).
- It is a monotonically increasing function of distance.
- For "small" distances, it is a linear function of distance (word "small" is given a precise meaning in Sec. 4).
- It is source independent.
- It is easily computable.

We would like to turn the reader's attention to an extremely important point regarding relations between mathematics and physics. It should be clearly realized that the same abstract mathematical system, Lobachevskian geometry in our case, may have several different physical representations. Physical objects themselves are totally irrelevant. What is important is the preservation of relations between the objects.

In this paper we show that an abstract Lobachevskian geometry when represented by a large scale vacuum (coordinate space) results in cosmological redshift. Another representation of an abstract Lobachevskian geometry, by Lobachevskian velocities space, is a mathematical expression of the Doppler effect. Yet in another representation on Lobachevskian plane, it results in the impedance match formulas, *i.e.* a Smith impedance chart.

For the above reasons we call the wavelength shift induced by Lobachevskian space:

- **Lobachevsky–Hubble (LH) shift**, since it was first given by Lobachevskian geometry and then experimentally discovered by Hubble.

The shift induced by Lobachevskian velocity space we call:

- **Lobachevsky–Doppler (LD) shift**, since it was first given by Lobachevskian geometry and then experimentally discovered by Doppler.

(Nikolay Ivanovich Lobachevsky (1793–1856), a Russian mathematician of Polish ancestry who made profound contribution in development of non-Euclidean geometry [2]. Edwin Hubble (1889–1953), an American astronomer, who in 1929 experimentally found the linearized version (Eq. (9)) of Lobachevsky's formula (Eq. (3)). Doppler, an Austrian physicist (1803–1853), who in 1842 experimentally observed the linearized version (Eq. (11)) of Lobachevski's formula (Eq. (3)) in velocity space. Both Hubble and Doppler were unaware of Lobachevsky's work.)

The Lobachevsky–Hubble cosmological redshift and the Lobachevsky–Doppler shift follow from different physical representations of an abstract Lobachevskian geometry. In our paper, we also briefly discuss aberration, which is shown to be information-wise equivalent to a wavelength shift, and we derive a new equation which relates redshift (LD shift) to aberration.

A word on notation: δ is Lobachevskian (normalized) distance, υ_{L} is Lobachevskian (normalized) velocity, d and β denote normalized distance and velocity, respectively, in the Euclidean model of Lobachevskian geometry.

Distances in Lobachevskian spaces and their Euclidean images are [6]: $d = \tanh \delta$, $\beta = \tanh \upsilon_{\mathrm{L}}$.

2. Geodesics and horospheres in Poincaré model of Lobachevskian geometry

By Lobachevskian (hyperbolic) space L^3, we always mean a 3-dimensional, real, **non-compact** space of constant negative curvature, equipped with a standard hyperbolic metric [1, 2, 6, 9, 10].

In the present paper we use a geometric and more intuitive approach which should be accessible to a wide audience. We represent (due to Poincaré) Lobachevskian space L^3 as an **interior** of a ball of radius R in Euclidean space E^3 [6]. This representation is also called a Euclidean model (one of several) of hyperbolic geometry. In the representation of Lobachevskian space in a Poincaré ball, geodesics γ are represented as arcs of Euclidean circles and straight lines orthogonal to the sphere S^2 which bounds L^3, and as Euclidean straight lines orthogonal to S^2. Each geodesic γ in Lobachevskian space is defined by its ends, *i.e.* by points where it meets the bounding sphere S^2. Two distinct geodesics γ_1, γ_2 are called **parallel** (or belonging to the same equivalence class $[\gamma]$) if they converge to the same point $p \in S^2$. Points on S^2 are in a one to one correspondence with the classes of equivalent geodesics, and they are at an **infinite** (hyperbolic) distance from any point in L^3 [9, 10]. In the Poincaré ball model of Lobachevskian space, one can imagine equivalence classes of geodesics as fountain shaped rays emerging from a point on the boundary S^2.

Surfaces orthogonal to the equivalence class(es) of geodesic $[\gamma]$, and tangent to the sphere at infinity S^2 are called **horospheres** [8, 10]. Any two horospheres Ω_1 and Ω_2 orthogonal to the same class of geodesic $[\gamma]$ are called **parallel**. The **distance** between two parallel horospheres is the length of a segment of a geodesic having endpoints on two parallel horospheres Ω_1 and Ω_2. The distance between two parallel horospheres is constant. Horospheres of Lobachevskian real three-dimensional space carry the geometry of the Euclidean plane [9].

The last thing we would like to mention is the choice of a **reference** point. Since Lobachevskian space is a **homogeneous space,** the choice of a reference from the mathematical point of view is irrelevant [9]. Nevertheless, since measurements are performed, the indication of reference point or "origin" is required. In the Poincaré ball model, the origin o is taken conveniently as o(1,0,0,0) in **homogeneous** coordinates, or as o(0,0,0) in **non-homogeneous** coordinates [6, 9].

The geodesic γ passing through the reference point o is called the **reference geodesic** γ_o. The horosphere Ω passing through the reference point o is called the **reference horosphere** Ω_o, see Fig. 3.

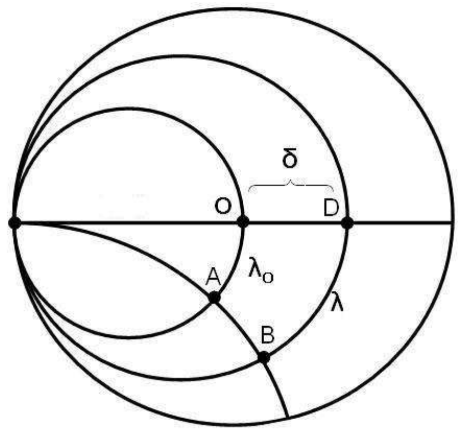

Fig. 3. Poincaré model of Lobachevskian geometry. The reference geodesic is the geodesic passing through point O. The reference horosphere is the horosphere passing through point O. Point O is arbitrary. Geodesics through O and A are parallel. Horospheres through O and D are parallel. Coordinates associated with the orthogonal grid of geodesic and horospheres are called horospherical coordinates.

In physics the origin o is interpreted as a point at which the source is located (source is "at rest"). Observers (detectors) are distributed arbitrarily over the entire Lobachevskian space L^3.

We parametrize the equivalence class of geodesic $[\gamma]$ by the real number z, $-\infty \leq z \leq +\infty$. So $\gamma = \gamma(z) \in [\gamma]$. We choose parameter z on a geodesic γ, at a reference point o, as $z_{\rm o} = 0$. Thus $\gamma({\rm o}) = \gamma(z_{\rm o}) = 0$. It is clear that the same parameter z parametrizes (indexes) the family of parallel horospheres orthogonal to $[\gamma]$, $\Omega = \Omega(z)$ and $\Omega(z_{\rm o}) = \Omega({\rm o}) = \Omega_{\rm o}$, see Fig. 3.

3. Fundamental formula of Lobachevskian geometry as a formula of cosmological red-shift

The fundamental formula of Lobachevskian geometry [6] gives an explicit measure of geodesic deviation *vs* distance traveled Δ and the curvature of the space K (Eq. (3)), see Fig. 3.

Theorem 1 *If $l_{\rm o}$ and l are two segments cut on two parallel horospheres $\Omega_{\rm o}$ and Ω, by corresponding parallel geodesics $\gamma_{\rm o}$, γ, then the ratio $\frac{l}{l_{\rm o}}$ is given by:*

$$\frac{l}{l_{\rm o}} = \exp(\pm\delta)\,, \tag{3}$$

where $\delta = K\Delta = \frac{\Delta}{R}$ is the normalized distance between parallel horospheres $\Omega_{\rm o}$ and Ω, and K is the curvature constant. In terms of a geodesic parameter z, the distance δ is given by: $\delta = |\gamma(z) - \gamma(z_{\rm o})| = |\gamma(z)|$ due to our convention on parametrization.

Looking at Eq. (3), it is not immediately apparent that it describes the Lobachevsky–Hubble cosmological shift and the Lobachevsky–Doppler shift as well. A few simple steps are needed to convert it to a more familiar form. First, we rewrite Eq. (3) in a different form, working with the + case in the exponent (the − case is handled similarly). Note that if $z > z_{\rm o}$, then a positive exponent implies $l > l_{\rm o}$. We have the following definition:

Definition 1: The fractional increase in length $\frac{l-l_{\rm o}}{l_{\rm o}}$ is called the red-shift z. (For a negative exponent, we get $\frac{l_{\rm o}-l}{l}$, which is blue-shift). Thus:

$$\delta = |\gamma(z)| = \ln\frac{l}{l_{\rm o}} = \ln\frac{l_{\rm o} + l - l_{\rm o}}{l_{\rm o}} = \ln(1+z)\,. \tag{4}$$

Definition 2: We say that the horosphere $\Omega_z = \Omega(z)$ is **red-shifted** with respect to horosphere $\Omega_{\rm o} = \Omega(z_{\rm o})$ if for real numbers z, $z_{\rm o}$ the inequality $z > z_{\rm o}$ holds, and we say that horosphere $\Omega_z = \Omega(z)$ is **blue-shifted** with respect to horosphere $\Omega_{\rm o} = \Omega(z_{\rm o})$ if the opposite inequality holds, *i.e.* $z < z_{\rm o}$.

In accordance with Definition 1, it is easy to see that by fixing the reference horosphere $\Omega_{\rm o}$, we divide the entire Lobachevskian space into two regions: red-shifted $z > z_{\rm o}$, and blue-shifted $z < z_{\rm o}$. At any point

$p \in \Omega_o \subset L^3$, on the reference horosphere $\Omega_o(z = 0)$, no wavelength shift will be detected in Lobachevskian space. Thus we can give another definition of a horosphere in Lobachevskian space:

Definition 3: A horosphere in Lobachevskian space is the locus of all observers (detectors) which will detect the same, $z = \text{const}$, red-shift (blue-shift) of a fixed monochromatic source. In particular, all detectors scattered across the reference horosphere will detect $z = 0$.

Some caution is needed in order to distinguish between hyperbolic and Euclidean measures. Horospheres of Lobachevskian space carry a Euclidean metric [1,6], and thus the intrinsic measure of segments l_o and l is Euclidean. On the other hand, the distance δ in Eq. (3) is the **hyperbolic** distance. Since we use a Euclidean model of hyperbolic space (analogues of S^2 imaged in E^2), one needs to convert hyperbolic distance to Euclidean distance d, given by the **hyperbolic tangent**: $d = \tanh \delta$ [6].

$$d = \tanh \delta = \tanh(\ln(1 + z))\,. \tag{5}$$

Here d is the normalized radial distance in the Poincaré ball: $d = \frac{D}{R}$, $0 \le d < 1$, R is the radius of the Poincaré ball, and D is un-normalized distance. The deformation introduced by hyperbolic tangent function, tanh( ), when mapping from L^3 to E^3 is quite analogous to the distortion introduced by the tangent function, tan( ), when mapping from S^2 into E^2. Intuitively, hyperbolic space is "roomier" than Euclidean space, while spherical space is "smaller" than Euclidean space. This is reflected by the character of the deformation. The Northern territories in a Mercator projection are enlarged, while unlimited functions in hyperbolic space are "clamped" by tanh(.) not to exceed unity. Fig. 4 shows pretty well the character of the distortion when Lobachevskian (hyperbolic) space and spherical space are projected into Euclidean space.

So far we have not specified a reference length. From an operational point of view it must be specified in a way which makes comparisons possible with the use of our instruments. A natural (and the only possibility to author's knowledge) choice, is to take the length $l_o = \lambda_o$ (reference wavelength), and $l = \lambda$ (detected wavelength). Recalling that $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, we immediately obtain:

$$d = \tanh(\ln(1 + z)) \qquad \text{equivalent to:} \qquad \frac{\lambda}{\lambda_o} = 1 + z = \sqrt{\frac{1+d}{1-d}}\,. \tag{6}$$

$d = \tanh(\ln(1+z))$ gives the normalized distance d in Lobachevskian space, viewed as an interior of a unit ball, *vs* the redshift z. In [3], we derived the same Eq. (5) in a different way starting from the basic definition of a horosphere in Lobachevskian space.

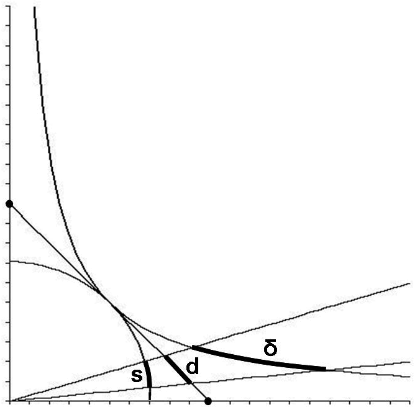

Fig. 4. Fig. 4 shows the nature of the deformation. Poincaré ball lies in the $x+y=a$ plane. It is clearly shown that the hyperbolic distance $\delta > d$, while the spherical distance $s < d$, where d is the Euclidean distance.

Eq. (5) agrees with Hubble's observations in that red-shift increases with the distance to a source. It fits experimental data **for all** z, $0 \leq z < \infty$. A plot of d *vs* z is given in our work [3]. A linearized version of Eq. (6), experimentally discovered by Hubble, is discussed in the next section.

We now discuss the Lobachevsky–Doppler shift. We recall that velocity space is a 3 dimensional real Lobachevskian space [3,9,10]. The fact that the space of velocities is a Lobachevskian space was first recognized by Feliks Klein [10] (1849–1925), a German geometer who first proposed to study geometries in unified way via group theory.

The signed distance function between two points A and B in Lobachevskian space is called the **relative** velocity υ_{L} of a point B with respect to a point A. The signed distance corresponds to two directions on a geodesic, and it accommodates the change in the direction of relative velocity. In the Poincaré ball model of Lobachevskian velocity space, the normalized distance $d = \tanh\delta$ is given by $\beta = \tanh\upsilon_{\mathrm{L}}$, where the constant c is the velocity of light in a vacuum (the radius of the Poincaré ball is c), and υ_{L} and β are relative velocities, intrinsic to Lobachevskian space L^3, and velocity β is the velocity in the Euclidean model of Lobachevskian geometry. Locally, Euclidean space mimics Lobachevskian space very closely.

Note that in the Poincaré ball model, velocities of photons are at an **infinite hyperbolic distance**, $\upsilon_{\mathrm{L}} = \infty$, from any point in Lobachevskian velocity space. One can say that photons populate the **boundary at infinity**, also called an **absolute** for Lobachevskian space. This is a mathematical expression of the outcome of the Michelson–Morley experiment which says that velocity of light in vacuum does not depend on the state of motion of a source (or detector).

The Lobachevsky–Doppler shift now follows trivially as just another representation of abstract Lobachevskian geometry. One needs only to substitute β for d in Eq. (6).

$$\beta = \tanh(\ln(1+z)) \qquad \text{equivalent to:} \qquad \frac{\lambda}{\lambda_{\rm o}} = z + 1 = \sqrt{\frac{1+\beta}{1-\beta}}\,. \tag{7}$$

The linearized Lobachevsky–Doppler shift follows from the expansion $\exp v_{\rm L}$ in power series.

$$z \approx \beta = \frac{1}{c} v = kv\,. \tag{8}$$

Here c is the radius of Lobachevskian velocity space, the slope $k = \frac{1}{c}$ is the curvature constant of velocity space, v non normalized Euclidean velocity, and $\approx$ means that higher then first order terms in expansion of $\exp v_{\rm L}$ are dropped, which is equivalent to the assumption that $\beta \ll 1$.

The reader can easily see that the Eq. (6) as well as Eq. (7) appears in two different form. A geometric form, which is on the left side, and on the right side as commonly seen in papers on physics. Geometric form of the Eq. (6) and (7) clearly shows that an origin of the redshift phenomenon is Lobachevskian (hyperbolic) geometry. On the other hand the source of physics due to Eq. (6) and (7) in the form on right, is entirely obscured.

4. Hubble's distance *vs* red-shift discovery as an experimental confirmation of the fundamental formula of Lobachevskian geometry. Calculation of the radius of the universe

The linearized Lobachevsky–Hubble shift distance formula for $d \ll 1$ follows directly from expanding the RHS of the fundamental formula (Eq. (4)) of Lobachevskian geometry: $1+z = \exp\delta$ into power series around zero and by taking only the linear term. For small x, $\tanh x \approx x$, and $d \approx \delta$. Thus, $1+z \approx 1+\delta \approx 1+d = 1+\frac{D}{R} = 1+KD$, or:

$$z \approx KD\,. \tag{9}$$

The same follows from a Euclidean image. Recall that for $x \ll 1$, $\ln(1+x) \approx x$, $\tanh x \approx x$, $d = \tanh(\ln(1+z)) \approx \tanh z \approx z$.

The linearized Eq. (9) precisely describes what Hubble discovered experimentally and has failed to recognize correctly **why the red-shift is a linear function of distance**. The slope of the graph in Eq. (9) is the curvature K of space. Its inverse $R = \frac{1}{K}$ is the radius of the hyperbolic universe in the Poincaré ball model.

By the end of 1840, non-Euclidean geometry was put in its final form by Lobachevsky as a logically coherent closed geometrical system [2], but it seems that Hubble was unaware of the work of Lobachevsky (and others geometers), and particularly he was not aware of Eq. (3), which appears in Lobachevski's works around 1830, and its linearized form (Eq. (9)), which he **rediscovered** experimentally a hundred years later.

4.0.1. Hubble's mistake as a fatal blow to cosmology

The Hubble velocity distance rule is an interesting example how **two independently correct facts**, *i.e.* the common Doppler shift and Hubble's experimental distance *vs* red-shift law (Lobachevskian linearized Eq. (9)), when "married" together resulted in an unfortunate conclusion. This happened because the **only** cause of red-shift that Hubble was aware of was due to **Lobachevskian velocity space,** *i.e.* the common Doppler shift. Thus looking at his experimental **distance-redshift** data $d = d(z)$, and by looking at the **redshift-velocity** plot $z = z(v)$ depicting a Doppler shift, Hubble **converted a measured red-shift to an equivalent velocity** via the Doppler formula, and thus he obtained a **distance–velocity** plot $d = d(z(v) = d(v)$ [8].

Contrary to his followers, Hubble himself was not entirely happy with his distance–velocity formula, which decisively contributed to the inflationary model of the universe. In the paper [*Astrophys. J.* **84**, 517H (1936)], jointly with Tolman, he wrote "The possibility that the redshift may be due to some other cause connected with the long time or distance involved in the passage of light from nebulae to observer, should not be prematurely neglected."

In a general setting and from a **logical** point of view, the existence of relative velocity is a **necessary** but **not sufficient** condition to record a wavelength shift. In Euclidean geometry *e.g.* wavelength shift uniquely implies existence of a relative velocity while in hyperbolic geometry it does not have a unique implication. Thus while the existence of relative velocity always results in a wavelength shift, the presence of a shift **may or may not imply** the existence of a relative velocity. Examples of this kind of situation are numerous. The mistake made by Hubble and his followers for almost one hundred years has paralyzed and still paralyses cosmology and related astronomical sciences as well.

The big bang came to life and acquired its legitimacy due to Hubble's erroneous reasoning. His genuine experimental data $d = d(z)$ were **manipulated** and the conclusion was "tailored" in the way shown above to get the velocity *vs* distance plot, in order to fit speculations following the FWR solution ($\frac{dR}{dt}\frac{1}{R}$ term) of Einstein field equations. We say **speculations** since it is well recognized that the RW metric and Einstein equations are conceptually and logically two independent entities and the result from melting them together may or may be not be taken for granted. The "cosmological constant" introduced by Einstein in order to get a static solution is actually the negative curvature of a Lobachevskian vacuum.

4.0.2. Euclidean geometry *vs* Lobachevskian geometry

From Eq. (5) and its linearized form Eq. (9), it follows that when the curvature constant $K = 0$ (in the case of Euclidean space), the space induced wavelength shift vanishes. $K = 0$ implies $z = 0$. In other words, Euclidean geometry **cannot** induce changes in wavelength of electromagnetic radiation. Horospheres in Euclidean space are Euclidean planes (plane waves) and geodesics are Euclidean straight lines orthogonal to planes, see Figs. 5 and 6.

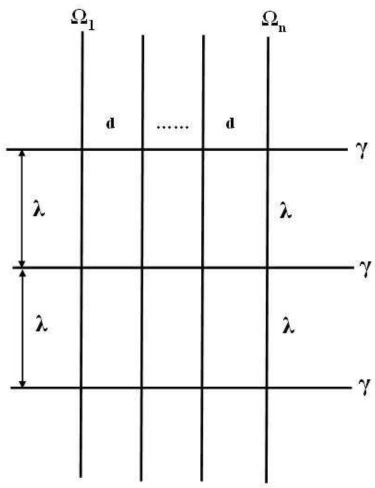

Fig. 5. The case of $K = 0$. In Euclidean space geodesics do not deviate. Segments cut by two geodesics on two parallel horospheres have the same length on the entire space E^3. Horospheres are simple Euclidean planes E^2. In mathematics this picture is called the foliation of E^3 by plane waves.

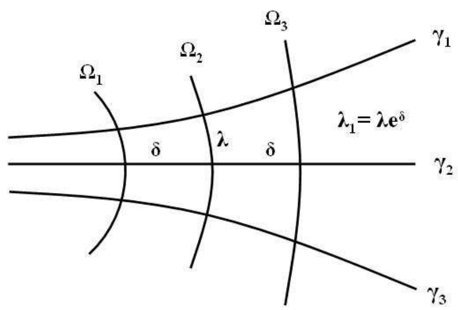

Fig. 6. This is the case of Lobachevskian (hyperbolic) space. Geodesics γ deviate at an exponential rate. This causes the **illusion** of space expansion since the separation between geodesics, which is λ at the source become λe^{δ} at the detector.

Many people, including Lobachevsky himself, suspected that the geometry of physical space may differ from Euclidean geometry. Lobachevsky himself tried to detect a curvature constant from the data on astronomical parallaxes, taking as a triangle the star Sirius and the Earth in it two opposite positions in orbit [2]. In [3], we calculated the distance at which the (negative) curvature of space will be directly recorded, assuming a resolving power of modern diffraction gratings at 10^6. Calculations done in [3] show why Lobachevski's attempt to detect curvature of space was unsuccessful.

From the above it follows that in a hyperbolic universe a measurement of a wavelength shift is **not sufficient** to distinguish between **space induced** and **velocity induced** contributions to the overall recorded shift. While a space induced shift increases monotonically with distance, it will be "smeared" by random blue/red shifts due to the random velocities of distant objects. This situation resembles a quantum mechanical rather than classical scenario. In quantum mechanics, a mixed state **cannot be uniquely decomposed** into pure state components, and the information of how the mixed state was produced is not recoverable. We illustrate the above with an example:

We model Lobachevskian spaces as an interior of the unit ball ($R=c=1$). Let us take the famous QSO PC 1247+3406 quasar with $z = 4.982$. We hope to gain some information from the distance data. What are our options? Since we know that a red-shift may result from distance in hyperbolic space and/or from the existence of a relative velocity we proceed as follows: Take a real parameter $p \in [0, 1]$ and split the observed red-shift z into two parts — a space induced $pz = x$ and a velocity induced $(1 - p)z = y$. Here we consider the case of outward bound velocity. The reader may repeat the calculation for inward bound velocity

$$z = pz + (1 - p)z \,. \tag{10}$$

Thus the generalized distance will be: $d_{\rm g} = \tanh(\ln(pz + (1 - p)z)$.

At $p = 0$, $d_{\rm g} = \tanh(\ln y)$. This means that the entire redshift is due to velocity, and the generalized distance means β. From Eq. (7), we conclude that QSO has relative receding velocity $v = 0.9225$. The distance to this object in this case is 0, which is unrealistic.

At $p = 1$, $d_{\rm g} = \tanh(\ln x)$. This means that entire redshift is due to distance only. Generalized distance is the distance d. From Eq. (5), we conclude that QSO is at a distance $d = 0.9225$, and it has no relative velocity with respect to us.

At $p \in (0, 1)$, (verteces excluded), we have a continuum of states parametrized by parameter $p \in (0, 1)$. No criteria exists how to select the "true" p, to split z by Eq. (10), or in other words we do not have any criteria how to uniquely decompose the mixture, a common situation in quantum mechanics. If due to some "miracle", we know for example that the "true" $p = 0.75$, then we may uniquely decompose the QSO recorded red-shift z as $0.75z + 0.25z = 4.982$. In this case we compute that the red-shift due to space is 3.7365 and redshift due to velocity is 1.2455. This in turn means that the object at a distance $d = 0.8663$ from us, and is moving away from us with a velocity $\beta = 0.2161$.

This example shows that in general, the distance to an object cannot be recovered from a red-shift measurement itself. Other techniques may be employed to evaluate the selection of p, if they are applicable. **At very high distances, where other techniques fail, the determination of distances from red-shift data itself is severely limited if not entirely impossible**.

Having genuine data on red-shift z, the radius of the universe in the Poincaré ball model, $R^{-1} = k$ can be easily computed if distance D is known from independent data

$$R = \frac{D}{\tanh(\ln(1+z))}\,. \tag{11}$$

Unfortunately, long range distance data are not reliable. The situation is even more complicated since light behaves differently in hyperbolic space than in Euclidean space. For example, the amplitude of an electromagnetic wave in Lobachevskian space varies exponentially with distance. This is seen directly from the solution of the Laplace Beltrami operator in Lobachevskian space [7], or more simply by considering the Lobachevskian polarization space of an EM wave, which is done on our work [4].

The intensity (amplitude squared) of recorded light may also decrease if there is an outward relative velocity, or increase if there is an inward relative velocity [4] (non-Euclidean fading/enhancement of light). Thus at very high distances $D \approx R$ and in the presence of random kinematical components, the evaluation of photometric distances may be quite complicated.

4.0.3. The new approach to detect gravitational waves

We would like to mention an interesting possibility which follows from the Lobachevsky–Hubble red-shift formula we discussed in [3]. We see that a shift in wavelength z is a function of distance and curvature, $z = z(k, D)$, Eq. (5). So far we have been preoccupied with $z = z(D, k = \text{const})$ *i.e.* with the dependence of the wavelength shift on distance while the negative curvature of space was assumed to be constant. Now, let us assume that due to some reason (*e.g.* passage of gravitational wave) the **local** curvature of some region in space varies as $k = k(x, t)$. This will cause a variable wavelength shift, or in other words frequency modulation (FM). This is a classical picture how a modulator works, however a modulator on the cosmic scale. The modulator is the hyperbolic space itself with variable curvature. The same effect will be true for AM modulation [4]. Thus by looking into space, a potential generator of gravitational waves may be located by recording AM and FM modulated light coming from that region. In that way we may find the presence of gravitational waves in the universe.

5. Lobachevskian reality and Euclidean perception. RPR ratio

In Figs. 7, 8, and 9, we plot the ratio of Lobachevskian distance δ to Euclidean distance d, for a fixed redshift z. We call this ratio the **reality to perception ratio** (RPR), and it measures the distortion of images from hyperbolic space as they are projected into Euclidean space.

$$\mathrm{RPR} = \frac{\delta}{d} = \frac{\ln(1+z)}{\tanh(\ln(1+z))}\,. \tag{12}$$

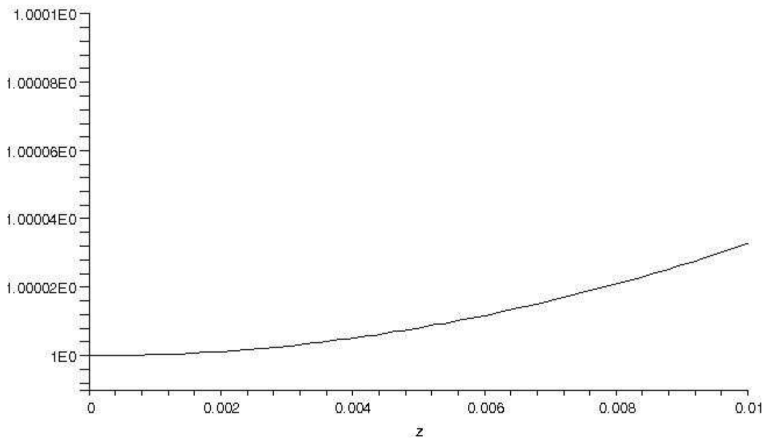

Fig. 7. For $z \ll 1$, the RPR ratio stays very close to 1. For example, for the nebulae NGC 4235 with the redshift around $z = 0.008$, we see an undeformed image. Deformation is less than 30 parts per million. In this case we may believe that the information is genuine assuming that there are no cancellations of space and velocity contributions to the overall z, as discussed in Sec. 4. In general z is a physical measure of departure of actual geometry from Euclidean one. For more on the relation between redshift z and the type of geometry see von Brzeski [5].

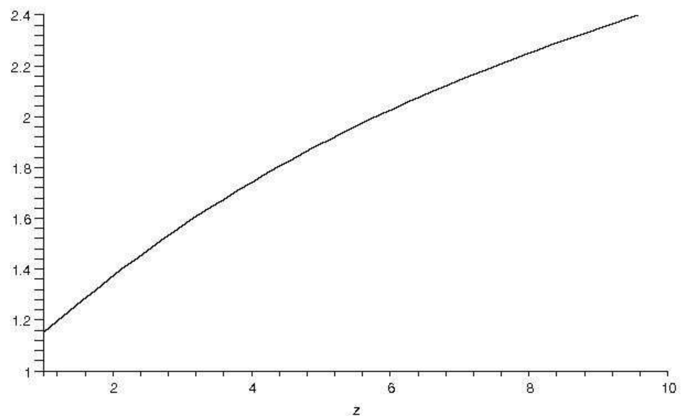

Fig. 8. For very high red-shifts, *e.g.* quasar PC 1247+3406, $z = 4.897$, the RPR ratio will be around 2. Assuming there is no kinematical contribution to the wavelength shift, quasar PC 1247+3406 is twice as far away as it seems to be. For even bigger red-shifts, the RPR ratio will grow like ln(.) since tanh(.) approaches 1.

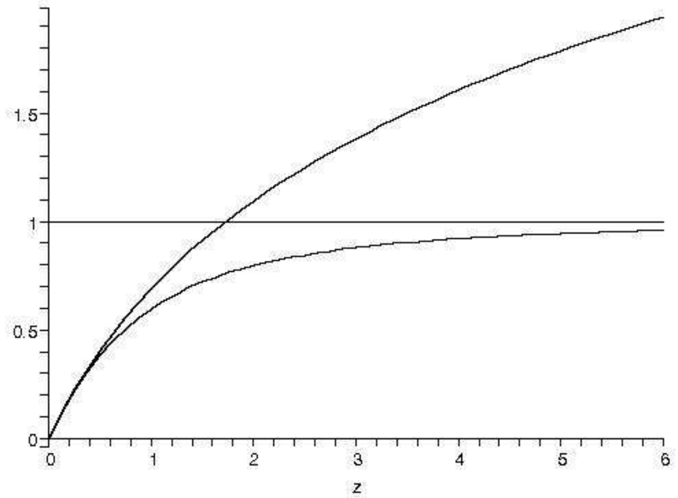

Fig. 9. Fig. 9 clearly shows the limited applicability of so called "relativistic" formulas. Lower curve $\tanh(\ln(1+z))$ presents the Euclidean image of Lobachevskian geometry. Upper curve $\ln(1+z)$ is intrinstic to negatively curved Lobchevskian space. Images from Lobachevskian spaces are so much compressed at the "ceiling" $y = 1$, that resolution on distance $y = d$ and/or velocity $y = \beta$ is *de facto* impossible for high z, *e.g.* $z > 6$. To get genuine information from the hyperbolic world at high redshifts we need a better tool (better way of mapping) than Einstein's Special Relativity. In fact, Special Relativity is just a kind of Mercator imaging from a 3D hyperboloid into Euclidean 3D space.

6. Aberration of light in hyperbolic spaces and aberration — redshift formula

The phenomenon of aberration, see Fig. 10, similarly to the phenomenon of red-shift, is also related to the **exponential divergence of geodesics in** Lobachevskian space. Before we proceed further we list some features of Lobachevskian geometry different from our Euclidean experience.

- First, since the length of the sides of a triangle in Lobachevskian geometry depends on angles (and *vice versa*), there are **no similar triangles** in Lobachevskian geometry [1, 2, 6].
- Second, two Lobachevskian spaces (of the same dimension) are **not isometric** unless they have the same curvature constant [6].

The aberration of light (change in angle) is **not an independent** phenomenon from the wavelength shift (change in length). This should be expected from the properties of a Lobachevskian triangle. Since the length of the sides of a triangle depends on its angles, it follows that the **change in length** will cause a **change in angles** and *vice versa*. Since a change in the red-shift z in Eq. (5) causes a change in distance d, and a change in distance d causes change in angles, a change in z will cause change in angles. Such a chain of conclusions is **not true in Euclidean geometry** as it allows for existence of similar triangles. In Euclidean geometry the notion of length is decoupled from the notion of an angle. We see that the information gained from a wavelength shift, in Lobachevskian space, is equivalent (the same number of bits) as the information gained from an angle change. The aberration of light is simply a geometrical statement about two triangles having one common vertex at infinity.

For simplicity we set $R = c = 1$ which makes Lobachevskian space and Lobachevskian velocity space isometric (indistinguishable in physics).

Consider a triangle in hyperbolic space with one vertex at infinity, Fig. 10. Recall that Lobachevskian (hyperbolic) distance δ is related to Euclidean distance d as $\tanh\delta = d$, and that the hyperbolic angle is also a Euclidean angle [2, 7]. After applying elementary hyperbolic trigonometry to a triangle with one vertex at infinity (sides AC and OC are parallel) [2, 3, 6] we obtain:

$$\cos\varphi = \tanh \mathrm{OA} = d_{\mathrm{AO}}, (\text{angle OAC} = \varphi)\,,$$

$$\cos\psi = \tanh \mathrm{OB} = d_{\mathrm{BO}}, (\text{angle OBC} = \psi)\,.$$

Thus the distance from A to B is:

$$d_{\mathrm{BA}} = \tanh(\mathrm{OB} - \mathrm{OA}) = \frac{\tanh \mathrm{OB} - \tanh \mathrm{OA}}{1 - \tanh \mathrm{OB} \tanh \mathrm{OA}} = \frac{\cos\psi - \cos\varphi}{1 - \cos\psi\cos\varphi}\,.$$

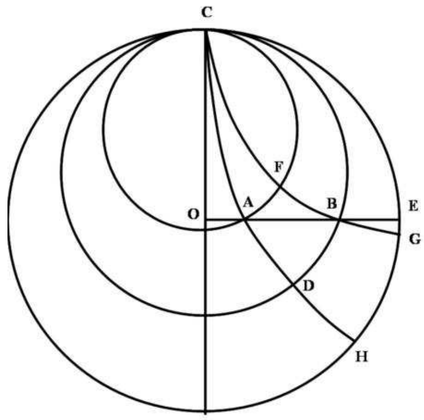

Fig. 10. Aberration of light in Lobachevskian space, either in a Lobachevskian universe or in a Lobachevskian velocity space.

We obtain the most general equation for the aberration of light in a Lobachevskian universe:

$$d_{\mathrm{BA}} = \frac{\cos\psi - \cos\varphi}{1 - \cos\psi\cos\varphi}\,. \tag{13}$$

It follows that an object immersed in a hyperbolic space will appear to us rotated by an angle which depends on the distance to the object and the space curvature constant.

We have already mentioned that in the Euclidean model of Lobachevskian velocity space, a distance d means relative velocity $\beta = \beta_{\mathrm{BA}}$ (velocity of B with respect to A). Thus, substituting $d \to \beta$ and solving Eq. (13) with respect to $\cos\varphi$ (or $\cos\psi$, depending on where the reference is located), one obtains the equation for aberration as commonly seen in textbooks on physics:

$$\cos\psi = \frac{\beta + \cos\varphi}{1 + \beta\cos\varphi}\,. \tag{14}$$

The clarity and simplicity of Lobachevskian physics cannot be underestimated. The power of geometric formulas is that they are **representation independent** and result in the same physics.

It is interesting to note that hyperbolic space induced rotation and velocity induced rotation may cancel or may add together. Thus an object close to the limiting sphere S^2, and moving with a velocity close to c will be seen from its "rear", *i.e.* it will be rotated by almost 180 degrees.

We now obtain the relationship between the red-shift and the object rotation angle. For simplicity, as before, we adopt $R = c = 1$. In a unit ball model, the normalized distance d will result in a red-shift of $\tanh\ln(1+z)$.

On the other hand, it will also produce an aberration accordingly to Eq. (13). Thus:

$$d = \tanh\left(\ln(1+z)\right) = \frac{\cos\psi - \cos\varphi}{1 - \cos\psi\cos\varphi}\,. \tag{15}$$

In particular if $\varphi = 90^\circ$, then:

$$\tanh\left(\ln(1+z)\right) = \cos\psi\,. \tag{16}$$

Eq. (16) gives the relation between the observed red-shift z and an angle ψ in abstract Lobachevskian geometry. The recorded wavelength shift z as well as the angle ψ may follow from the Lobachevskian universe and/or from the Lobachevskian velocities space.

7. Conclusions and remarks

On the basis of a three-dimensional real Lobachevskian geometry, we presented a geometrical analysis from which cosmological red-shift and related phenomena follow in natural way. The presented equations give correct numerical values for their respective physical quantities. The new Eqs. (15) and (16) which relate red-shift to aberration might be useful in astronomical observations.

Our presentation of Lobachevsky–Hubble cosmological redshift (5), the Lobachevskian–Doppler effect (7), and aberration was done in rigorous way on a purely geometrical basis of Lobachevskian three-dimensional real geometry with all entities clearly defined. At present, the widely adopted view explains cosmological red-shift using the vague concept of physical **space inflation**. For example, observations tell us that space within galaxies, which are rather diffuse objects, do not expand. Thus, where is the "border line" in space which divides expanding space from non expanding space?

Next, we are told that inflation itself is due to some rather mysterious event, which was sarcastically named by Fred Hoyle (to ridicule the whole concept), as the big bang.

Instead, we offer an alternative solution based on simple Lobachevskian geometry. We believe that looking at experimental data and Eq. (5), a much simpler solution (**minimum complexity solution**) is to admit that the space between distant sources and our spectrographs is negatively curved, *i.e.* it is a Lobachevskian three-dimensional space causing the recorded shifts. In other words what we see through our telescopes is the fundamental formula of Lobachevskian geometry: Eq. (3). Experiments confirm our model.

From the analysis performed, the importance of the **range of applicability** of some mathematical notions follows. For example, someone who only saw a map of the Earth as in Fig. 2, and had no prior knowledge where this map came from, and what mechanism was used in mapping process, will in good faith believe that Greenland is as big as the USA. His or her conclusions about geography made from the distorted image will be necessarily false.

Similarly, making conclusions about the geography of the universe based on the so called "relativistic" formulas in the form of RHS expression in Eq. (7) (and Eq. (6) as well), is misleading since we did not know that we were looking at **distorted formulas** of a precise Eq. (3) of non-Euclidean geometry projected into Euclidean space–space in our vicinity! Conclusions based on a distorted formula will inevitably lead to the inconsistencies and/or paradoxes for projections from regions of high distances $d \simeq 1$ in space or high distances $\beta \simeq 1$ in velocities space. Of course, as long as we stay "close to equator", (which means going local, *i.e.* $d \ll 1$, $\beta \ll 1$) distortion will be negligible within the required range of precision. Nevertheless we have to be aware that we are still dealing with the distorted images. This rises the serious question of applicability of the Special Relativity in the range $d \simeq 1$, $\beta \simeq 1$.

One may ask a legitimate question of how the experimentally detected cosmic microwave background radiation (**CMBR)** is related to Lobachevbreak skian geometry (Lobachevskian universe)?

The answer is that in Lobachevskian space, CMBR is identified with the homogeneous space of horospheres which is **dual** [7,9] to Lobachevskian space. In our work [3] we showed that a horosphere in Lobachevskian space, as far as physics is concerned, is a surface of **constant phase** of an electromagnetic horospherical wave. In other words, it is a **horospherical wavefront**. Radiation represented by horospherical wavefronts homogeneously fills the entire Lobachevskian universe. Therefore, assuming a hyperbolic universe, we **have to have** CMBR with its properties of homogeneity and isotropy! It follows "automatically" from Lobachevskian geometry.

Horospherical waves are solutions of the Laplace–Beltrami operator (wave operator) in Lobachevskian space. Their properties are well known and well understood. Thus, there is entirely no need to associate CMBR with the big bang — an event which itself cannot be understood and deliberated in scientific terms.

In Lobachevskian space filled only with radiation CMBR would be **perfectly isotropic**. In the presence of matter however, which on local scales is distributed rather randomly, a small anisotropy in the properties of CMBR might be present due to local conditions. This was already recorded by COBE. More about the space of horospheres can be found in [7,9].

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