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HARD SCATTERING AND ELECTROWEAK CORRECTIONS AT HIGH ENERGIES*

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After a brief recollection of joint scientific work with Staszek Jadach, recent results on electroweak radiative corrections for scattering processes in the TeV region are presented. The status of the four-fermion scattering amplitudes is discussed, with emphasis on logarithmically enhanced contributions in two-loop approximation. Predictions for the production of γ, Z and W with large transverse momenta together with a jet are presented. For p_{T} above 1 TeV the electroweak corrections may well reach several tens of percent. A similar situation is observed for top-antitop quark production at large invariant mass of the $t\bar{t}$ system.

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1. Introduction

This year's Cracow Epiphany Conference serves a twofold purpose: Like every year it brings together colleagues from Cracow and abroad to stimulate discussions on recent developments in theoretical an experimental particle physics. However, in addition, this year we are celebrating Staszek Jadach's sixtieth anniversary, which gives us the opportunity to look back in time into our common past and to honor his scientific achievements.

My personal recollections go back to the Cracow–Munich meetings of the years 1976 and later, which were at that time initiated by Leo van Hove, Andrzej Białas and Kacper Zalewski. This opened for us fascinating new possibilities, at the scientific level, for new projects and collaborations, but also to get acquainted with the situation at the other side of the "iron curtain" and to get to know new colleagues and form new friendships. My friendship and collaboration with Staszek dates back to this time. Several years later we were both involved in the preparations for LEP. That is where our joint

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scientific papers start, the first ones documenting the Monte Carlo generator TIPTOP, constructed to simulate top quark production in electron–positron annihilation, a reaction which at that time was of course still considered as one of the Standard Model processes of LEP [1–3].

Later, during Staszek's stay at the Max-Planck Institute in the years 1986 and 1987 our attention shifted more and more to precision physics that started to become one of the many interesting options for LEP and SLC. In particular, the possibility of measurements with polarized beams offered the opportunity to determine the weak mixing angle with unprecedented precision, and complementary precision calculations were called for, in order to fully exploit this unique possibility [4].

Of course, instead of polarized beams, the study of tau polarization in the final state was another interesting possibility and indeed turned out to be a nearly equally powerful option. The study of semileptonic decays was not only particularly suited for the analysis of tau polarization, at the same time a richness of hadron physics was just waiting to be uncovered. To provide an adequate tool to experimentalists, the Monte Carlo generator TAUOLA was constructed (together with Zbyszek Wąs), simulating the decay of polarized tau leptons into a multitude of final states [5]. Both the conceptual and the phenomenological aspects were quite exciting, and [5], together with the subsequent developments [6,7], has turned out to be a gold mine ever since, as far as physics results are concerned and in terms of citations. Clearly, I have profited a lot from Staszek's enormous expertise in Monte Carlo programs and I am grateful for this experience.

Time goes on, and our interests have shifted. Nevertheless, we are all urgently waiting for the start of the LHC and have invested a lot of effort into its preparation. During the rest of the talk I will, therefore, briefly describe some topics in electroweak physics which have been investigated with my colleagues in Karlsruhe, and which are specifically devoted to LHC experiments.

Historically, weak phenomena have manifested themselves at low energies through effective four-fermion interactions, as expected for reactions with energy transfer far smaller than the mass of the exchanged bosons. Although much information about the structure of the charged and neutral currents was collected in the course of time, the description through this effective four-fermion coupling remained valid and the existence of the W and Z bosons was only demonstrated in the early eighties at the CERN proton–antiproton collider.

From then on a second period started, where the characteristic energies were comparable to the masses of the weak gauge bosons. Unification of electromagnetic and weak interactions became manifest and electroweak precision physics culminated in the LEP and SLC experiments.

The picture is about to change again, and with the turn-on of LHC a third period will soon emerge. Compared to characteristic energies in the TeV region, the masses of W and Z can be considered as small, "infrared" parameters and powers of $\mathcal{L}$, the large logarithm of the ratio s/m_W^2, may appear in the evaluation of exclusive and inclusive cross sections. These large logarithms may at least partially compensate the smallness of the weak coupling constant and lead to (negative) enhanced corrections.

In exclusive reactions the dominant "leading" logarithms are given by $(\alpha_W/\pi\mathcal{L}^2)^n$, where $\alpha_W/\pi \approx 0.01$ and $\mathcal{L}^2 \approx 20$ to 40 and n denotes the loop order. In addition these corrections are multiplied by the sum of the eigenvalues of the quadratic Casimir operator characterizing the representations of the external particles. All these leading terms are negative, and for the LHC and in one-loop approximation the effects may easily accumulate to ten to thirty percent, which makes the need for a sufficiently precise control of the two-loop terms evident.

These considerations have triggered a wave of interest in this so-called Sudakov asymptotic regime [8–31]. In Karlsruhe we have, on one hand, explored four-fermion processes (recently also W-pair production) in great detail, demonstrating the importance of subleading logarithms and evaluating these up to the N^{3}LL-approximation. On the other hand we have, in one-loop approximation (including only a limited part of subleading terms), studied a number of processes of phenomenological importance, like W or Z plus jet production at large transverse momenta, and top quark pair production. Results for all these reactions will be reviewed in the following.

2. Form factors and four-fermion scattering at two loops

Let us start with the analysis of four-fermion scattering where the investigation of the subleading terms has been performed at the most detailed level. In Refs. [16,17] we have extended the leading logarithmic (LL) analysis of [15]. The next-to-leading logarithmic (NLL) and next-to-next-to-leading logarithmic (NNLL) corrections to the high energy asymptotic behavior of the neutral current four-fermion processes have been resummed to all orders using the *evolution equation* approach discussed below. Only the light quark case was considered and the mass difference between the neutral and charged gauge bosons was neglected. On the basis of this result the logarithmically enhanced part of the phenomenologically important two-loop corrections to the total cross section and to various asymmetries was obtained including the $\ln^n(s/M_{W,Z}^2)$ terms with $n = 2, 3, 4$. The results up to NLL have been confirmed by the explicit one-loop [12,19,21] and two-loop [22–25,28] calculations. The subleading logarithms in the TeV region are comparable to the leading terms due to their large numerical coefficients. Thus, the calculation of the remaining two-loop quadratic and linear logarithms was necessary to control the convergence of the logarithmic expansion. These are of special interest both from the phenomenological and conceptual point of view, because, in contrast to the higher powers of the logarithm, the N^{3}LL terms are sensitive to the details of the gauge boson mass generation. The first results beyond the NNLL approximation have been obtained in [24, 29, 30].

The complete result can be found in [31], where the calculation of the two-loop logarithmic corrections to the neutral current four-fermion processes was completed and the previously neglected effects of the gauge boson mass difference incorporated. The two-loop logarithmic terms were derived within the *expansion by regions* approach [33–36] by inspecting the structure of singularities of the contributions of different regions. The calculation was significantly simplified by taking the exponentiation of the logarithmic corrections in the Sudakov limit into account. This property naturally appears and can be fully elaborated within the evolution equation approach [37–39]. To identify the pure QED infrared logarithms which are compensated by soft real photon radiation, the *hard* evolution equation which governs the dependence of the amplitudes on s was combined with the *infrared* evolution equation [15] which describes the dependence of the amplitude on an infrared regulator.

Let us now recall the strategy and the main results of these investigations in more detail:

The Form Factor and Evolution Equations

Starting from the form factor in Born approximation

$$\mathcal{F}_{\mathrm{Born}} = \bar{\psi}(p_2)\gamma_\mu\psi(p_1) \tag{1}$$

the all-order resummation of leading and subleading logarithms is based on the evolution equation

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int\limits_{M^2}^{Q^2}\frac{dx}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}\,, \tag{2}$$

with the solution

$$\mathcal{F} = F_0(\alpha(M^2))\exp\left\{\int\limits_{M^2}^{Q^2}\frac{dx}{x}\left[\int\limits_{M^2}^{x}\frac{dx'}{x'}\gamma(\alpha(x'))+\zeta(\alpha(x))+\xi(\alpha(M^2))\right]\right\}. \tag{3}$$

To obtain all non-power suppressed terms up to two loops corresponds to the reconstruction of all terms of the form $\alpha^2\mathcal{L}^i$ with $i = 0,\ldots,4$. In arbitrary orders this is quivalent to the N^{4}LL approximation, collecting all terms of the form $\alpha^n \mathcal{L}^{2n-i}$, again with $i = 0, \ldots, 4$. From Eq. (3) it is evident that the NNLL approximation can be obtained from the evaluation of the anomalous dimensions $\zeta(\alpha), \xi(\alpha)$ to one loop, $\gamma(\alpha)$ to two loop (from a massless two-loop calculation), $F_0(\alpha)$ to one loop and the lowest two coefficients of the β-function. To derive the N^{3}LL approximation, the linear logarithms have to be calculated in an explicit evaluation of the two-loop form factor and, finally, N^{4}LL requires even the constant two-loop term, which is available for the form factor in the Abelian theory only. Since these considerations are equally applicable to the four-fermion process (which are governed by the same soft and collinear logarithms) the two-loop form factor has been evaluated in [29, 31, 32] in the high energy limit. The set of fermionic, Abelian and non-Abelian two-loop vertex corrections is shown in Fig. 1, 2, 3, respectively.

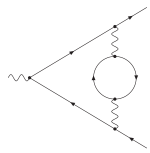

Fig. 1. Fermionic vertex correction.

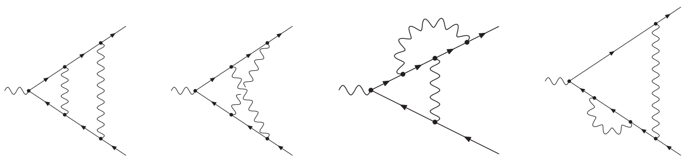

Fig. 2. Abelian vertex corrections.

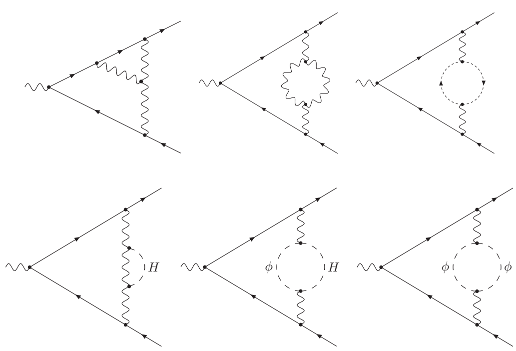

Fig. 3. Non-Abelian vertex corrections.

Defining

$$\mathcal{F}(M,Q) = \mathcal{F}_{\rm Born}\left[1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \ldots\right] \tag{4}$$

and $\mathcal{L} = \log(Q^2/M^2)$, the high energy behavior in the Abelian and SU(2) case respectively, is given by

$$\begin{aligned} f^{(2)}_{\rm Abelian} &= \frac{1}{2}\mathcal{L}^4 - 3\mathcal{L}^3 + \left(8 + \frac{2}{3}\pi^2\right)\mathcal{L}^2 - \left(-24\zeta_3 + 9 + 4\pi^2\right)\mathcal{L} + 256\,{\rm Li}_4\left(\frac{1}{2}\right) \\ &\quad + \frac{32}{3}\ln^4 2 - \frac{32}{3}\pi^2\ln^2 2 - \frac{52}{15}\pi^4 + 80\zeta_3 + \frac{52}{3}\pi^2 + \frac{25}{2}\,, \\ f^{(2)}_{\rm SU(2)} &= \frac{9}{32}\mathcal{L}^4 - \frac{19}{48}\mathcal{L}^3 - \left(\frac{463}{48} - \frac{7\pi^2}{8}\right)\mathcal{L}^2 + \left(29 - \frac{11}{24}\pi^2\right)\mathcal{L}\,, \end{aligned} \tag{5}$$

where $M_{\rm Higgs} = M_{\rm gauge\,boson} \equiv M$ was adopted for the SU(2) case and three doublets of fermions were assumed. The relative size of the corrections for this SU(2) "toy model" is shown in Fig. 4. The gauge boson induced terms are clearly dominant, the Higgs and light fermion terms small. Comparing the relative importance of LL versus NLL terms and so on, one finds an oscillation behavior and considerable compensations between the different powers of the logarithms.

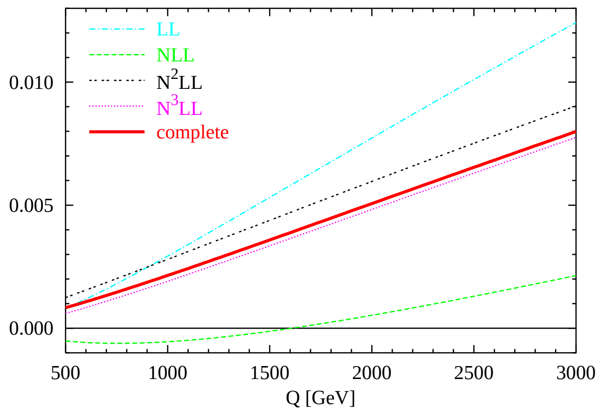

Fig. 4. Leading and subleading logarithmic contributions to the two-loop form factor for the SU(2) toy model.

Four Fermion scattering

The behavior of four-fermion amplitudes is again best discussed in the toy model of a massive SU(2) theory. The isospin and chiral decomposition needed to describe this process is characterized by the amplitudes:

$$\begin{aligned}
\mathcal{A}^{\lambda} &= \bar{\psi}_2 t^a \gamma_\mu \psi_1 \bar{\psi}_4 t^a \gamma_\mu \psi_3 \,, \\
\mathcal{A}^{\lambda}_{LL} &= \bar{\psi}_{2\mathrm{L}} t^a \gamma_\mu \psi_{1\mathrm{L}} \bar{\psi}_{4\mathrm{L}} t^a \gamma_\mu \psi_{3\mathrm{L}} \,, \\
\mathcal{A}^{d}_{LR} &= \bar{\psi}_{2\mathrm{L}} \gamma_\mu \psi_{1\mathrm{L}} \bar{\psi}_{4\mathrm{R}} \gamma_\mu \psi_{3\mathrm{R}} \,.
\end{aligned} \tag{6}$$

Adopting this basis and defining a reduced amplitude $\tilde{A}$ (actually a vector in the isospin/chiral basis) by splitting off the collinear logarithms

$$\mathcal{A} = \frac{ig^2}{s} \mathcal{F}^2 \tilde{\mathcal{A}} \,, \tag{7}$$

it is possible to define a matrix evolution equation

$$\frac{\partial}{\partial \ln Q^2} \tilde{\mathcal{A}} = \chi(\alpha(Q^2)) \tilde{\mathcal{A}} \,, \tag{8}$$

which can be solved iteratively. The matrix χ of the "soft" anomalous dimensions is obtained from the one-loop result for four-fermion scattering in the high energy limit, if one is interested in the NNLL terms only. To obtain the four-fermion cross section in N^{3}LL approximation, the form factor is required up to N^{3}LL and the matrix χ up to two loops. The latter can be obtained from the hard contribution to the single pole part of four fermion scattering [31]. For the SU(2) toy model discussed already above and for fermions with identical isospin in initial and final state ($u\bar{u} \to u'\bar{u}'$) the two loop part of the cross section is given by

$$\sigma^{(2)} = \left[\frac{9}{2} \mathcal{L}^4 - \frac{449}{6} \mathcal{L}^3 + \left(\frac{4855}{18} + \frac{37\pi^2}{3} \right) \mathcal{L}^2 \right] \sigma_{\mathrm{B}} \,, \tag{9}$$

Again large cancellations between different powers of the logarithm are observed.

Electroweak Theory

Two important aspects have to be taken into account, when moving from SU(2) to the full electroweak theory:

(i) The appearance of the massless photon leads to infrared singularities which must be separated from the complete result and eventually canceled against those from real radiation. The matrix χ and the collinear factor is modified accordingly, following the prescription of [15].

(ii) The mass difference between M_W and M_Z must be taken into account and, starting from N^{3}LL, details of the gauge boson mass generation start to matter.

The relative size of the logarithmically enhanced terms of the corrections for $e^+e^- \to q\bar{q}$ is shown in Fig. 5 for one- and two-loop contributions, respectively. Similar techniques have been applied to W-pair production in electron–positron annihilation [40]. In this case a strikingly different behavior of longitudinal *versus* transverse polarized W's is observed, which, using the equivalence-theorem, is easily traced to the different isospin-representation of the Goldstone modes ($I = \frac{1}{2}$) and the gauge bosons ($I = 1$). Furthermore, as a consequence of the direct and strong coupling of the longitudinal W to the top quark, the proper treatment of the mixing of the $W - tb$ system is important for this interesting case.

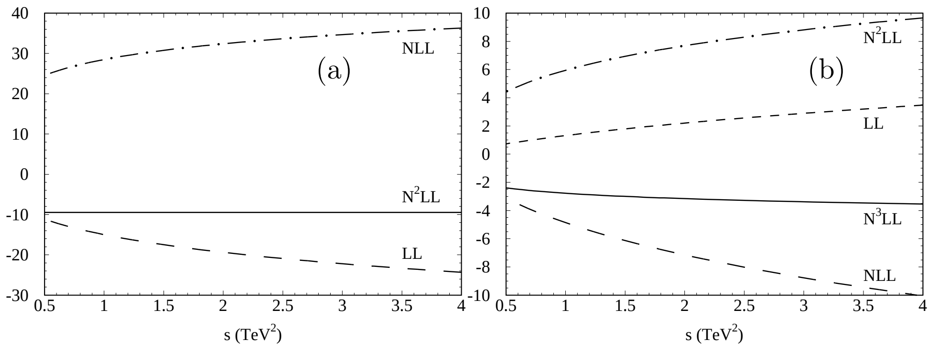

Fig. 5. Separate logarithmic contributions to $R(e^+e^- \to q\bar{q})$ in % to the Born approximation: (a) the one-loop LL ($\ln^2(s/M^2)$, long-dashed line), NLL ($\ln^1(s/M^2)$, dot-dashed line) and N^{2}LL ($\ln^0(s/M^2)$, solid line) terms; (b) the two-loop LL ($\ln^4(s/M^2)$, short-dashed line), NLL ($\ln^3(s/M^2)$, long-dashed line), NNLL ($\ln^2(s/M^2)$, dot-dashed line) and N^{3}LL ($\ln^1(s/M^2)$, solid line) terms.

3. Z, photon and W production at large transverse momenta

In view of the enormous luminosity expected for the LHC, the production of Z, W or photons at large transverse momenta, recoiling against a quark or gluon jet will be observed with sizable event rates. Indeed, gauge bosons with p_{T} up to 2 TeV will be observed, corresponding to scattering energies of up to 4 TeV. In this region large (negative) electroweak corrections are expected. This has motivated a series of investigations of these reactions, moving systematically from Z and photon [41–44] to W production [45–47]. In addition to the full evaluation of the one-loop terms the NLL and NNLL two-loop corrections are presented in [42–46].

Compact analytical formulae were obtained in [42–46]. The virtual corrections can be decomposed into "Abelian" and "non-Abelian" parts which can be traced back to the original SU(2)×U(1) structure of the theory. For

W production furthermore, the photonic corrections do not form a separate gauge invariant subset and must be included in the analysis, together with real radiation, *i.e.* $W+\gamma$+jet final states. Their contribution can only be obtained from a Monte Carlo generator.

Let us, in a first step, consider Z-production. For small $\sqrt{\hat{s}}$ up to 200 GeV the corrections are practically irrelevant, below 0.3%. For $\sqrt{\hat{s}} = 4$ TeV, however, they amount up to 40%. In this region the dominant logarithmic terms are given by

$$H_1^{\mathrm{A}} \overset{\mathrm{NLL}}{\sim} -\left[\log^2\left(\frac{|\hat{s}|}{M_W^2}\right) - 3\log\left(\frac{|\hat{s}|}{M_W^2}\right)\right] H_0^A\,, \tag{10}$$

$$H_1^{\mathrm{N}} \overset{\mathrm{NLL}}{\sim} -\left[\log^2\left(\frac{|\hat{t}|}{M_W^2}\right) + \log^2\left(\frac{|\hat{u}|}{M_W^2}\right) - \log^2\left(\frac{|\hat{s}|}{M_W^2}\right)\right] H_0^N\,, \tag{11}$$

with the remaining subleading terms below 2.5%. Using arguments discussed in [20, 21, 25] also the dominant two-loop terms can be predicted, which lead to a slight reduction of the corrections by about 5% at the highest energies. The relative one-loop and dominant two-loop corrections for Z and γ production are shown in Fig. 6 and compared to the expected statistical precision at LHC, assuming an integrated luminosity of 200 fb^{-1}, and full efficiency for leptonic decay modes. The corresponding results for W^+ and W^- production are shown in Fig. 7.

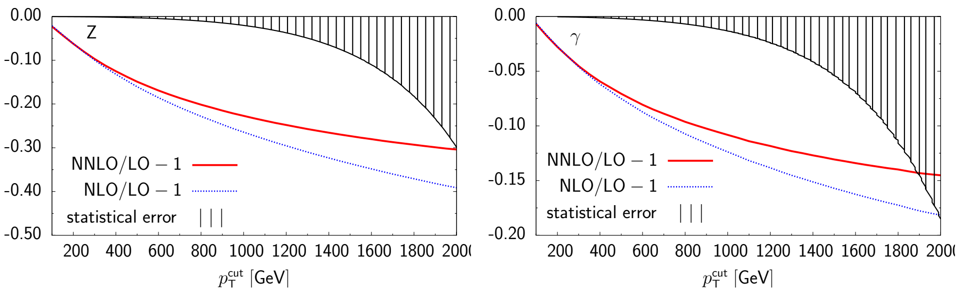

Fig. 6. Relative NLO (solid) and NNLO (dotted) corrections wrt. the LO prediction and statistical error (shaded area) for the unpolarized integrated cross section for $pp \to Zj$ (left) and $pp \to \gamma j$ (right) at $\sqrt{s} = 14$ TeV as a function of $p_{\mathrm{T}}^{\mathrm{cut}}$.

The theory uncertainty in predicting the absolute cross section originates from uncalculated higher orders in QCD and uncertainties in the parton distribution functions. On the experimental side the measured boson transverse momentum may be smeared, which leads to a corresponding smearing of the steeply falling p_{T} distribution. QCD uncertainties could be significantly reduced by considering ratios of W^+, W^-, Z and γ distributions (Fig. 8).

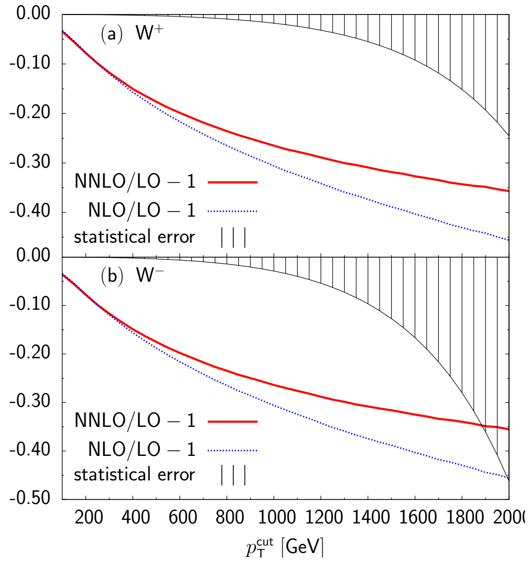

Fig. 7. Unpolarized integrated cross section as a function of $p_{\rm T}^{\rm cut}(W)$ for W^+ (a) and W^- (b) production: estimated statistical error (shaded area) and relative electroweak corrections in NLO (dotted) and NNLO (solid) approximation.

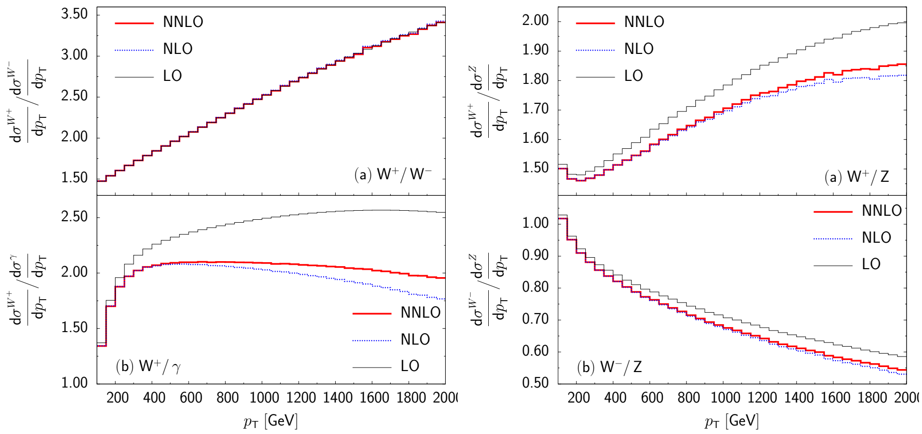

Fig. 8. Ratio of the transverse momentum distributions for W^+/W^-, W^+/γ, W^+/Z and W^-/Z at the LHC: LO (thin solid), NLO (dotted) and NNLOO (thick solid) predictions.

These ratios are shown again in Born approximation, including the full one-loop and as a third option, including the dominant two-loop terms. The W^+/W^- ratio remains practically unchanged, the W/Z and W/γ ratios are shifted by 10 to 25% for the largest transverse momenta.

4. Top production

As a third topic, clearly connected to the start of the LHC in the course of this year, let us consider the impact of electroweak corrections on top-quark pair production. An early discussion can be found in [48,49] where the main issues were addressed for the first time. Later, with the advent of LHC, it was pointed out [50,51] that the results for the quark induced process had been incomplete, with mixed QCD-electroweak box contributions connected to real and virtual gluon emission (Fig. 9) being of the same order as the generic electroweak corrections (Fig. 10).

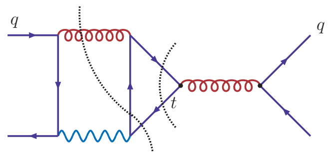

Fig. 9. Mixed QCD and electroweak corrections to $q\bar{q}$ induced top quark production.

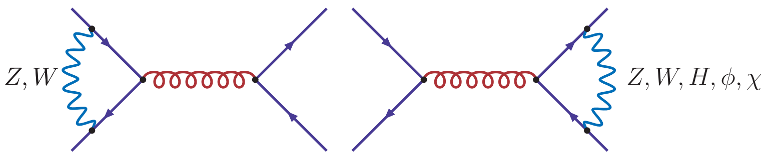

Fig. 10. Generic weak corrections to $q\bar{q}$ induced top quark production.

The corrections to the gluon-induced process were analyzed in [53,54] where some discrepancies were observed relative to earlier work [35,48,52]. Representative diagrams are shown in Fig. 11. Of specific interest are the s-channel contributions from Z, χ and the Higgs boson. If kinematically allowed, the Higgs propagator would lead to a resonant behavior, damped, of course, by the finite width of the Higgs boson. The Z and χ amplitudes exhibit a smooth behavior in the full kinematic range. The relative size of the corrections are shown in Fig. 12 for quark and gluon induced processes as functions of $\sqrt{\hat{s}}$ from threshold up to 1 TeV. Not surprisingly, again large negative corrections are observed for large energies. Close to threshold, however, one also obtains sizable corrections which are strongly dependent on m_H, with a difference between large m_H and $m_H = 120\,\text{GeV}$ of 5 to 10%. This is the consequence of the attractive Yukawa potential resulting from a light Higgs boson exchange.

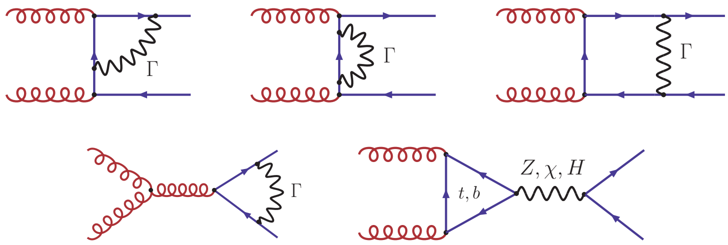

Fig. 11. Representative weak corrections to $gg \to t\bar{t}$.

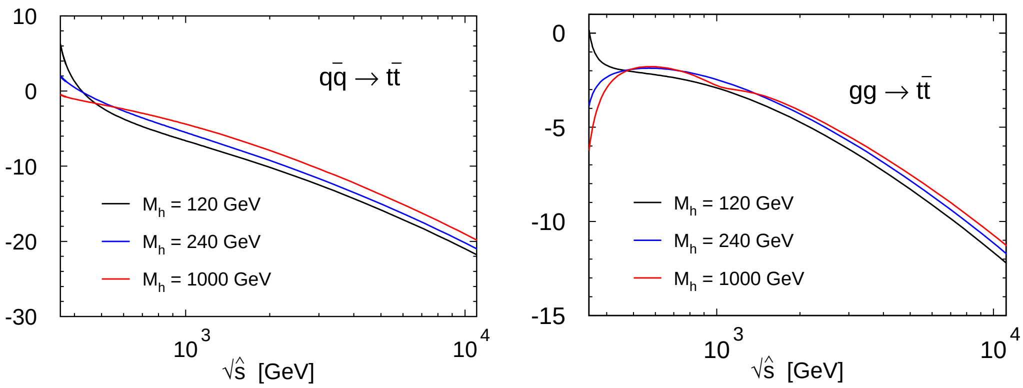

Fig. 12. Relative corrections for $q\bar{q}$ (left) and gg (right) induced processes as functions of the partonic energy $\sqrt{\hat{s}}$ for different Higgs boson masses.

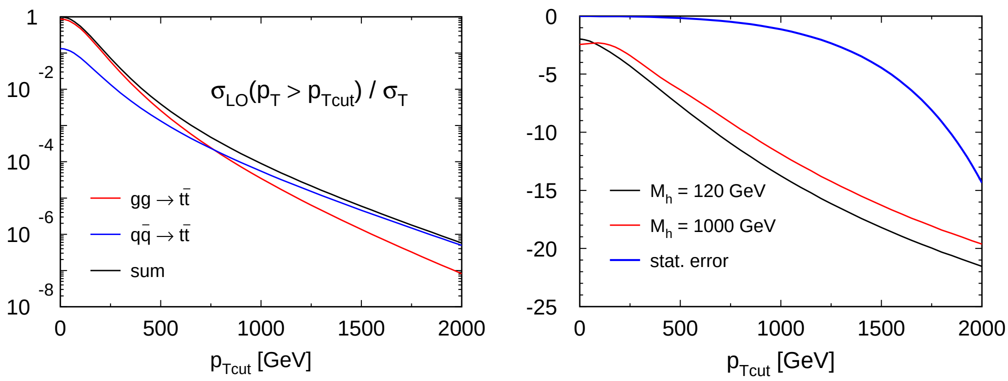

Fig. 13. Left: Fraction of events from $q\bar{q}$ and gg induced processes with $p_{\rm T} > p_{\rm T\,cut}$ as functions of $p_{\rm T\,cut}$, compared to the total $t\bar{t}$ production cross section. Right: Relative correction for the cross section for $t\bar{t}$ production with $p_{\rm T} > p_{\rm T\,cut}$. Also shown is the anticipated statistical error at the LHC.

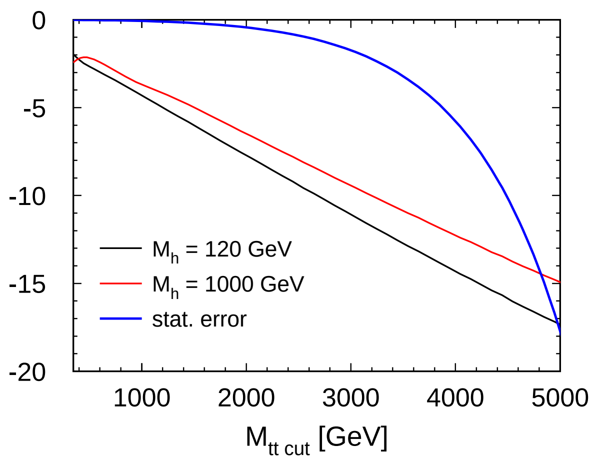

Fig. 14. Relative correction for the cross section for $t\bar{t}$ production with $M_{t\bar{t}} > M_{t\bar{t}\mathrm{cut}}$. Also shown is the anticipated statistical error at the LHC.

To arrive at a prediction for the LHC the partonic cross sections must be convoluted with the parton distribution functions. As shown in Fig. 13, the gluon induced process is dominant at low p_{T}, the $q\bar{q}$ induced processes for p_{T} above 700 GeV. This is reflected in the size of the corrections which are shown in Fig. 13 and compared to the anticipated statistical error, assuming an integrated luminosity of 200 fb^{-1}. The corresponding results for the distribution on the invariant mass of the $t\bar{t}$ system are shown in Fig. 14.

5. Summary

The LHC will be the first collider exploring the energy region where parton scattering energies will be significantly larger than the masses of the gauge bosons. This leads us into a region where, depending on the choice of the observables, electroweak corrections may become large, of the order 10 to 30%. For precise predictions the logarithmically enhanced terms are required in one and two-loop approximation. At present, detailed studies of two loop N^{3}LL effects are available for the fermion form factor, for four-fermion processes and in N^{2}LL for W-pair production. Studies involving the full one-loop corrections and, partially, the dominant two-loop terms, have been performed for Z, W and γ production at large transverse momenta and, similarly, for top quark pair production. It will be exciting to see the first of these reactions soon.

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