Vol. 39 (2008) ACTA PHYSICA POLONICA B No 7

UNSTABLE QUARK–GLUON PLASMA AT LHC*

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(Received May 9, 2008)

Coupling of the quark–gluon plasma from the early stage of heavy-ion collisions is argued to be significantly weaker at LHC than at RHIC. For this reason, the role of instabilities — the pre-equilibrium plasma is unstable with respect to chromomagnetic modes — will be enhanced. The instabilities isotropize the system and speed up the process of equilibration. A possibility to observe direct signals of the instabilities is considered.

PACS numbers: 12.38.Mh

1. Introduction

A prospect of heavy-ion program at Large Hadron Collider (LHC) to be initiated at CERN soon, makes one to wonder about new features of nucleus–nucleus interactions when the collision energy is increased nearly 30 times when compared to the highest accessible energy at Relativistic Heavy Ion Collider (RHIC) at BNL. The problem actually attracted a lot of attention and numerous predictions were formulated [1]. In my lecture, however, I would like to focus on one specific aspect of heavy-ion collisions — the dynamical role of chromomagnetic Weibel instabilities which, I expect, will be strongly enhanced at LHC.

A successful experimental program at RHIC provided a convincing evidence that the quark–gluon plasma (QGP), which is produced at the early stage of relativistic heavy-ion collisions, equilibrates fast — presumably within the time interval as short as 1 fm/c — and later on it behaves as a nearly ideal fluid [2]. Both features can be easily explained assuming that QGP is strongly coupled [3]. However, it is quite probable that the plasma

* Presented at the Cracow Epiphany Conference on LHC Physics, Cracow, Poland, 4–6 January 2008.

at the pre-equilibrium stage of the collision is rather weakly coupled due to the asymptotic freedom regime achieved at a very high energy density. Actually, the very recent analysis [4] of the RHIC data on jet quenching has shown that even the equilibrium QGP is not so strongly coupled, as the inferred coupling constant $\alpha = 0.3$. And it has been argued that the weakly coupled plasma can actually equilibrate fast, for a review see [5], and it can behave as a fluid of low viscosity [6]. This is possible because the pre-equilibrium plasma is unstable with respect to the chromomagnetic plasma modes due to the anisotropy of momentum distribution. The instabilities isotropize the system and efficiently speed up the equilibration process [5]. The spontaneously generated magnetic fields can be responsible for the so-called anomalous viscosity which make the plasma behave as a nearly ideal fluid [6]. However, the instabilities are operative if the plasma is weakly coupled. In the following sections I argue that conditions for the instabilities are much more favorable at LHC than at RHIC and I speculate about possible direct signals of the unstable modes.

2. Coupling of QGP at LHC

I argue here that, due to the increase of energy density, the coupling of QGP at the pre-equilibrium stage of relativistic-heavy-ion collisions is significantly weaker at LHC than at RHIC.

The (thermal) energy density and temperature of the plasma at the moment when it reaches local thermodynamic equilibrium are estimated as $\epsilon_{\rm T}^{\rm RHIC} \approx 30$ GeV/fm^3 and $T^{\rm RHIC} \approx 350$ MeV (the lower index T stands for 'thermal') at the highest RHIC energy ($\sqrt{s} = 200$ GeV per N–N collision) [2,7]. The analogous quantities for LHC ($\sqrt{s} = 5500$ GeV per N–N collision) are expected to be $\epsilon_{\rm T}^{\rm LHC} \approx 130$ GeV/fm^3 and $T^{\rm RHIC} \approx 500$ MeV [7,8]. The increase of the temperature is not very big but noticeable and it influences the plasma coupling. In the ideal gas of massless particles $\epsilon_{\rm T} = 3p$, where p is the gas pressure. Therefore, the dimensionless quantity $(\epsilon_{\rm T} - 3p)/T^4$ is often treated as a measure of the interaction strength in the plasma. The QCD lattice calculations show that $(\epsilon_{\rm T} - 3p)/T^4 \approx 3$ at $T = 350$ MeV and it is reduced to about unity at $T = 500$ MeV [9]. Thus, the early stage plasma is closer to the non-interacting gas at LHC than at RHIC.

However, we are mostly interested in the pre-equilibrium plasma and we would like to get an estimate of the coupling constant $\alpha \equiv g^2/4\pi$ which at RHIC energies is usually chosen to be $\alpha^{\rm RHIC} \approx 0.3$ [10]. As already mentioned, the very recent analysis of the RHIC data on jet quenching supports correctness of this choice [4]. Let us first get an idea about the energy density in the center-of-mass frame just after the collision. In the central collisions of nuclei of mass number A, we estimate it as

$$\epsilon_0 = \frac{kA\sqrt{s}}{\pi R^2 l}\,, \tag{1}$$

where k is the inelasticity — the fraction of initial energy, which goes to particle production, R is the radius of colliding nuclei and l is the length of the cylinder where the energy is released. Assuming that $k = 0.5$ independently of energy [11] and taking $A = 200$, $R = 7$ fm and $l = 1$ fm, one obtains $\epsilon_0^{\rm RHIC} \approx 130$ GeV/fm^3 for $\sqrt{s} = 200$ GeV and $\epsilon_0^{\rm LHC} \approx 3600$ GeV/fm^3 for $\sqrt{s} = 5500$ GeV. One wonders why $\epsilon_{\rm T}$ is so much smaller than ϵ_0 (4 times for RHIC and 28 for LHC). The system's expansion during the pre-equilibrium phase is partially responsible for the decrease but it is far more important that the thermal energy density does *not* include the energy related to a collective motion which is very large.

The evolution of α with energy density can be roughly estimated, using the celebrated formula of running coupling constant

$$\alpha(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda_{\rm QCD}^2)}\,, \tag{2}$$

where Q is the characteristic momentum transfer, $\Lambda_{\rm QCD} = 200$ MeV is the QCD scale parameter and $N_f = 3$ is the number of flavours. Referring to the dimensional argument, I replace Q^2 by $a\sqrt{\epsilon_0}$ with the parameter $a = 4.12$, which is chosen in such a way that Eq. (2) with $\epsilon_0 = \epsilon_0^{\rm RHIC}$ gives $\alpha^{\rm RHIC} = 0.3$. Then, I obtain $\alpha^{\rm LHC} = 0.16$ for $\epsilon_0 = \epsilon_0^{\rm LHC}$. The reduction appears to be quite significant but the actual numerical value of α is, obviously, not very reliable. However, I can safely conclude this section that the lattice calculations and the asymptotic freedom argument both suggest noticeably weaker coupling of the early stage QGP at LHC than at RHIC.

3. Instabilities at LHC

Occurrence of Weibel instabilities requires an anisotropy of parton momentum distribution. The condition is trivially fulfilled, as initially the parton momentum is strongly elongated along the beam — its shape is prolate — and in the course of system's expansion it becomes locally squeezed along the beam — its shape is oblate. And the Weibel modes are present in both configurations.

The anisotropy can be quantified by means of the parameter $2\langle p_{\rm L}^2\rangle/\langle p_{\rm T}^2\rangle$, where $p_{\rm L}$ and $p_{\rm T}$ are parton longitudinal and transverse momenta and $\langle\ldots\rangle$ denotes inclusive averaging. As discussed in [14], the parameter is well approximated by the formula

$$\frac{2\langle p_{\rm L}^2\rangle}{\langle p_{\rm T}^2\rangle} = e^{2\sigma_y^2} - 1\,, \tag{3}$$

where partons are assumed to be massless and σ_y is the width of Gaussian rapidity distribution. To obtain σ_y at LHC, I use the Landau model parameterization [15]

$$\sigma_y^2 = \ln\left(\sqrt{s}/2m_p\right), \tag{4}$$

where m_p is the proton mass. Eq. (4) gives $\sigma_y = 2.2$ at the top RHIC energy which agrees well with the pion data [16]. For $\sqrt{s} = 5500$ GeV, the formula (4) predicts $\sigma_y = 2.8$ which in turn gives $2\langle p_{\rm L}^2\rangle/\langle p_{\rm T}^2\rangle = 7 \times 10^6$ (at the top RHIC energy $2\langle p_{\rm L}^2\rangle/\langle p_{\rm T}^2\rangle = 2\times 10^4$). So, initially there will be a huge anisotropy which will locally decay in the course of system's expansion. As shown in [14], it takes about 5 fm$/c$ to make the local distribution oblate ($2\langle p_{\rm L}^2\rangle/\langle p_{\rm T}^2\rangle < 1$), if the system evolves solely due to the free streaming. An actual evolution of the momentum distribution will be faster, as not only the free streaming but interactions in the parton system will tend to reduce the momentum anisotropy, but one expects that during the first, say, 1–2 fm$/c$ the unstable modes will be operative if their growth rates are sufficiently large. As I explain below, this requires a weak coupling and high energy density of the system.

If QGP is really weakly coupled, the characteristic inverse time of processes driven by inter-parton collisions is $t_{\rm hard}^{-1} \sim g^4 \ln(1/g)p$ or $t_{\rm soft}^{-1} \sim g^2 \ln(1/g)p$, depending whether the momentum transfer in a collision is of order p or gp with p being a typical parton momentum which in the equilibrium plasma can be identified with the temperature T [12]. The characteristic inverse time of mean-field collective phenomena, in particular the growth rate of instabilities is of order gp [5]. Therefore, there is a good separation of the time scales provided $g^2 \ll 1$. Then, the instabilities are much faster than the inter-parton collisions which are responsible for dissipative processes. The collisions slow down the growth of the unstable modes and there is an upper limit on the collisional frequency beyond which no instabilities exist [13]. Therefore, the Weibel instabilities require a sufficiently weak coupling of the plasma.

While the weak coupling guarantees that the instabilities are faster than the collisions, the instabilities should be also much faster than the characteristic time of the system's temporal evolution. Then, the spontaneously generated chromomagnetic fields will reach large values as a result of many e-foldings of their amplitudes. As already mentioned, the growth rate of the instabilities γ is of order gp [5]. If p is identified with $\epsilon_0^{1/4}$, where ϵ_0 is the initial energy density discussed earlier, $p^{\rm LHC} = 2.3$ GeV (for top RHIC energy $p^{\rm RHIC} = 1.0$ GeV). As the coupling constant g is actually of order of unity, the instabilities are presumably faster than the characteristic time of the system's temporal evolution ($\gamma^{-1} \sim 0.1$ fm$/c$), but it is unclear whether the instabilities are fast enough to avoid a strong damping caused by the inter-parton collisions.

Although my estimates do not allow to draw a firm conclusion that the Weibel instabilities will indeed play an important role in the pre-equilibrium QGP at LHC, my estimates clearly show that the conditions will be much more preferable than those at RHIC. And if the instabilities already exist at RHIC, as the fast thermalization suggests, they should play a prominent role at LHC. So, let me discus how the Weibel instabilities can manifest themselves.

4. Fast equilibration

The instabilities speed up the process of equilibration as they effectively isotropize the momentum distribution. It should be stressed here that inter-parton collisions are not very effective in changing parton's momenta, because the one-gluon-exchange cross section, as the Rutherford one, is strongly peaked at small momentum transfers. The characteristic time of collisional izotropization coincide with the earlier introduced $t_{\rm hard}$ and it is too long in the weakly coupled plasma to comply with the fast equilibration. The radiative parton collisions are more effective in redistributing parton's momenta but the processes are suppressed by an extra power of α. The magnetic fields associated with the unstable modes do the job really fast.

To explain the mechanism of isotropization I assume that the momentum distribution is strongly elongated along the beam (z) direction. The colour currents, which initiate the Weibel instability as a random fluctuation and then grow when the instability develops, flow in the z direction. The wave vector of the fastest unstable mode lies in the x–y plane. I assume that it points in the x direction. Then, the magnetic field generated by the currents is oriented in the y direction and the Lorentz force, which acts on partons flying along the z axis, pushes them in the x direction where there is a momentum deficit. Having a superposition of many unstable modes with their wave vectors in the x–y plane, the instabilities produce approximately axially symmetric transverse momentum distribution.

The system isotropizes not only due to the effect of the Lorentz force but also due to the momentum carried by the growing field. When the magnetic and electric fields are oriented along the y and z axes, respectively, the Poynting vector points in the direction x that is along the wave vector. Thus, the momentum carried by the fields is oriented in the direction of the momentum deficit. The numerical simulations [17,18], which, however, were performed for the oblate not prolate momentum distribution, indeed show that growth of instabilities is accompanied by the system's fast isotropization.

The isotropization should not be confused with the equilibration. Obviously, the instabilities cannot equilibrate the system but once the instabilities have redistributed parton's momenta, soft parton–parton collisions, which are much more frequent than the hard ones, complete the processes of equilibration.

5. Signals of instabilities

Since the Weibel instability is a phenomenon, which occurs at the pre-equilibrium phase of QGP, later temporal evolution of the plasma, including equilibration and hadronization, conceals its presence and it is difficult to point a direct signal of the instabilities which can be observed in the final state of heavy-ion collisions.

Recently it has been argued [19] that the experimentally observed longitudinal broadening of jets, which are quenched in the plasma, can be attributed to the interaction of jet particles with colour fields generated by the unstable modes. I would like to briefly discuss two possible signals of the instabilities related to one another.

5.1. Elliptic flow fluctuations

The so-called elliptic flow, which is caused by an initially asymmetric shape of the interaction zone of colliding nuclei, is sensitive to the collision early stage. The phenomenon is successfully described by the hydrodynamic model, see *e.g.* [2], which, in principle, requires that the system under study is in a local thermodynamical equilibrium. However, an approximate hydrodynamic behaviour occurs, as argued in [20], when the momentum distribution of liquid components is merely isotropic in the local rest frame. The point is that the structure of the ideal fluid energy-momentum tensor *i.e.* $T^{\mu\nu} = (\varepsilon + p)\, u^\mu u^\nu - p\, g^{\mu\nu}$, where ε, p and u^μ is the energy density, pressure and hydrodynamic velocity, respectively, holds for an arbitrary, though isotropic momentum distribution. ε and p are then not the energy density and pressure but the moments of the distribution function which are equal the energy density and pressure in the equilibrium limit. Since the tensor $T^{\mu\nu}$ obeys the continuity equation $\partial_\mu T^{\mu\nu} = 0$, one gets an analogue of the Euler equation. However, due to the lack of thermodynamic equilibrium there is no entropy conservation.

Usually, non-equilibrium fluctuations are significantly bigger than the equilibrium fluctuations of the same quantity. Therefore, sizeable fluctuations of the elliptic flow due to the pre-equilibrium stage of quasi-hydrodynamic evolution were predicted [21, 22]. Large fluctuations of the elliptic flow have been indeed observed at RHIC [23, 24] but the effect is now commonly understood as a result of fluctuations of the eccentricity of the interaction zone as suggested in [25]. Thus, it is assumed that the whole effect of the observed elliptic flow fluctuations comes from the eccentricity fluctuations while the hydrodynamic evolution is assumed to be fully deterministic and thus, it does not contribute to the elliptic flow fluctuations[1].

Although the calculations of the eccentricity fluctuations reproduce well the experimentally observed elliptic flow fluctuations, see *e.g.* [27], the eccentricity fluctuations seem to me significantly overestimated. The nucleons of colliding nuclei are treated as essentially independent from each other, and consequently a smooth shell structure of a nucleus is ignored. It is even more important that a significant contribution to the eccentricity fluctuations comes from collisions of nucleons from a nucleus periphery. Transverse positions of these collisions are usually well separated from the positions of interactions of other nucleons [28]. In my opinion, partons produced in these isolated nucleon–nucleon collisions do not participate in the hydrodynamic evolution of the system and do not contribute to the elliptic flow. If the isolated collisions are excluded, the eccentricity fluctuations are reduced, and an extra source of fluctuations is needed to explain the data. The extra fluctuations come, I suppose, from the quasi-equilibrium hydrodynamic evolution.

5.2. Azimuthal fluctuations

As argued in the previous subsection, the instability driven equilibration leads to significant elliptic flow fluctuations because the quasi-hydrodynamic evolution starts when the system is merely isotropic but not fully equilibrated. However, the instabilities can also be directly responsible for flow-like effects. In the prolate momentum configuration, which is characteristic for the pre-equilibrium stage of the quark–gluon plasma [14], the wave vector of the fastest unstable modes is randomly oriented in the transverse plane. As explained in Sec. 4, the momentum is transported along the wave vector. Therefore, I expect a collective radial flow which exhibits strong azimuthal fluctuations [22]. In contrast to the elliptic flow, the transverse flow caused by the instabilities is *not* correlated with the reaction plane. In particular, the flow should occur in exactly central collisions when the elliptic flow is absent for the symmetry reasons.

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[1] Very recently the observation of large elliptic flow fluctuations has been retracted by STAR collaboration [26]. It is claimed now that the previously given magnitude of the fluctuations should be taken only as an upper limit due to the difficulties to disentangle the elliptic flow fluctuations and the contributions which are not correlated with the reaction plane. Since the effects of instabilities are not associated with the reaction plane, the retraction [26] does not much influence the considerations presented here.

6. Conclusions

The Weibel instabilities seem to be present in relativistic heavy-ion collisions at RHIC and the estimates show that the conditions for the instabilities will be much more preferable at LHC than at RHIC. Therefore, I expect that the role of instabilities will be strongly enhanced.

The instabilities provide a plausible mechanism responsible for a surprisingly short equilibration time and the fast isotropization is a distinctive feature of the mechanism. Since the Weibel instabilities occur at the pre-equilibrium phase of QGP, later temporal evolution of the plasma conceals their presence and it is difficult to indicate direct signals. However, careful studies of the evolution of jets, of the azimuthal fluctuations of radial flow and of the elliptic flow fluctuations will hopefully provide an evidence that the pre-equilibrium QGP is indeed unstable.

I am grateful to Uli Heinz and Constantin Loizides for helpful correspondence and to Art Poskanzer and Xin-Nian Wang for fruitful discussions.

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