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HIGHER ORDER CORRECTIONS TO HEAVY FLAVOUR PRODUCTION IN DEEP INELASTIC SCATTERING* **

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In the asymptotic limit $Q^2 \gg m^2$, the non-power corrections to the heavy flavour Wilson coefficients in deep-inelastic scattering are given in terms of massless Wilson coefficients and massive operator matrix elements. We start extending the existing NLO calculation for these operator matrix elements by calculating the $\mathcal{O}(\varepsilon)$ terms of the two-loop expressions and performing first investigations of the three-loop diagrams.

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1. Introduction

In deep-inelastic scattering, the differential cross-section with respect to the Bjørken-variable x and the virtuality of the photon Q^2, can be expressed in terms of the unpolarised structure functions $F_2(x,Q^2)$ and $F_{\rm L}(x,Q^2)$, and the polarised structure functions $g_1(x,Q^2)$, $g_2(x,Q^2)$. For small values of x, the contributions of charm to $F_2(x,Q^2)$, $F_2^{c\bar{c}}(x,Q^2)$, are of the order of 20–40%, and therefore deserve and need a more detailed investigation. So far, there exist NLO — 2-loop — heavy flavour corrections to $F_2^{p,d}(x,Q^2)$ in the whole kinematic range, calculated in a semi-analytic way in x-space [1]. A fast implementation for complex Mellin N-space was given in [2]. One observes that $F_2^{c\bar{c}}(x,Q^2)$ is very well described by an asymptotic result for $F_2^{c\bar{c}}(x,Q^2)|_{Q^2\gg m^2}$ for $Q^2 \gtrsim 10\, m_c^2$. For these higher values of Q^2, one can calculate the heavy flavour Wilson coefficients, the perturbative part of the structure functions $F_2(x,Q^2)$ and $F_{\rm L}(x,Q^2)$, analytically, which has been

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done for $F_2(x,Q^2)$ to 2-loop order in [3,4] and for $F_{\rm L}(x,Q^2)$ to 3-loop order in [5]. First steps towards an asymptotic 3-loop calculation for $F_2^{c\bar{c}}(x,Q^2)$ have been done by the present authors by calculating the first $\mathcal{O}(\varepsilon)$ terms of the 2-loop diagrams [6], contributing to 3-loop heavy flavour Wilson coefficients via renormalisation. We report here on further steps towards a full 3-loop calculation for the moments of the heavy flavour Wilson coefficients.

2. Heavy flavour Wilson coefficients in the limit $Q^2 \gg m^2$

On the twist-2 level, the structure functions can be expressed as a convolution of perturbatively calculable Wilson coefficients and the non-perturbative parton densities. We consider here the heavy flavour contributions to these Wilson coefficients, the heavy flavour Wilson coefficients. In the region $Q^2 \gg m^2$, one can use the massive renormalisation group equation to obtain all non-power corrections to these heavy flavour Wilson coefficients as convolutions of massless Wilson coefficients $C_k(Q^2/\mu^2)$ and massive operator matrix elements (OMEs) $A_{ij}(\mu^2/m^2)$ [1]. The light Wilson coefficients are known by now up to three loops [7] and carry all the process dependence. The operator matrix elements, on the other hand, are universal, process-independent objects, which are calculated as flavour decomposed operators in the light-cone expansion between partonic states. Both objects have an expansion in $\alpha_{\rm s}$.

3. Massive operator matrix elements

In order to perform the 3-loop calculation of the OMEs, one has to first calculate the bare quantities and then to renormalise them, where they need to be mass- and charge-renormalised and contain ultraviolet (UV) and collinear divergences. The mass renormalisation is done in the on-shell scheme [8,9], whereas the charge renormalisation is done using the $\overline{\rm MS}$ scheme. After mass- and charge-renormalisation, the remaining UV-divergences are accounted for by operator renormalisation via Z-factors, Z_{ij}, and the collinear divergences via mass factorisation, multiplying by Γ_{ij}. The Z-factors are given by the generic formula:

$$
\begin{aligned}
Z_{ij}(N,a_{\rm s},\varepsilon) = {}& \delta_{ij} + a_{\rm s} S_\varepsilon \frac{\gamma_{ij,0}}{\varepsilon} + a_{\rm s}^2 S_\varepsilon^2 \left\{ \frac{1}{\varepsilon^2}\left[\frac{1}{2}\gamma_{im,0}\gamma_{mj,0} + \beta_0\gamma_{ij,0}\right] + \frac{1}{2\varepsilon}\gamma_{ij,1} \right\} \\
& + a_{\rm s}^3 S_\varepsilon^3 \left\{ \frac{1}{\varepsilon^3}\left[\frac{1}{6}\gamma_{in,0}\gamma_{nm,0}\gamma_{mj,0} + \beta_0\gamma_{im,0}\gamma_{mj,0} + \frac{4}{3}\beta_0^2\gamma_{ij,0}\right] \right. \\
& \left. + \frac{1}{\varepsilon^2}\left[\frac{1}{6}(\gamma_{im,1}\gamma_{mj,0} + 2\gamma_{im,0}\gamma_{mj,1}) + \frac{2}{3}(\beta_0\gamma_{ij,1} + \beta_1\gamma_{ij,0})\right] + \frac{\gamma_{ij,2}}{3\varepsilon} \right\},
\end{aligned}
$$

which has to be adapted for the various flavour decomposed combinations. The indices i, j here either run over $i, j \in \{q, g\}$ or denote the non-singlet combinations. The pure-singlet Z-factor is given by: $Z_{qq}^{\rm PS} = Z_{qq} - Z_{\rm NS}$. $\gamma_{ij,k}$ are the $(k+1)$-loop anomalous dimensions and β_i denote the expansion coefficients of the β-function. The transition functions Γ_{ij} remove the collinear singularities.

Let us consider, *e.g.*, the renormalised matrix element A_{Qg}, where $\hat{A}_{Qg}$ denotes the mass- and charge-renormalised expression. One finds then:

$$
\begin{aligned}
A_{Qg} &= Z_{qq}^{-1}\hat{A}_{Qq}^{\rm PS}\Gamma_{qg}^{-1} + Z_{qq}^{-1}\hat{A}_{Qg}\Gamma_{gg}^{-1} + Z_{qg}^{-1}\hat{A}_{gq,Q}\Gamma_{qg}^{-1} + Z_{qg}^{-1}\hat{A}_{gg,Q}\Gamma_{gg}^{-1}\,,\\
A_{Qg}^{(2)} &= \hat{A}_{Qg}^{(2)} + Z_{qq}^{-1,(1)}\hat{A}_{Qg}^{(1)} + Z_{qg}^{-1,(1)}\hat{A}_{gg,Q}^{(1)} + Z_{qg}^{-1,(2)} + \left(\hat{A}_{Qg}^{(1)} + Z_{qg}^{-1,(1)}\right)\Gamma_{gg}^{-1,(1)}\,.
\end{aligned}
$$

As the general expression in the first line already indicates, there is a mixing with $\hat{A}_{Qq}^{\rm PS}$ and $\hat{A}_{gg,Q}$ from $\mathcal{O}(\alpha_{\rm s}^3)$, while $\hat{A}_{gq,Q}$ starts contributing from $\mathcal{O}(\alpha_{\rm s}^2)$ and therefore at $\mathcal{O}(\alpha_{\rm s}^4)$ to $\hat{A}_{Qg}$. For $A_{Qg}^{(2)}$ given in the second line, one finds the $\mathcal{O}(\varepsilon)$ term of the gluonic one-loop OME, $\overline{a}_{Qg}^{(1)}$, entering the two-loop expression via renormalisation, as described for example in [3, 4, 6]. In the same way, for the renormalisation of $A_{Qg}^{(3)}$, the $\mathcal{O}(\varepsilon)$-term of the two-loop expression $\overline{a}_{Qg}^{(2)}$ is needed — as are the terms $\overline{a}_{gg}^{(2)}$, $\overline{a}_{Qq}^{(2),\rm PS}$ due to the above mentioned operator mixing, and the $\mathcal{O}(\varepsilon^2)$ of the one-loop $A_{Qg}^{(1)}$. The calculation of these $\mathcal{O}(\varepsilon)$ terms is a first step towards a 3-loop calculation, *cf.* Sec. 3.1., while first calculations of 3-loop diagrams for fixed Mellin N are described in Sec. 3.2.

As a last remark, note that we consider charm quark contributions here, while for heavier quarks decoupling [10] has to be applied.

3.1. Two-loop diagrams to $\mathcal{O}(\varepsilon)$ for general Mellin N

Our calculation is performed in Mellin space, where the convolution of functions becomes a simple product. The $\mathcal{O}(\varepsilon)$ terms for the unpolarised gluonic OMEs, as for the pure-singlet and non-singlet cases, have been given in [6] for general Mellin N. The corresponding polarised contributions are to be published soon. The calculation is performed in two ways: on the one hand, we rewrote the OMEs in terms of Mellin–Barnes integrals and used the package MB [11] to obtain numeric results, serving as a check for the analytic results, which have been obtained expressing the OMEs as generalised hypergeometric functions. Expanding these functions in ε, one has to sum the expression for the desired order, which we did using integral techniques and SIGMA [12]. The results are then given in terms of nested harmonic sums [13, 14], to which we applied algebraic and analytic relations [15, 16] to find the most compact representation possible.

The term $\hat{A}_{gg,Q}^{(2)}$ has been newly calculated and is given up to order $\mathcal{O}(\varepsilon)$ by:

$$
\begin{aligned}
\hat{A}_{gg,Q}^{(2)} = & T_{\rm F}C_{\rm A}\Biggl\{\frac{1}{\varepsilon^2}\Biggl(-\frac{32}{3}S_1+\frac{64(N^2+N+1)}{3(N-1)N(N+1)(N+2)}\Biggr) \\
& +\frac{1}{\varepsilon}\Biggl(-\frac{80}{9}S_1+\frac{16P_1}{9(N-1)N^2(N+1)^2(N+2)}\Biggr) \\
& +\Biggl(-\frac{8}{3}\zeta_2S_1+\frac{16(N^2+N+1)\zeta_2}{3(N-1)N(N+1)(N+2)} \\
& -4\frac{56N+47}{27(N+1)}S_1+\frac{2P_3}{27(N-1)N^3(N+1)^3(N+2)}\Biggr) \\
& +\varepsilon\Biggl(-\frac{8}{9}\zeta_3S_1-\frac{20}{9}\zeta_2S_1-\frac{S_1^2}{3(N+1)}+\frac{16(N^2+N+1)\zeta_3}{9(N-1)N(N+1)(N+2)} \\
& +\frac{P_5}{81(N-1)N^4(N+1)^4(N+2)}+\frac{4P_1\zeta_2}{9(N-1)N^2(N+1)^2(N+2)} \\
& -2\frac{328N^4+256N^3-247N^2-175N+54}{81(N-1)N(N+1)^2}S_1+\frac{2N+1}{3(N+1)}S_2\Biggr)\Biggr\} \\
& +T_{\rm F}C_{\rm F}\Biggl\{\frac{1}{\varepsilon^2}\Biggl(\frac{16(N^2+N+2)^2}{(N-1)N^2(N+1)^2(N+2)}\Biggr)+\frac{1}{\varepsilon}\Biggl(\frac{4P_2}{(N-1)N^3(N+1)^3(N+2)}\Biggr) \\
& +\Biggl(\frac{4(N^2+N+2)^2\zeta_2}{(N-1)N^2(N+1)^2(N+2)}-\frac{P_4}{(N-1)N^4(N+1)^4(N+2)}\Biggr) \\
& +\varepsilon\Biggl(\frac{4(N^2+N+2)^2\zeta_3}{3(N-1)N^2(N+1)^2(N+2)}+\frac{P_2\zeta_2}{(N-1)N^3(N+1)^3(N+2)} \\
& +\frac{P_6}{4(N-1)N^5(N+1)^5(N+2)}\Biggr)\Biggr\}
\end{aligned}
$$

$$
\begin{aligned}
P_1 &= 3N^6+9N^5+22N^4+29N^3+41N^2+28N+6, \\
P_2 &= N^8+4N^7+8N^6+6N^5-3N^4-22N^3-10N^2-8N-8, \\
P_3 &= 15N^8+60N^7+572N^6+1470N^5+2135N^4+1794N^3+722N^2-24N-72, \\
P_4 &= 15N^{10}+75N^9+112N^8+14N^7-61N^6+107N^5+170N^4+36N^3-36N^2-32N-16, \\
P_5 &= 3N^{10}+15N^9+3316N^8+12778N^7+22951N^6+23815N^5+14212N^4+3556N^3 \\
& \quad -30N^2+288N+216, \\
P_6 &= 31N^{12}+186N^{11}+435N^{10}+438N^9-123N^8-1170N^7-1527N^6-654N^5 \\
& \quad +88N^4-136N^2-96N-32.
\end{aligned}
$$

We agree to constant order with the result of [17]. Even the all order ε result, which is solely given in terms of Euler Γ and ψ, could be derived. This expression is also needed in the context of the variable flavour number scheme.

In the unpolarised case, all 2-loop $\mathcal{O}(\varepsilon)$ terms are now known. In the polarised case, the calculation proceeds in the same way and we calculated so far the gluonic, pure-singlet and non-singlet terms, which will be published soon [6].

3.2. Fixed values of N at three loops

As a next step towards a full $\mathcal{O}(\alpha_s^3)$ calculation, we started calculating unpolarised three-loop OMEs $A_{ij,Q}^{(3)}$ for fixed values of Mellin N. The contributing OMEs are: singlet: $\{A_{Qg},\ A_{gg,Q},\ A_{gq,Q}\}$, pure-singlet: $A_{Qq}^{\rm PS}$, non-singlet: $\{A_{qq,Q}^{\rm NS,+},\ A_{qq,Q}^{\rm NS,-},\ A_{qq,Q}^{\rm NS,v}\}$, where we have operator mixing between the singlet and pure-singlet terms. The first object of investigation is the gluonic $A_{Qg}^{(3)}$: The necessary three-loop diagrams are generated using QGRAF [18], where the operator product expansion has been implemented up to insertions of operators with three and four gluonic lines. The number of diagrams contributing to $A_{Qg}^{(3)}$, *e.g.*, is 1478 diagrams with one and 489 diagrams with two quark loops, where at least one of the loops is heavy.

The steps for the calculation of these self-energy type diagrams with one additional scale set by the Mellin variable N, are the following: The diagrams are genuinely given as tensor integrals due to the operators contracted with the light-cone vector Δ, $\Delta^2 = 0$. The idea is, to first undo this contraction and to develop a projector, which, applied to the tensor integrals, provides the results for the diagrams for a specific (even) Mellin N under consideration. So far, we implemented the projector for the first 4 contributing Mellin moments N, $N = 2, \ldots, 8$, where the color factors are calculated using [19]. The diagrams are then translated into a form, which is suitable for the program MATAD [20], which does the expansion in ε for the remaining massless and massive three-loop tadpole-type diagrams. We have implemented these steps into a FORM [21] program and tested it against two-loop results and the all-order ε result of $A_{gg,Q}^{(2)}$ and found agreement. We then turned to a subset of the 3-loop diagrams, the diagrams $\propto T_{\rm F}^2$. The contributions $\propto d_{abc}d^{abc}$ are found to vanish. Currently we investigate $T_{\rm F}^2 C_{\rm F}$, $T_{\rm F}^2 C_{\rm A}$.

4. Conclusions and outlook

We calculated the $\mathcal{O}(\varepsilon)$ contributions to heavy flavour Wilson coefficients for general Mellin variable N at $\mathcal{O}(\alpha_s^2)$, as a first step towards a $\mathcal{O}(\alpha_s^3)$ calculation. Furthermore, we installed a program chain to calculate the corresponding 3-loop diagrams to $\mathcal{O}(\alpha_s^3)$, with the help of MATAD. This chain is now existing and we expect first results in the near future.

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