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FULLY UNINTEGRATED PARTON CORRELATION FUNCTIONS*

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We summarize recent progress in the formulation of QCD factorization theorems in terms of parton correlation functions and discuss their relevance to LHC physics. We describe open problems and directions for future work.

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1. Introduction

Perturbative Quantum Chromodynamics (QCD) predictions for the Large Hadron Collider (LHC) require precise knowledge of the universal parton distribution functions (PDFs) of the perturbative QCD factorization theorems (*e.g.* Refs. [1]). There is also a need for a deeper understanding of the type of non-perturbative factors that appear in extensions of factorization beyond the usual collinear approach. In these proceedings, we discuss important issues in factorization that are needed for a more complete understanding of PDFs when the details of final state kinematics are important. For illustration, we focus on deep inelastic scattering.

The PDFs of interest to LHC physics can be classified according to three types:

- **Integrated PDFs:** The integrated PDFs depend only on the longitudinal momentum fraction of the struck parton. The standard definition for the quark PDF is,

$$f_j(x_{\mathrm{Bj}},\mu)=\int \frac{dw^-}{4\pi} e^{-ix_{\mathrm{Bj}}p^+w^-} \left\langle p \left| \bar{\psi}(0,w^-,\mathbf{0}_{\mathrm{t}})V_w^\dagger(n)\gamma^+V_0(n)\psi(0) \right| p \right\rangle . \tag{1}$$

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$x_{\rm Bj}$ is the usual Bjørken-x variable, ψ is the coordinate space quark field operator, and $V_0(n)$ is a Wilson line needed for exact gauge invariance:

$$V_w(n) = P \exp\left(-ig \int_0^\infty d\lambda\, n \cdot A(w + \lambda n) \right) , \tag{2}$$

where P is the path-ordering operator. The Wilson line points in the light-like direction,

$$n = (0, 1, \mathbf{0}_{\rm t}) \ . \tag{3}$$

The definition in Eq. (1) contains ultra-violet divergences and requires renormalization — the application of renormalization group methods leads to the familiar DGLAP evolution equation.

- **Unintegrated PDFs:** In certain interactions, the transverse component of parton momentum becomes important. In these situations, the definition above must be generalized. Options for doing this consistently are summarized in Ref. [2]. One possibility is to use the obvious generalization of the above definition:

$$\begin{aligned} P(x, \boldsymbol{k}_t, \mu) \ = \ & \int \frac{dw^- d\boldsymbol{w}_t}{16\pi^3} e^{-ixp^+ w^- + i\boldsymbol{k}_t \cdot \boldsymbol{w}_t} \\ & \times \left\langle p \left| \bar{\psi}(0, w^-, \boldsymbol{w}_t) V_w^\dagger(n') I_{n';w,0} \gamma^+ V_0(n') \psi(0) \right| p \right\rangle . \end{aligned} \tag{4}$$

However, the Wilson line must now point in a slightly non-light-like direction to avoid the appearance of divergences from gluons with infinite rapidity in the out-going quark direction,

$$n' = \left(-e^{-|2y|}, 1, \mathbf{0}_{\rm t} \right) , \qquad y \gg 0 \, . \tag{5}$$

In addition to this, it was pointed out in [3] that for exact gauge invariance, a transverse Wilson line is needed to connect the points at light-cone infinity. Hence, we have inserted the factor,

$$I_{n',w,0} = P \exp\left(-ig \int_C dz^\mu A_\mu(z) \right) , \tag{6}$$

where C is a line connecting the points at infinity. For more discussion of consistent operator definitions of $k_{\rm t}$-unintegrated PDFs, see recent work in [4].

- **Fully Unintegrated PDFs:** Finally, it has been observed [5] that calculations for certain observables require knowledge of all components of parton four-momentum to avoid large errors. This requires using the *fully* unintegrated PDFs that we will discuss in these proceedings. A formalism for setting up factorization with fully unintegrated PDFs is discussed in recent work [6] where operator definitions are given. In the fully unintegrated approach, one needs to include non-perturbative factors for both the initial and final states (even in inclusive processes). For example, a soft factor is needed in the final state. Collectively, we refer to all non-perturbative factors as parton correlation functions (PCFs).

In describing the different types of PDFs, we have emphasized the use of operator definitions because the universality of such objects is central to the use of factorization theorems in phenomenology. We stress this point in Fig. 1 where we show the flow of information from past experiments to new predictions. For most totally inclusive quantities, factorization using the integrated PDFs such as Eq. (1) with the classic factorization theorems is sufficient. For more inclusive quantities, however, we must generalize to unintegrated or fully unintegrated parton correlation functions. Using such objects in factorization theorems presents a number of complications which we discuss in the next section.

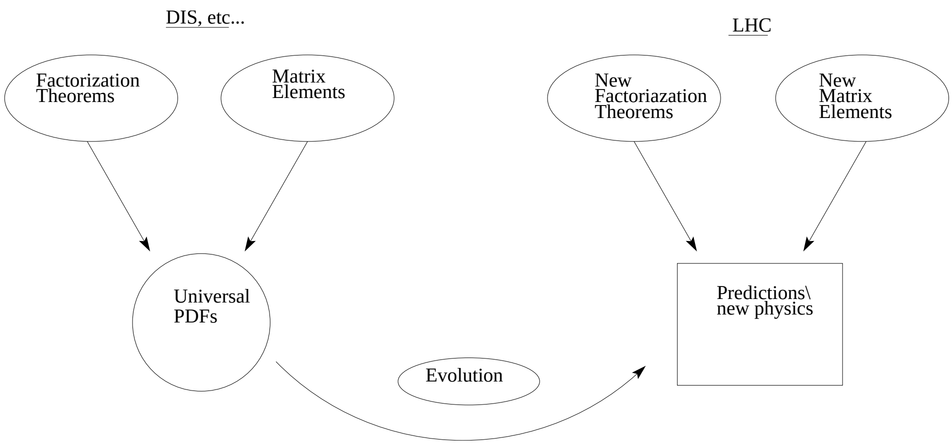


Fig. 1. Flow-chart showing the use of factorization theorems in phenomenology. Factorization theorems allow universal PDFs to be extracted from data for processes such as DIS. Those same PDFs can then be used (via pQCD evolution) to make predictions for new experiments.

2. Parton correlation functions

2.1. Kinematical approximations

The classic factorization theorem for DIS relies on a number of kinematic assumptions, as can be seen by a consideration of graphs like Fig. 2(a) where we make no approximation on parton momenta. For now we neglect

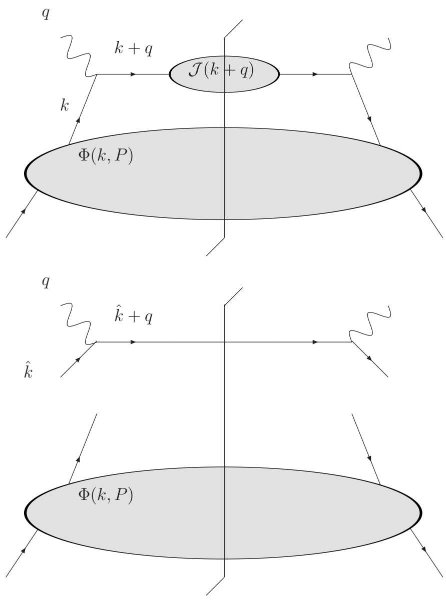


Fig. 2. (a) Unapproximated graph for deep inelastic single jet production. (b) Factorized handbag diagram with kinematic approximations.

the appearance of soft and collinear gluons, but we maintain the bubble that represents the outgoing final state quark jet. Fig. 2(a) gives, for the unapproximated hadronic tensor,

$$W^{\mu\nu}(q,P) = \sum_j \frac{e_j^2}{4\pi} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\gamma^\mu \mathcal{J}(k+q)\gamma^\nu \Phi(k,P)] \ , \tag{7}$$

where the sum is over quark flavors. To obtain the usual factorized form for the handbag diagram, we must make the following set of kinematical approximations:

- Inside the target bubble, make the replacement,

$$k \longrightarrow \left(x_{\mathrm{Bj}} P^+, k^-, \boldsymbol{k}_{\mathrm{t}}\right) \ . \tag{8}$$

- Inside the outgoing jet bubble, make the replacement,

$$l = k + q \longrightarrow \left(l^+, \frac{Q^2}{2x_{\mathrm{Bj}}P^+}, \mathbf{0}_{\mathrm{t}} \right) . \tag{9}$$

- Use parton model kinematics *inside the hard vertex*,

$$k \longrightarrow \hat{k} \; = \; \left(x_{\mathrm{Bj}}P^+, 0, \mathbf{0}_{\mathrm{t}} \right) , \tag{10}$$

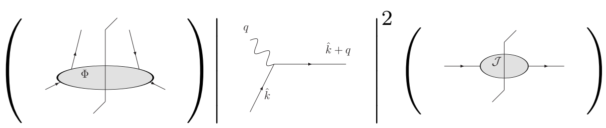
$$l \longrightarrow \hat{l} \; = \; \left(0, \frac{Q^2}{2x_{\mathrm{Bj}}P^+}, \mathbf{0} \right) . \tag{11}$$

In addition to these replacements, we also keep the largest term in an expansion over Dirac matrices. After these approximations (and a change of variables), we can move the integral over the plus component of momentum through the rest of the integrand to obtain,

$$\begin{aligned} W^{\mu\nu}(q,P) \; &\approx \; \sum_j \frac{e_j^2}{4\pi} \left[\int \frac{dk^- d^2\boldsymbol{k}_t}{(2\pi)^4} \varPhi^+(x_{\mathrm{Bj}}P^+, k^-, \boldsymbol{k}_t; P) \right] \\ &\quad \times \mathrm{Tr}\left[\gamma^\mu \gamma^+ \gamma^\nu \gamma^-\right] \left[\int dl^+ \mathcal{J}^-(l^+, q^-, \mathbf{0}_t) \right] . \end{aligned} \tag{12}$$

Or graphically,

$$W^{\mu\nu}(q,P) \approx \sum_j \frac{e_j^2}{4\pi}$$



. (13)

Note that the small components of four-momentum cannot be set to zero in the target and jet bubbles. We set the last factor in Eqs. (12), (13) to one by unitarity. Up to a counter-term needed to remove the ultra-violet divergence, the first factor corresponds to the lowest order term in an expansion of the definition in Eq. (1) for the fully integrated PDF. The middle term is just the squared amplitude for a virtual photon to scatter off an on-shell quark.

Thus, Eq. (12) can be reduced to the usual hand-bag diagram, as shown in Fig. 2(b). Note that whereas the integration over k_{t} in Eq. (7) is constrained by four-momentum conservation, the integration over k_{t} in Eq. (12) runs over all values and gives rise to the usual ultra-violet divergence associated with the evolution of the PDF.

Thus, for certain regions of phase space, there is a large mismatch between the kinematics of the initial and final states. For studies involving the details of the final states, a more exact account of over-all kinematics is needed. However, as the discussion of Fig. 2(a) illustrates, the usual collinear factorization theorem requires kinematic approximations. In particular, the kinematic approximations in Eqs. (8)–(11) are needed to eliminate the explicit appearance of the final state jet bubble in Fig. 2(a), and they are needed to correctly identify the PDF in a way that is consistent with Eq. (1).

2.2. Generalization to fully unintegrated PCFs

The approach to factorization proposed in [6] involves leaving *all* integrations over parton four-momentum undone. This allows us to keep exact initial and final state kinematics from the beginning. The steps that allow the final state bubbles (such as the out-going quark jet bubble in Fig. 2(a) to be simplified cannot be performed without introducing large mismatches between final and initial state kinematics. Thus, to correctly address the issue of factorization with exact over-all kinematics, it is necessary to start with the most general type of graph contributing to hard scattering with a single out-going quark jet. That is, we must consider graphs with the topology of the reduced diagram in Fig. 3. Schematically, the aim is to make

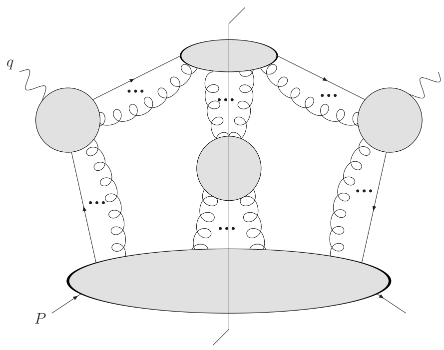


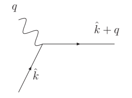
Fig. 3. Reduced diagram illustrating the structure of the most general type of diagram contributing to deep inelastic single jet production. The out-going target and quark jets are connected by soft gluons via a soft bubble (the circle shown in the center). The hard bubbles are connected to the outgoing target and jet bubbles by arbitrarily many collinear gluons.

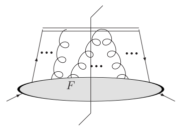
approximations to the hard part of the interaction such that, after applying Ward identities to the sum of graphs with the topology of Fig. 3, the cross section reduces to the form,

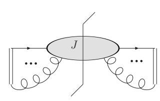
$$\sigma = C \otimes F \otimes J \otimes S + \mathcal{O}\left(\frac{\Lambda}{Q}\right) . \tag{14}$$

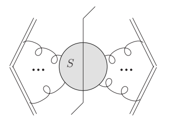
Here, σ represents the total cross section (or a similar object like a structure function). The factors on the right are a hard scattering coefficient, C, a fully unintegrated parton distribution function, F, a jet factor, J, and a soft factor S. The last term symbolizes corrections suppressed by factors of order Λ/Q. For a factorization formula like Eq. (14) to be complete, one must identify operator definitions for the fully unintegrated parton correlation functions, F, J, and S.

Graphically, Eq. (14) takes the form,

$W^{\mu\nu}(q,P) \approx \sum_j \frac{e_j^2}{4\pi} \Bigg|$  $\Bigg|^2$

$\times \Bigg($  $\Bigg)$

$\Bigg($  $\Bigg)$

$\Bigg($  $\Bigg)$. (15)

In the factors representing the PCFs, the double lines represent Wilson lines that appear in the definitions. Arbitrarily many collinear/soft gluons are shown connecting the Wilson lines to the target/jet/soft bubbles. (Double counting subtraction terms are needed to get the correct expressions for the PCFs and hard scattering coefficients. For simplicity, we do not show the subtraction terms explicitly.) A procedure for obtaining Eqs. (14), (15) was worked out in [6] for the case of an Abelian theory. (The result is highly suggestive of a similar result for the non-Abelian case, though the use of non-Abelian Ward identities is not yet well-enough understood for an explicit proof.) There, explicit operator definitions for the PCFs in Eq. (14) are given. An example is the fully unintegrated quark PDF, determined in [6] to be,

$$\tilde{F}(w, y_p, y_s, \mu) = \langle p|\bar{\psi}(w)V_w^\dagger(n_s)I_{n_s;w,0}\frac{\gamma^+}{2}V_0(n_s)\psi(0)|p\rangle_{\rm R}\,, \qquad (16)$$

where the direction of the Wilson line in this formula is,

$$n_s = (-e^{-y_s}, e^{y_s}, \mathbf{0}_{\rm t})\,, \qquad (17)$$

and $y_s \approx 0$. The subscript R indicates that renormalized operators are used. Similar definitions are also given in [6] for S and J.

The advantage of the fully unintegrated approach is its generality. Issues involving final state kinematics can be addressed more rigorously than in more standard approaches. The disadvantage is that it is much more complicated. Eq. (14) involves three PCFs, all involving multiple parameters.

Each of these PCFs needs to be fitted to experimental data. Furthermore, the arbitrary cutoffs on rapidity in the definitions of the PCFs mean that we will need evolution equations in several variables, in addition to the usual evolution in the hard scale, μ. Another complication is the issue of how the PCFs of the fully unintegrated formalism are related to other objects, such as the k_{t}-unintegrated PDFs or the fully integrated PDFs.

3. Outlook and open problems

3.1. Next-to-leading order

A consistent formulation of factorization for a single outing jet allows, in principle, for a determination of next-to-leading order (NLO) hard scattering coefficients in a double-counting subtraction formalism, since one now knows what to subtract from higher order graphs. A subtractive approach to higher order hard scattering coefficients already exists for the case of scalar-ϕ^3 theory in six dimensions [7] in the context of Monte Carlo event generators. For the less trivial case of an Abelian gauge theory, the results of [6] allow, in principle, for the construction of the subtractive formalism in [7] to be extended to the case of a gauge theory. One complication that arises in the treatment of NLO hard scattering coefficients is the issue of mapping the exact parton used to evaluate the PCFs to the approximated momenta used to evaluate the hard scattering amplitude in a way that is consistent with factorization. One such prescription was given in [7] for the scalar-ϕ^3 theory in six dimensions. A very similar prescription is likely to work in the case of a gauge theory, though a more explicit treatment of high orders in the fully unintegrated treatment is needed. However, even without a complete formulation of factorization with fully unintegrated PCFs, it is possible that using a mapping prescription that is consistent with the subtraction technique will lead to improved calculations.

4. Extension to other processes

For the fully unintegrated approach to be useful, one must be able to constrain the PCFs with experimental data. For this to be possible, the PCFs must appear in the factorization theorems for other processes. It is likely possible to prove factorization for certain processes, such as e^+e^- annihilation. For other processes, such as those involving pp collisions, one has to contend with various factorization breaking effects [8], which may require some modification to the direct approach discussed here.

I would like to thank the organizers of this very interesting conference. I would also like to thank the Polish Academy of Sciences and the Ministry of Science and Higher Education. This talk summarizes work done in collaboration with John Collins and Anna Staśto. Feynman diagrams were made using Jaxodraw [9]. This work was supported by the US DOE under grant number DE-FG02-90ER-40577.

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