VISCOSITY AND BOOST INVARIANCE AT RHIC AND LHC* **

Piotr Bożek

The H. Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences
Radzikowskiego 152, 31-342 Kraków, Poland
and
Institute of Physics, Rzeszów University
Al. Rejtana 16 A, 35-959 Rzeszów, Poland
Piotr.Bozek@ifj.edu.pl

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We consider the longitudinal hydrodynamic evolution of the fireball created in a relativistic heavy-ion collision. Nonzero shear viscosity reduces the cooling rate of the system and hinders the acceleration of the longitudinal flow. As a consequence, the initial energy density needed to reproduce the experimental data at RHIC energies is significantly reduced. At LHC energies, we expect that shear viscosity helps to conserve a Bjorken plateau in the rapidity distributions during the expansion.

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1. Introduction

In heavy-ion collisions at relativistic energies a fireball of dense and hot matter is created. In later stages, the fireball expands and eventually hadronizes. In order to deduce the properties of that medium, including a possible phase transition to the quark–gluon plasma, a careful modeling of the evolution of the system is needed. With the assumption of local thermal equilibrium, the description of the dynamics of the system can be undertaken within the relativistic hydrodynamics [1]. Hydrodynamic calculations reproduce the spectra of particles in the transverse momentum, the collective flow, and the Hanbury–Brown–Twiss radii measured at Relativistic Heavy

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Ion Collider (RHIC) [2]. Such calculations are essential in determining the correct equation of state of the matter under such extreme conditions, the initial conditions of the system and the freeze-out time.

Until recently, hydrodynamic calculations were performed assuming the applicability of the ideal fluid limit. It means that local equilibration processes are instantaneous, and no entropy is produced in the hydrodynamic stage. Relaxing the ideal fluid assumption amounts to the introduction of viscous effects in the evolution. For the fireball created at RHIC and Large Hadron Collider (LHC) energies, the most important dissipative effect in the hydrodynamic evolution is due to the shear viscosity [3–8]. A nonzero shear viscosity coefficient causes the saturation of the elliptic flow, a stronger transverse flow and can lead to a significant entropy production. Most of the calculations of the hydrodynamic evolution of the fireball with viscosity are done in the transverse directions only, with boost-invariance assumed in the beam (longitudinal) direction. However, shear viscosity is also important for the longitudinal expansion of the fireball [9]. In the following, we present results of calculations in a $1+1$ dimensional geometry of a non-boost-invariant expanding fluid with viscosity. The cooling rate and the acceleration of the longitudinal flow are reduced, and the entropy is produced in the expansion.

2. Longitudinal expansion with viscosity

Relativistic hydrodynamics with viscosity can be formulated consistently, without violating the causality [10]. The hydrodynamic equations

$$\partial_\mu T^{\mu\nu} = 0 \tag{1}$$

are modified; the energy-momentum tensor $T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \pi^{\mu\nu}$ is composed of the energy-momentum tensor of an ideal fluid and a stress tensor $\pi_{\mu\nu}$ describing deviations from local equilibrium. Assuming the dependence of the densities only on the time t and the longitudinal coordinate z and restricting the flow velocity $u^\mu = (\gamma, 0, 0, \gamma v)$ only to the beam direction, one can write the hydrodynamic equations for a fluid with shear viscosity as [9]

$$\begin{aligned} (\epsilon + p) DY &= -\mathcal{K}p + \mathit{\Pi} DY + \mathcal{K}\mathit{\Pi}\,, \\ D\epsilon &= (\epsilon + p)\mathcal{K}Y - \mathit{\Pi}\mathcal{K}Y\,, \\ D\mathit{\Pi} &= \frac{\left(\frac{4}{3}\eta\mathcal{K}Y - \mathit{\Pi}\right)}{\tau_\pi}\,, \end{aligned} \tag{2}$$

where

$$\begin{aligned} D &= u^\mu\partial_\mu = \cosh(Y-\theta)\partial_\tau + \frac{\sinh(Y-\theta)}{\tau}\partial_\theta\,, \\ \mathcal{K} &= \sinh(Y-\theta)\partial_\tau + \frac{\cosh(Y-\theta)}{\tau}\partial_\theta\,. \end{aligned}$$

$\theta = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$ is the space-time rapidity and $Y = \frac{1}{2}\ln\left(\frac{E+zp_z}{E-p_z}\right)$ is the kinematic rapidity of a fluid element. Eqs. (2) involve four unknown function of θ and of the proper time $\tau = \sqrt{t^2 - z^2}$: the energy density ϵ, the pressure p, the rapidity Y of the fluid element, and the shear correction Π (in the 1+1 dimensional geometry the stress tensor $\pi^{\mu\nu}$ reduces to one independent scalar function Π). One additional relation is given by the equation of state connecting ϵ and p. We use a realistic equation of state by Chojnacki and Florkowski combining lattice QCD results and a hadron gas model [11]. The ratio of the shear viscosity coefficient η to the entropy of the fluid s is not known. We perform calculation for several values $\eta/s = 0.1$–0.3 of this parameter. For the relaxation time of dissipative corrections we take $\tau_\pi = 6\eta/Ts$ [12]. The hydrodynamic equations (2) are solved in the τ–θ plane starting from initial conditions at $\tau_0 = 1$ fm/c

$$\begin{aligned} \epsilon(\tau_0, \theta) &= \epsilon_0 \exp\left(-\frac{\theta^2}{2\sigma^2}\right) , \\ Y(\tau_0, \theta) &= \theta , \end{aligned} \tag{3}$$

i.e. a Gaussian initial energy density profile and the Bjorken collective longitudinal flow of the fluid.

3. RHIC results

Eqs. (2) are solved numerically and the freeze-out hypersurface of constant temperature $T_{\rm f} = 165$ MeV is extracted. At the freeze-out hypersurface, hadrons are emitted according to the Cooper–Frye formula [4, 14]

$$\frac{dN_{\rm visc}}{dy} = \frac{dN}{dy} + \frac{d\delta N}{dy} . \tag{4}$$

with the usual thermal emission contribution of the form

$$\begin{aligned} \frac{dN}{dy} &= \frac{S}{4\pi^2} \int\limits_{-\theta_{\rm max}}^{\theta_{\rm max}} \Big(\tau(\theta)\cosh(y-\theta) - \tau^{'}(\theta)\sinh(y-\theta)\Big) \\ &\quad \times (2m\xi + 2\xi^2 + m^2)\xi \exp\left(-\frac{m\cosh(y - Y_{\rm f}(\theta))}{T_{\rm f}}\right) d\theta , \end{aligned} \tag{5}$$

where m is the meson mass, $Y_{\rm f}(\theta) = Y(\tau(\theta), \theta)$ is the fluid rapidity at the freeze-out hypersurface, and

$$\xi = \frac{T_{\rm f}}{\cosh(y - Y_{\rm f}(\theta))} ; \tag{6}$$

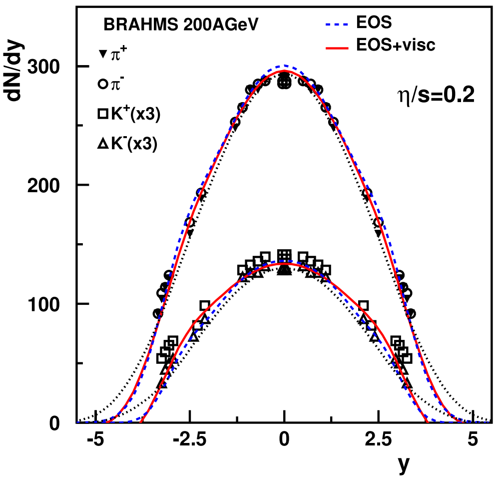

Fig. 1. Rapidity distributions for pions and kaons calculated with the ideal fluid hydrodynamics (dashed line) and using the viscous hydrodynamics with $\eta/s = 0.2$ (solid line) [9]. The dotted line denotes the results of the viscous hydrodynamic evolution, but neglecting the viscous corrections to the particle emission at freeze-out (Eq. (7)). Data are from the BRAHMS Collaboration [13].

and an additional term due to the viscous corrections [4]

$$\begin{aligned}\frac{d\delta N}{dy} &= \frac{S}{4\pi^2}\int\limits_{-\theta_{\max}}^{\theta_{\max}} \Big(\tau(\theta)\cosh(y-\theta) - \tau^{'}(\theta)\sinh(y-\theta)\Big) \\ &\times \left[12\xi^5 + 5\xi^3 m^2 + 12\xi^4 m + \xi^2 m^3 - \sinh(y - Y_{\mathrm{f}}(\theta))\right. \\ &\times \left.(24\xi^5 + 12\xi^3 m^2 + 24\xi^4 m + 4\xi^2 m^3 + \xi m^4)\right] \\ &\times \frac{\Pi}{2T^2(\epsilon+p)} \exp\left(-\frac{m\cosh(y - Y_{\mathrm{f}}(\theta))}{T_{\mathrm{f}}}\right) d\theta\,. \end{aligned} \tag{7}$$

At the freeze-out temperature, most of the pions and kaons come from secondary decays of heavier resonances. This effect is taken into account by a factor, equal to the ratio of all mesons to primary mesons [15], multiplying the calculated distributions. The parameters of the initial distribution σ and ϵ_0 are adjusted to reproduce the pion and kaon rapidity distributions measured by the BRAHMS Collaboration [13]. With the increase of the shear viscosity of the fluid, two effects can be observed (Fig. 2) in the retuned initial state of the fireball:

- a reduced initial energy density of the fireball

- and a small increase of the width of the initial distribution σ with increasing viscosity.

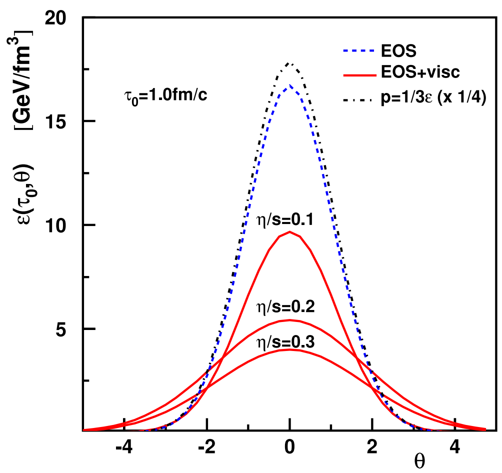

Fig. 2. Initial energy density distributions for the ideal fluid hydrodynamic evolution with a realistic equation of state (dashed line), for several viscous hydrodynamic evolutions (solid lines), and for the ideal fluid with a relativistic gas equation of state (dashed-dotted line) [9]. All distributions after the hydrodynamic evolution and freeze-out give pions distributions close to the BRAHMS measurements [13].

The two above mentioned effects are related to the change of the dynamics induced by the shear viscosity. The reduction of the initial energy density at central rapidities is related to the reduced cooling rate in the viscous evolution. It originates from the smaller work of the fluid in the longitudinal viscous expansion, due to the change of the effective pressure from p to $p - \Pi$ (first equation in (2)). The second effect is related to the acceleration of the longitudinal flow, as given by the second equation in (2). Gradients of the pressure in the longitudinal direction cause the acceleration of the flow, which becomes faster than the Bjorken one [16]. In the viscous evolution, the gradients of the pressure p are reduced by the gradients of the shear correction Π. As a result, the flow at the freeze-out is still Bjorken-like for the viscosity $\eta/s = 0.2$ (Fig. 3), and is significantly slower than for the ideal fluid. Fast moving fluid elements emit hadrons far in the forward and background rapidities, to counteract this effect a narrower initial energy density distribution in rapidity must be assumed for the ideal fluid.

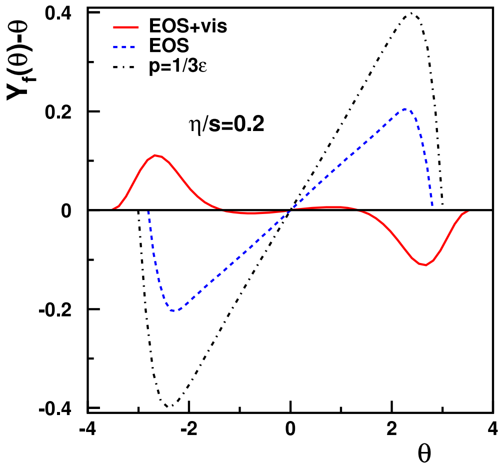

Fig. 3. Difference between the flow rapidity of the fluid and the Bjorken flow taken at the freeze-out hypersurface, calculated for the evolution with the shear viscosity coefficient $\eta/s = 0.2$ (solid line), for the ideal fluid with a realistic equation of state (dashed line) and for the ideal fluid with a relativistic gas equation of state (dashed-dotted line) [9].

4. Predictions for LHC

For the forthcoming LHC Pb–Pb heavy-ion experiments, we assume arbitrarily that the particle multiplicities at central rapidities would increase twice compared to Au–Au at the highest RHIC energies. This constraint serves to fix the initial energy density ϵ_0 for the hydrodynamic evolution. At RHIC energies, experimental results show that a Bjorken scaling plateau at central rapidities, if existing at all, is very narrow [9, 16, 17]. At LHC energies, we consider two different scenarios: a Gaussian initial energy density distribution in the space-time rapidity (similar as for RHIC), or a distribution with a plateau of width $2\sigma_p$ at central rapidities

$$\epsilon(\tau_0, \theta) = \epsilon_0 \exp\left(-(\theta - \sigma_p)^2 \Theta(|\theta| - \sigma_p)/2\sigma^2\right) . \tag{8}$$

The initial energy density distributions for the two scenarios are shown in Fig. 4, both for the ideal fluid and for the viscous hydrodynamics with $\eta/s = 0.2$. The initial energy density is smaller for nonzero viscosity. The reduction of the initial density accounts for the slower cooling and the entropy production in the later viscous evolution.

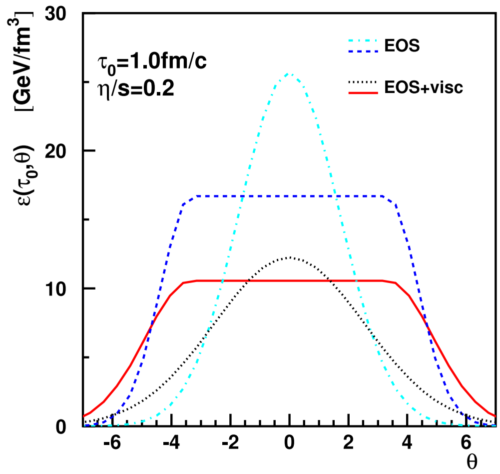

Fig. 4. Initial energy density distributions for ideal fluid hydrodynamic evolutions (dashed and dashed-dotted lines) and for viscous hydrodynamic evolutions (solid and dotted lines). All distributions after the hydrodynamic evolution and freeze-out give the same pion density at central rapidity.

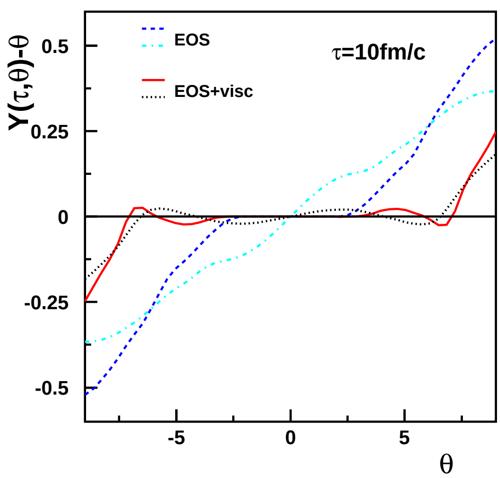

Fig. 5. Difference between the flow rapidity of the fluid and the Bjorken flow taken at $\tau = 10$ fm/c, calculated for evolutions with shear viscosity coefficient $\eta/s = 0.2$ (solid and dotted lines) and for ideal fluid evolutions (dashed and dashed-dotted lines) for LHC initial conditions (Fig. 4).

The scenario with a Bjorken scaling plateau in the range of 8 units of central rapidities leads to different results than the one with the Gaussian initial density profile. Within the plateau region, the Bjorken scaling flow is stable, the pressure gradient in space-time rapidity is zero. During the evolution, the plateau region is becoming narrower, as the gradient of the pressure at the edges of the plateau starts to destroy the scaling form of the density and of the flow. If the shear viscosity is large, the rate at which the Bjorken scaling region is reduced is smaller than in the ideal fluid evolution. As a consequence, at the freeze-out, a substantial region with Bjorken scaling of the density and flow survives, if it is present in the initial state and if the shear viscosity is large (Fig. 5). For the final meson distribution a plateau at central rapidities is possible only for the viscous evolution from an initial condition with a plateau in the initial energy density distribution (solid line in Fig. 4).

5. Summary

We calculate the evolution of the matter created in relativistic heavy-ion collisions in the longitudinal direction both for the ideal and for the viscous fluid hydrodynamics [9]. Starting with Gaussian profiles of the energy density in space-time rapidity and with a Bjorken scaling longitudinal flow, the hydrodynamic evolution reduces the energy density and accelerates the longitudinal flow. The rate of these processes is governed by gradients of the effective pressure $p - \Pi$, therefore nonzero shear viscosity corrections Π reduce the cooling rate and, at the same time, make the longitudinal flow to stay closer to the initial Bjorken scaling flow. At RHIC energies, comparison to meson rapidity distributions of the BRAHMS Collaboration [13] allows to constraint the parameters of the initial state. Depending on the value of the shear viscosity coefficient one gets a reduction of the initial energy density by a factor 2–3 for $\eta/s = 0.1$–0.2. At LHC energies, the same effects take place for similar Gaussian initial conditions. Assuming an initial distribution with a plateau at central rapidities, where the Bjorken scaling solution applies, the dynamics is different. Within the plateau region the Bjorken flow is stable, both in the ideal and viscous hydrodynamics. Nonzero viscosity helps to preserve the Bjorken plateau in a wider region of rapidities trough the evolution; reduced gradients in the hydrodynamic equations make smaller the rate at which the Bjorken plateau diminishes. For $\eta/s = 0.2$, the plateau region remaining till freeze-out is wide enough to survive the hadron emission process, and could be visible as a plateau in the final meson distribution in the kinematic rapidity.

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