

RADIATIVE CORRECTIONS TO DIS<sup>\* \*\*</sup>

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Early deep inelastic scattering (DIS) experiments at SLAC discovered partons, identified them as quarks and gluons, and restricted the set of the candidate theories for strong interactions to those exhibiting the asymptotic freedom property. The next generation DIS experiments at FNAL and CERN confirmed the predictions of QCD for the size of the scaling violation effects in the nucleon structure functions. The QCD fits to their data resulted in determining the momentum distributions of the point-like constituents of nucleons. Interpretation of data coming from all these experiments and, in the case of the SLAC experiments, even an elaboration of the running strategies, would not have been possible without a precise understanding of the electromagnetic radiative corrections. In this note I present my personal recollection of the important milestones, achieved in the period preceding the HERA era, in the high precision calculations of the radiative corrections to DIS, and in the development of the methods of their experimental control. I present subsequently the measurement strategies and discuss the advanced radiative correction tools used in the initial phase of the HERA experimental program, with an emphasis on their role in the first, model independent, measurement of the partonic densities in the small- $x_{Bj}$  region.

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**1. Introduction**

DIS cross-sections, in particular those measured using the charged lepton beams, contain large contributions coming from the higher order QED processes. These contributions must be subtracted in order to interpret the measurements in terms of the nucleon structure functions.

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Let me recall the following two examples which illustrate the importance of the radiative corrections to DIS. The first one is the measurement of the nucleon photo-absorption cross-sections for longitudinally and transversely polarised virtual photons. The kinematical dependence of their ratio,  $R$ , is driven by the spin of the nucleon constituents. It is sensitive to the diquark–quark structure of the nucleons and, for the large four-momentum transfer  $Q$ , to the intrinsic transverse momenta of the quarks. The precision of measuring the kinematical dependence of  $R$  was improved by the SLAC E140 experiment [1] to such an extent, that the overall measurement errors were no longer determined by the experimental ones, but by the uncertainties in the size of the radiative corrections [2]. Another illustrative example of the importance of radiative corrections to DIS is the comparison of the size of the QCD scaling violation effects and the radiative correction effects in the proton structure function  $F_2$  at small  $x_{\text{Bj}}$ . In this region the effects of the radiative corrections are particularly large — they are of the same magnitude as those reflecting a change by 200 MeV of the  $\Lambda_{\text{QCD}}$  value [3].

What is the main reason for such a large sensitivity of the DIS observables to the size of the radiative corrections? In each of the early electron-beam DIS scattering experiments at SLAC and DESY, as well as in the next generation muon-beam DIS experiments at CERN and FNAL, the kinematic variables were reconstructed using the initial lepton energy, the scattered lepton momentum and its scattering angle. Within such a reconstruction scheme the *apparent* value of the four-momentum transfer from the lepton to the nucleon,  $Q_{\text{r}}$ , could differ significantly from the *true* four-momentum transfer,  $Q_{\text{t}}$ , in the presence of untagged, hard photon radiation processes. Due to a steep rise of the DIS cross-section in the region of small  $Q_{\text{t}}$  the contribution of the hard-photon radiative processes were found to be of similar size as the contribution of the Born process. This “kinematical bias” was the dominant source of the large radiative corrections.

At HERA, where the leptonic radiative corrections were expected to be even larger, three complementary strategies of reducing their impact on the overall measurement precision were planned [3] and realized [4]. The first one was to apply the same correction procedure as for the early SLAC experiments. This procedure required a precise modelling of the Born cross-section in the small  $Q_{\text{t}}^2$  region, and the development of dedicated methods of its experimental control, *e.g.* by using the Compton events [5]. The second one was to use the capacity of the HERA detectors in direct (zero-angle photon calorimeter), or indirect (hadronic energy flow) detection of hard photons radiated in the angular region collinear to the incoming electron direction. The third one was to determine the event kinematic variables using (entirely or partially) the hadronic final state observables.

Each of the above three methods required high-precision calculations of the radiative corrections and their subsequent implementation in a form which was easily applicable to the data analysis procedures. This note brings into light important milestones in this domain. It tries to pay a tribute to those who provided indispensable tools for the DIS experimental program. Their painstaking work, often hardly visible in the bulk of the published experimental results was of primordial importance for the analysis of the DIS data. It enabled a precision mapping of the partonic distribution functions — a base for understanding the wide-band-partonic beams which will be used by the LHC experimental program.

## 2. The tools and the methods for the SLAC, FNAL and CERN DIS experiments

The early SLAC experiments were confronted with the following three challenges. Firstly, in order to increase the observed event rate, the collision data were collected with relatively thick (couple-of-percent-radiation-length-long) targets. Therefore, the radiative corrections to DIS (internal radiative corrections) had to be calculated simultaneously with the target-length-dependent external radiative corrections. In the SLAC kinematic region the effect of internal radiative corrections was roughly of the same magnitude as the effect of the external ones. The calculations of the latter was controlled experimentally by using the variable thickness targets [6]. Secondly, the deep inelastic cross-section for absorption of transverse and longitudinal photons were not known at the time when the first measurements were done. The unfolding of the radiatively corrected observables required thus a global iterative procedure, and could not be confined to the local correction factors. Thirdly, and perhaps most importantly, the available computing power was limited and several approximations had to be made while calculating the size of radiative corrections.

Given all these constraints, the early DIS experiments at SLAC were using various peaking approximations based on the formulae of Mo and Tsai [7]. The combined effect of the external and the internal radiative corrections was calculated using the equivalent radiator method in which the radiative corrections were approximated by adding two hypothetical radiators, of the  $Q^2$ -dependent length, one in front and one behind the vertex position. In the numerical integration procedures the energy peaking approximation was used. Such an approximation was indispensable to reduce the integration domains to the strips along the energies of the initial and the scattered electron.

For the next generation SLAC experiments, and for the muon-beams CERN and FNAL experiments, the above approximations turned out to be the dominant overall precision-limiting factors. Therefore, new methods had

to be developed. From the theoretical side, the most important development for the precision DIS program came from Bardin and his Dubna group [8]. They calculated the full set of internal radiative corrections including the hadronic corrections, the electroweak effects, and the soft photon exponentiation effects. These calculations, implemented within their TERAD86 program, provided a very important tool to verify the precision of several approximations, present in the calculations of the internal radiative corrections for the SLAC DIS experiments. Several precision-limiting approximations in the procedures based on the Mo-Tsai formulae were identified [9, 10]. In addition, the increased power of the SLAC computing facilities allowed avoidance of the approximations made in the calculations of the combined effects of the internal and external corrections. The improvement in the precision of the radiative corrections achieved using new calculation procedures [9] triggered the reanalysis of the early SLAC DIS data resulting in a consistent picture of the structure functions measured at SLAC and at CERN.

### 3. The tools and the methods for the HERA DIS program

For the HERA research program, novel theoretical tools and novel experimental methods of controlling the processes of hard photon emission had to be developed in order to cope with large radiative corrections.

Already before the startup of HERA, experimentalists were well equipped with the new generation computer-programs for radiative corrections (both for the analytical calculations and for the Monte Carlo generation of radiative processes) [11], and with the HERA-specific methods of application of these corrections to the experimental data [3].

The TERAD91 program by Bardin *et al.* [12] had its roots in the TERAD86 program applied successfully to the earlier DIS experiments. It included, as well, the code for the analytical calculations of the complete electroweak corrections to the NC and CC scattering in the quark-parton model (DISEPNC and DISEPCC). The EPRC91 program by Spiesberger *et al.* [13], was a package of programs for the calculations of complete electromagnetic and weak corrections to the DIS NC and CC scattering. The above two programs were tested using identical input structure functions and shown to agree with each other at the level below 1%.

The HERA experiments H1 and ZEUS were the first DIS experiments capable of measuring the hadronic flow associated with the CC and NC collisions. This allowed us to introduce novel reconstruction methods for the DIS kinematical variables and novel methods of applying the radiative corrections to the measured observables. One of the most important aspect of these methods was to go beyond the classical scheme, in which the radiative

corrections were applied at the last analysis step — *i.e.* after correcting for the measurements effects — by introducing the methods in which unfolding of the experimental effects was done simultaneously with correcting the data for radiative effects. In order to implement such methods the development of the Monte Carlo generators involving radiative processes was indispensable.

The first attempt in this direction was the LESKO-C generator by Jadach [14]. This Monte Carlo generator was based on the collinear approximation for the processes of bremsstrahlung from the polarised initial lepton. This program evolved to the LESKO-F program [15]. It included the complete  $O(\alpha)$  QED radiative corrections to the lepton line for the NC deep-inelastic scattering. Later, in the LESKO-YFS generator, by Placzek and Jadach, the multi-photon radiation processes were added [16]. An interface of these programs to the LEPTO [22] program for parton cascades and fragmentation, the FRANEQ program by Placzek [17], allowed the generation of complete final states.

The main limitation of these generators was that they were applicable only to the processes of large four-momentum transfer to the hadronic system,  $Q_t$  — for which protons could be considered as composed of quarks and gluons. Since the reconstruction of this quantity required modelling of perturbative and non-perturbative QCD effects, and was sensitive to a precise modelling of the detector response to the produced hadrons, this program had a limiting power for high-precision unfolding of the radiative corrections to electron inclusive observables. Nevertheless, these generators were of great importance in the early phase of development of the experimental methods of controlling the radiative processes at HERA [18, 19] and while developing a novel method of measuring the proton longitudinal structure function [20].

The most important and the most widely used Monte Carlo generator for the HERA experimental program, HERACLES, was developed by Spiesberger *et al.* [21]. The HERACLES program allowed a separate treatment of the Born term and the radiative terms (soft and hard bremsstrahlung and the corresponding virtual corrections). It included the Compton part and quarkonic radiation. Users were allowed to use any parameterisation of the input structure functions including the longitudinal structure function. For those of users for whom the quark-parton modelling of the proton structure was sufficient, the interface DJANGO to the LEPTO and JETSET programs [22] was provided.

The HERA experiments were the first DIS experiments in which the radiative hard photons could be identified and measured [4]. Events in which photons were emitted in the angular range of  $\sim 0.45$  mrad with respect to the incoming electron direction, and events in which the radiative photon transverse momentum balanced (to a similar precision) that of the scattered

electron, were used to measure the machine luminosity, and to monitor the beam position and divergence at the HERA interaction points. Direct and indirect (using the hadronic flow observables) detection of radiative photons allowed an experimental cross-check of the size of radiative corrections and to measure the proton structure functions in the kinematical regions not accessible to the standard methods.

#### 4. Radiative corrections for the first HERA data

One of the first and most cited HERA result based on the data collected in the 1992 run was the first measurement of the proton charge-structure in the small- $x_{Bj}$  region [23]. The observed strong rise of the differential cross-section, if interpreted in terms of the leading twist partonic distributions indicated that, in the small- $x_{Bj}$  region, the proton momentum was carried mostly by gluons. The H1 collaboration strategy for the first measurement at HERA of the proton structure [23] anticipated that both the physics, and the detector and machine performance, could not be precisely modelled at the time of the initial measurement.

Therefore, dedicated measurement methods based on a direct control of the experimental errors and on the factorisation of the radiative correction effects from the detector performance effects had to be invented. The important aspect of the initial measurement strategy was to determine the differential cross-sections using independent methods having different sensitivity not only to the size of radiative corrections but also to the assumed shape of the differential cross-section in the unmeasured region. Such a strategy, similar to the dedicated strategies of the first DIS experiments at SLAC, assured a precise control of the radiative corrections in spite of the fact that, for the measurements based on the electron variables, they were exceeding 100% of the measured values. The implementation of such strategies was possible owing to the effort of the DESY radiative correction group<sup>1</sup> involving both the theorists and the experimentalists who prepared highly efficient tools and strategies.

#### 5. Conclusions

Precision measurements in high-energy physics, as long as they require modelling of the underlying phenomena using the framework of the perturbative quantum field theories, will always need a symbiotic effort of theorists and experimentalists. One of the most obvious targets for such a collaborative work is to prepare the high-precision theoretical tools to control the

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<sup>1</sup> The DESY radiative correction group was convened by Hubert Spiesberger and by the author of this note.

higher-order radiative corrections. Their “user-market” quality is measured presently by a simple criterion: to which extent such tools encapsulate their sophisticated technical aspects while being robust and versatile. The tools for the DIS program fulfilled perfectly this quality requirement. The measure of their success is, paradoxically, that they became, being uncontroversial, invisible in the results of the HERA experiments. The challenge for elaboration of advanced EW and QCD radiative correction tools for the LHC experimental program is, first of all, to preserve the interest of theorists to work in such a “shadow” activity which is as much unexposed as it is useful for the community of experimentalists. Staszek Jadach’s group is one of the groups of “last Mohicans” staying in this field. What must be stressed is that, for a high-precision scrutiny of the Standard Model at LHC, these tools must be elaborated in an undissociated way with preparation of the dedicated measurement strategies in which the theoretical (modelling) uncertainties and the experimental errors could be controlled independently. The latter aspect, of lesser importance for the LEP and HERA programs, is of extreme importance and urgency at LHC where the modelling of the partonic wide-band-beams and higher-order QCD correction involves a sufficient freedom to “swallow” the experimental manifestation of a wide class of novel phenomena.

These notes are dedicated to Staszek Jadach as a contribution to the celebration of his 60th birthday.

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