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REVIEW OF APPLICATIONS OF YFS-STYLE RESUMMATION IN QUANTUM FIELD THEORY VIA MONTE CARLO METHODS*

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We review the application of exact, amplitude-based, YFS-style resummation in quantum field theory via Monte Carlo methods.

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1. Preface

It is with great pleasure that I present this review of the application of YFS-style [1] exact, amplitude based resummation via Monte Carlo methods on the occasion of the $60^{\rm th}$ birthday of Prof. S. Jadach, my friend and collaborator since 1985. In the review, we intend to highlight some of the many pioneering contributions which Prof. Jadach has made to this important subject. We are all grateful to him for all that he has taught us about the subject.

2. Introduction

The theoretical foundation of the subject of this discussion is the pioneering paper by Yennie, Frautschi, Suura published already in 1961 [1]. In this paper, the exact result for the processes $f_1(p_1) + f_2(p_2) \to f_3(p_3) + f_4(p_4) + n(\gamma)$ is given as

$$d\sigma_{\rm exp} = e^{2\alpha{\rm ReB}+2\alpha\tilde{\rm B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j} \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(p_1+p_2-p_3-p_4-\sum k_j)+D} \times \bar{\beta}_n(k_1,\ldots,k_n) \frac{d^3p_3}{p_3^0} \frac{d^3p_4}{p_4^0}\,, \quad (1)$$

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where the hard photon residuals, $\bar{\beta}_n(k_1,\ldots,k_n)$, as defined in Ref. [1], are free of infrared singularities to all orders in α. We use an obvious notation for the 4-momenta $\{p_i\}$ for the scattering charged particles $\{f_i\}$ and the infrared functions B, $\tilde{B}$, and D are as defined in Ref. [1]. The exactness of (1) is essential for precision theory applications.

The presentation is organized as follows. In the next sections, we review the applications of (1). We discuss in this connection the period before precision electroweak (EW) physics at LEP/SLC, the era of precision EW physics, the applications of the QCD extension of (1) for precision LHC physics and recent results obtained from applications of the extension of (1) for quantum general relativity. We conclude with some discussion of possible future applications.

3. Applications: comparative observations

The original applications [1] of (1) were at the precision of the leading term, the $\bar{\beta}_0$-level, in which one retains only the $n=0$-term therein. The 4-momentum conservation in (1) is then treated exactly, which necessitates integration over the y-dependent exponential factors therein. This was done in Ref. [1] already, with the result, for example, for initial state radiation (ISR) in e^+e^- annihilation

$$d\sigma_{\rm exp} \cong \gamma F_{\rm YFS}(\gamma)(1-z)^{\gamma-1}\sigma_{\rm B}\, dz\,, \tag{2}$$

where we have defined $z=s'/s$, $\gamma=(2\alpha/\pi)(\ln\frac{s}{m^2}-1)$, and

$$F_{\rm YFS}(\gamma)=\frac{e^{-C\gamma}}{\Gamma(1+\gamma)}\,. \tag{3}$$

Here, $C=0.5772\ldots$ is Euler's constant and $\sigma_{\rm B}$ is the respective Born cross section. Only the leading terms in γ are then retained in this $\bar{\beta}_0$-level approximation [1]. The accuracy is expected to be in the $\lesssim 10\%$ regime, which is quite adequate for applications in which there were errors on $\sigma_{\rm B}$ that could be much larger. It is also important to note that these early applications of (1) were (semi-)analytical in nature.

The LEP1/SLC, LEP2 era marked the application of (1) to precision predictions from quantum field theory via exact Monte Carlo methods. The collaboration in this connection between the author and Staszek (Prof. Jadach) started in the 1985–1986 time frame as a result of a Radiative Corrections Workshop organized at SLAC by Prof. G. Feldman, who at that time was a spokesman for the MkII Collaboration at the SLC. We were both invited to participate in that workshop and as a result we began discussion of the feasibility to realize the exact result (1) by Monte Carlo methods[1]. The key

[1] This was a long and technical discussion, some of it done on walks in the Tatra Mountains at a Zakopane Summer School, for example.

issue, after much successful discussion on other issues, such as our reduction procedure [2], *etc.*, was the realization by Monte Carlo methods of the factor e^D in (1). The pioneering solution was given by Jadach in Ref. [3]. The title of the paper, "Yennie–Frautschi–Suura Soft Photons in the Monte Carlo Event Generators", underscores how important it was to the Jadach–Ward approach to precision theory for quantum field theory predictions for physical processes: it opened the way to use the exact result (1) via Monte Carlo methods so that arbitrarily precise predictions could be obtained on an event-by-event basis. The solution presented in Ref. [3] is to date the only such solution known and thus is a true testament to the genius of its creator.

With the complete set of ingredients now in place to realize (1), we published in 1988 in Ref. [2] the first realistic MC for precision SLC/LEP1 physics, YFS1, an exact $\mathcal{O}(\alpha)$, YFS-exponentiated multiple photon MC for $e^+e^- \rightarrow f\bar{f} + n(\gamma),\;\; f \neq e$. Here, the modifier "YFS" denotes that the exponentiation is the resummation given by (1). As we discuss in Ref. [2], the precision tag for YFS1 in Z physics is $\lesssim 1\%$. This was followed in 1989 with the publication in Ref. [4] of the first realistic exact $\mathcal{O}(\alpha)$, YFS-exponentiated multiple photon MC for $e^+e^- \rightarrow e^+e^- + n(\gamma)$ at low angles, BHLUMI 1.0, for Z physics, where the primary applications were precision luminosity predictions. Again, the precision tag is $\lesssim 1\%$.

The large number of Z's at LEP1 (2×10^7 were detected) necessitated per mille level theory precision in order that the theoretical error would not compromise the outstanding experimental error in the attendant tests of the EW and QCD theories. We therefore developed the YFS2 and YFS3 level MC realizations of (1) in Refs. [5,6], wherein the precision tags are 0.1% for initial state radiation and for the combination of initial state and final state radiation, respectively.

Continuing in this way, working as well with our collaborators M. Melles, W. Placzek, E. Richter-Was, M. Skrzypek, Z. Was and S. Yost, we have developed the following YFS MC event generators, all realizations of (1): KORALZ3.8,4.04 [7] with 0.1% precision tag on $2f$ production at the Z regime in LEP1/SLC; BHLUMI 2.01,2.30,4.04 [8] for the LEP1/SLC luminosity process small angle Bhabha scattering with the final precision tag of 0.061%(0.054%), according as one does not (does) implement the soft pairs effect from either Ref. [9,10]; and BHWIDE [11] for the large angle Bhabha scattering with precision tag 0.2% at the Z regime at LEP1/SLC.

The advent of LEP2, and its attendant 2×10^5 W pairs, created the need for precision predictions for W-pair productions and decay, the $4f$ background processes, radiative return Z production as well as the need for reliable $2Z$ production predictions. We developed [12] the new coherent realization of (1) to treat the Z-radiative return events at high precision by treating the real emission IR singularities at the level of amplitude in complete analogy with the original treatment of the virtual IR singularities by Yennie, Frautschi and Suura in Ref. [1]. We refer to this form of the theory as the CEEX theory. It is realized in the event generator KK MC [13], which gives 0.2% precision on radiative return $2f$ production at LEP2 energies. In addition, for LEP2 our collaboration developed the MC's YFSWW3 [14] with 0.4% precision on WW production, KoralW(1.02,1.42) [15] with 1.0% precision on the $4f$ background processes, KoralW1.51 [16], the concurrent KoralW&YFSWW3 MC, with 0.4% on $4f$ production near the WW regime, and YFSZZ [17] with 2% precision for ZZ production. These are all state-of-the-art results for LEP2 based on the rigorous MC realization of (1) on an event-by-event basis. We also determined [18] the precisions of BHWIDE and BHLUMI at LEP2 as 0.4% and 0.122%, respectively. We now present some exemplary results based on these seminal calculations.

3.1. Exemplary results

The MC KoralZ was a workhorse for LEP1,2 physics. As an example of its many applications, we illustrate with the analysis by the ALEPH Collaboration [19] of their data on μ-pair production from 20 GeV to 136 GeV:

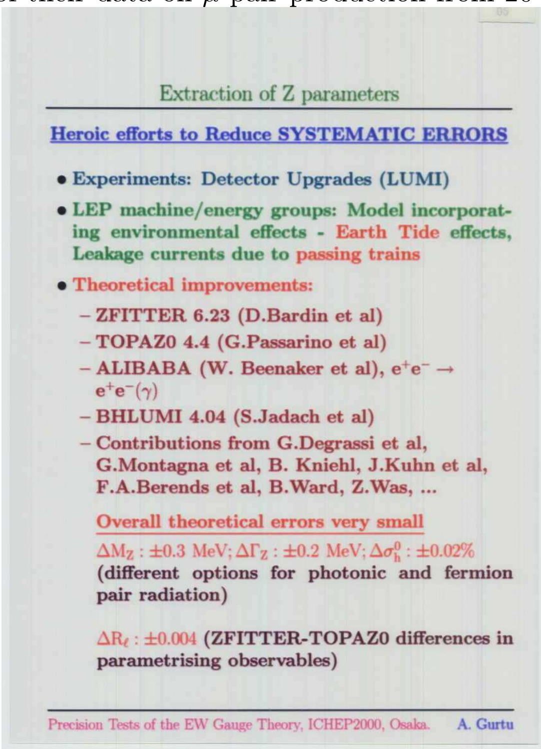

Fig. 1. Summary talk of EW theory progress on Z physics as presented by Gurtu at ICHEP2000, see Ref. [20].

we quote from Ref. [19], "In order to study the effect of the experimental cuts, more than 2×10^6 events were produced with full detector simulation, using the DYMU3[8] and KORALZ 4.0 [9] Monte Carlo event generators for the exclusive and inclusive analysis, respectively, at several nominal LEP energies. Radiation of hard photons in the initial and final state is treated at $\mathcal{O}(\alpha)$ by DYMU3 and at $\mathcal{O}(\alpha^2)$ by KORALZ 4.0. In KORALZ the radiation of soft photons is included at all orders by exponentiation". This is one of many examples.

In Fig. 1, we show the summary of the progress on precision EW theory as presented by Gurtu in his review for ICHEP2000 at Osaka [20]. We see in the figure that he shows BHLUMI4.04 as a key element in these improvements which allowed the proper exploitation of the LEP data for precision SM tests.

For BHWIDE, there are also many examples of its seminal role in establishing the precision comparison between the Standard Model EW theory and the LEP data. We show in Fig. 2 the results presented by De Bonis [21] at ICHEP02, where he shows that BHWIDE gives outstanding agreement with the LEP observations of large angle Bhabha scattering.

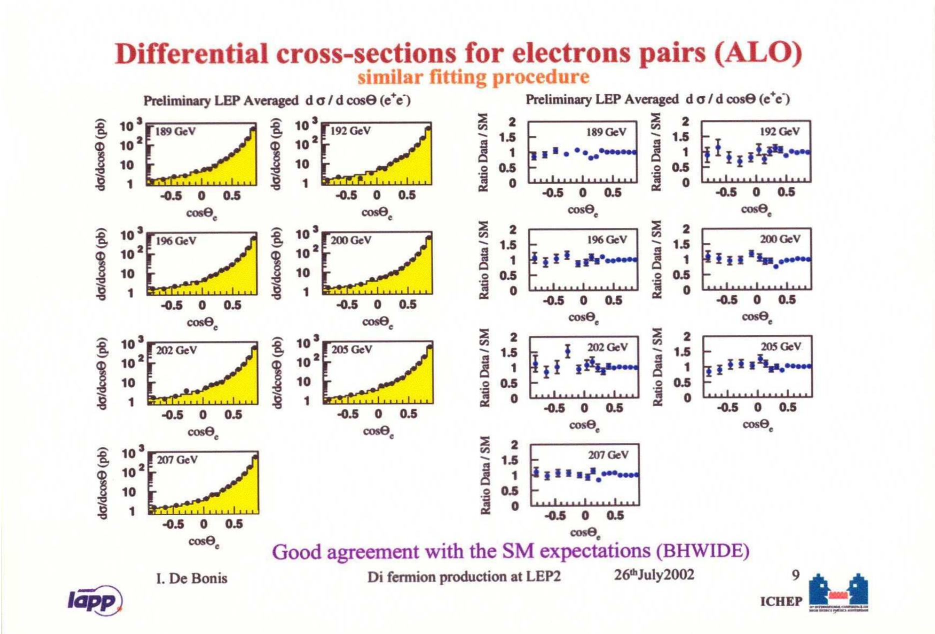

Fig. 2. Comparison of BHWIDE with precision LEP data as presented by De Bonis at ICHEP2002, see Ref. [21].

For YFSWW and KK MC, there are also many examples of their seminal role in precision LEP physics. To illustrate, we use again an example for from Ref. [20] as shown in Fig. 3 which summarizes the progress in theory for $2f$ and $4f$ processes at LEP1,2 for ICHEP2000. The MC YFSZZ is also featured in Fig. 3, as it provided state-of-the-art simulations for the Z-pair production data at LEP2. We see then in Figs. 4, 5 that the YFSWW3, along with RacoonWW [22], did indeed establish the proper normalization and simulation of the LEP2 WW pair production as predicted by the 't Hooft–Veltman non-Abelian gauge theory renormalization theory [23] and that YFSZZ did indeed provide state-of-the-art Z-pair production simulation for the LEP2 data.

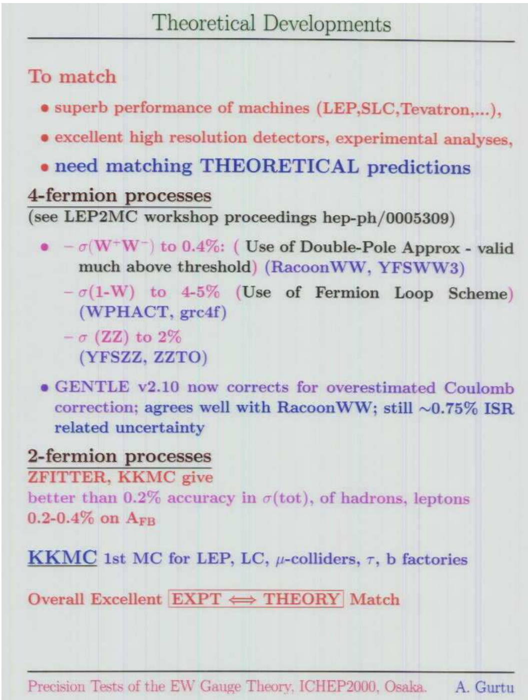

Fig. 3. Comparison of YFSWW3 and RacoonWW with precision LEP2 data as presented by Gurtu at ICHEP2000, see Ref. [20].

The Monte Carlo KoralW has played an essential role in the $4f/WW$ data analysis as well, providing as it did, precision simulation of the background processes for W-pairs as we have indicated. This is illustrated in Fig. 5. What we have illustrated are examples that indicate the broad effect that the Monte Carlo realization of (1) has had on tests of the SM using precision LEP data.

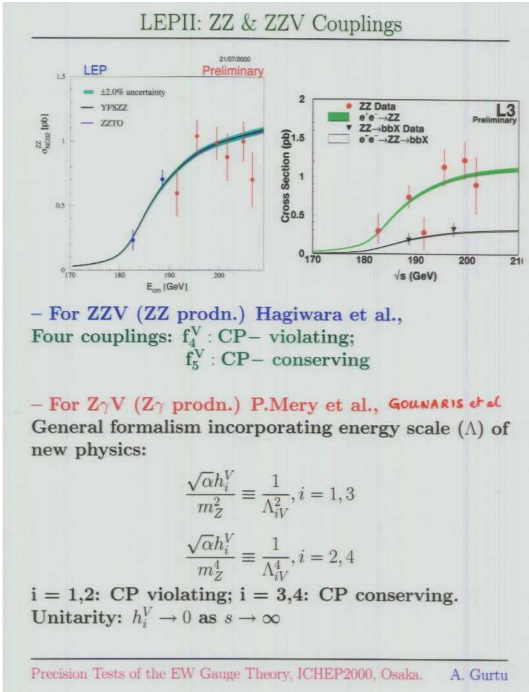

Fig. 4. Comparison of YFSZZ with LEP2 Z-pair production data as presented by Gurtu at ICHEP2000, see Ref. [20].

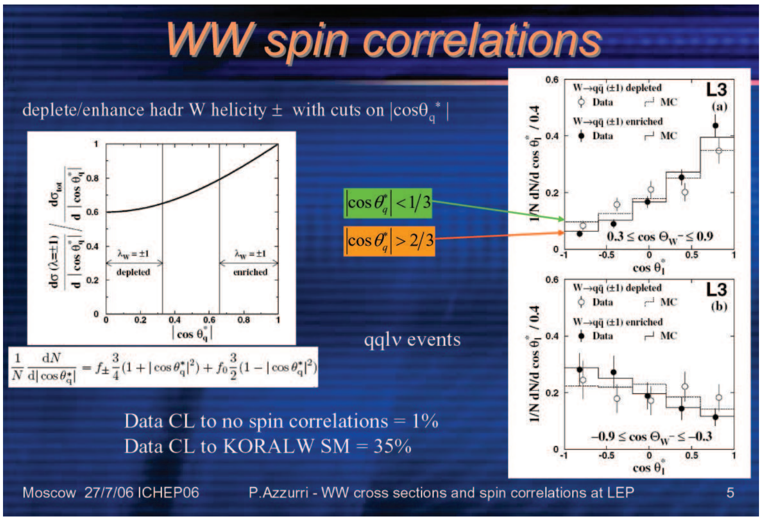

Fig. 5. Comparison of KoralW with LEP2 $WW/4f$ spin correlation production data as presented by Azzurri at ICHEP2006, see Ref. [24].

Indeed, these precision calculations, which we need to emphasize employed as well the pioneering EW libraries of Refs. [25] in isolating some of the purely weak exact results in the residuals $\bar{\beta}_n$, have played essential roles in determining the degree of agreement between then SM non-Abelian loop corrections to precision observables and the value of these effects as measured by LEP data. This is illustrated in Fig. 6 as it is presented in Ref. [26] at ICHEP06. The many consequences of the latter comparison, such as its implications for the mass of the still-sought SM Higgs particle — a main objective for discovery at LHC, are illustrated in Fig. 7. The precision comparison between the SM expectations and the LEP data establish the

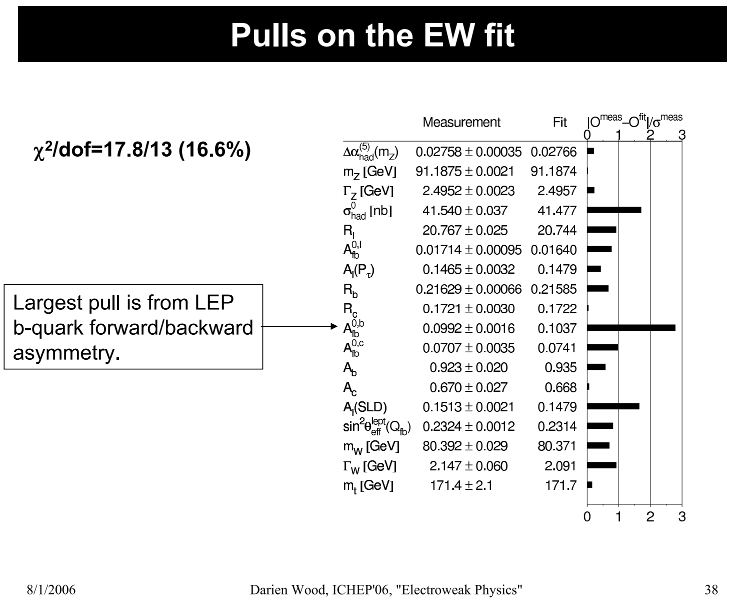

Fig. 6. Comparison of precision EW data with the SM theory as presented by Wood at ICHEP2006, see Ref. [26].

correctness of the 't Hooft–Veltman renormalization theory for non-Abelian gauge theories at the one-loop level and gives us confidence that the origin of EW symmetry breaking, as it is represented by the Higgs boson, is within reach of LHC experimentation. In addition, when the precise value of the running $\alpha_{\rm s}(Q)$ is extracted for the the LEP data and compared with data at lower energies [27], one also obtains experimental proof of the running of the latter coupling as predicted by the asymptotic freedom discovery of Gross, Wilczek [28] and Politzer [29]. The Royal Swedish Academy [30] has emphasized these points in awarding the 1999 Nobel Prize in Physics to Profs. G. 't Hooft and M. Veltman, with the citation:

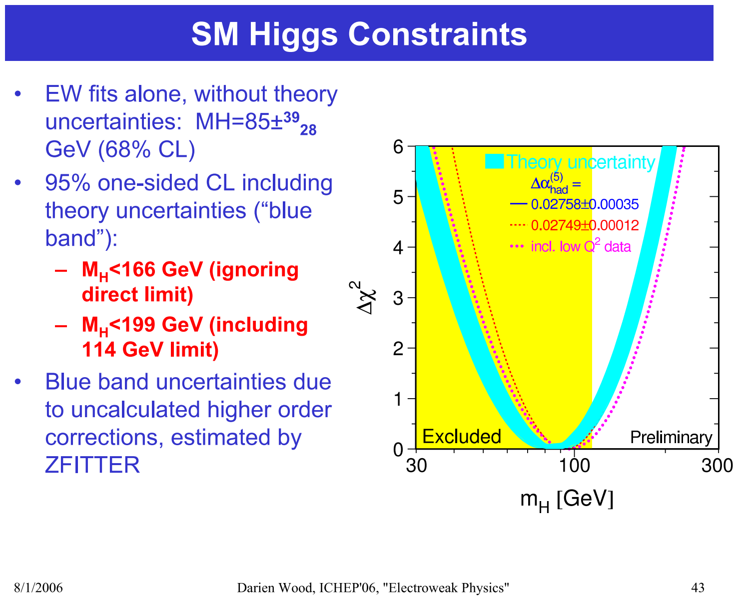

Fig. 7. Implications for the mass of the SM Higgs particle from the SM EW fit to precision LEP data as presented as presented by Wood at ICHEP2006, see Ref. [26].

> "... for elucidating the quantum nature of the electroweak interactions in physics ... The theory's predictions verified ... large quantities of W and Z have recently been produced under controlled conditions at the LEP accelerator at CERN. Comparisons between measurements and calculations have all the time showed great agreement, thus supporting the theory's predictions ..."

and the 2004 Nobel Prize in Physics to Profs. D.J. Gross, F. Wilczek and H.D. Politzer, with the citation

> "...The theory has been tested in great detail, in particular during recent years at the European Laboratory for Particle Physics, CERN, in Geneva ...".

Prof. Jadach and his collaborators have made via YFS-based MC methods an essential contribution to the realization of the two respectively cited precision studies.

4. QCD and QED⊗QCD extension

Already at the start of the preparations for the physics program for the now canceled SSC, we moved our attention to the application of the analog of (1) to the QCD theory in Refs. [31]. This development has resulted in the QCD resummation formula [32], for the processes $f_1(p_1) + f_2(q_1) \rightarrow f_3(p_2) + f_4(q_2) + n(G)$,

$$d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j} \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_j)+D_{\rm QCD}} \times \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \frac{d^3p_2}{p_2^0} \frac{d^3q_2}{q_2^0}\,, \tag{4}$$

where now the hard gluon residuals $\tilde{\bar{\beta}}_n(k_1,\ldots,k_n)$ are free of all infrared divergences to all orders in $\alpha_{\rm s}(Q)$. The functions ${\rm SUM_{IR}(QCD)}, D_{\rm QCD}$, together with the attendant basic infrared functions $B^{nls}_{\rm QCD}, \tilde{B}^{nls}_{\rm QCD}, \tilde{S}^{nls}_{\rm QCD}$ are specified in Ref. [32]. Here, Q is the relevant hard scale. We have shown that (4) leads to an independent cross check of the size of threshold resummation effects in $t\bar{t}$ production at FNAL at the 1% level as found in Ref. [33]. More recently, realizing that for LHC physics the EW corrections can be significant in a 1% error budget, we have extended the result (4) to the simultaneous resummation of QED and QCD, QED⊗QCD resummation [34]

$$d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3k'_{j_2}}{k'_{j_2}} \int \frac{d^4y}{(2\pi)^4} \times e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{\rm QCED}} \times \tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m) \frac{d^3p_2}{p_2^0} \frac{d^3q_2}{q_2^0}\,, \tag{5}$$

where the new YFS [1, 2] residuals, defined in Ref. [34], $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n; k'_1,\ldots,k'_m)$, with n hard gluons and m hard photons, represent the successive application of the YFS expansion first for QCD and subsequently for QED. The functions ${\rm SUM_{IR}(QCED)}, D_{\rm QCED}$ are determined from their analogs ${\rm SUM_{IR(QCD)}}), D_{\rm QCD}$ via the substitutions

$$\begin{aligned} B^{nls}_{\rm QCD} &\rightarrow B^{nls}_{\rm QCD} + B^{nls}_{\rm QED} \equiv B^{nls}_{\rm QCED}\,, \\ \tilde{B}^{nls}_{\rm QCD} &\rightarrow \tilde{B}^{nls}_{\rm QCD} + \tilde{B}^{nls}_{\rm QED} \equiv \tilde{B}^{nls}_{\rm QCED}\,, \\ \tilde{S}^{nls}_{\rm QCD} &\rightarrow \tilde{S}^{nls}_{\rm QCD} + \tilde{S}^{nls}_{\rm QED} \equiv \tilde{S}^{nls}_{\rm QCED} \end{aligned} \tag{6}$$

everywhere in expressions for the latter functions given in Refs. [32]. The residuals $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$ are free of all infrared singularities. The result in (5) is a representation that is exact and that can therefore be used to make contact with parton shower MC's without double counting or the unnecessary averaging of effects such as the gluon azimuthal angular distribution relative to its parent's momentum direction.

Indeed, from the result (5) and the standard formula for the hadron cross section

$$d\sigma = \sum_{i,j}\int dx_1 dx_2 F_i(x_1)F_j(x_2)\, d\hat{\sigma}_{\rm exp} \tag{7}$$

we have immediately two issues to address: shower/ME matching, which we do preferably by shower-subtracted residuals, $\tilde{\bar{\beta}}_{m,n} \to \hat{\tilde{\bar{\beta}}}_{m,n}$, as presented in Ref. [35], and for MC stability, IR-improved DGLAP-CS theory [36], a new exponentiated scheme for the respective kernels, P_{AB}, reduced cross sections, and parton distributions,

$$\begin{aligned} F_1,\ \hat{\sigma}\ &\to\ F'_i,\ \hat{\sigma}' \\ \text{for}\ \ P_{qq}\ &\to\ P^{\rm exp}_{qq} = C_F F_{\rm YFS}(\gamma_q)e^{\frac{1}{2}\delta_q}\frac{1+z^2}{1-z}(1-z)^{\gamma_q}\,,\ etc., \end{aligned} \tag{8}$$

giving the same value for the respective hadron cross section σ, with improved MC stability.

In addition, other technical checks are now open, such as the issue of setting all quark masses m_q to zero in the ISR at $\mathcal{O}(\alpha_{\rm s}^n)$, $n \geq 2$ due to the theorem in Refs. [37, 38], according to which there is a lack of Bloch–Nordsieck cancellation of IR singularities unless $m_q = 0$. We show in Ref. [39] that the result (4) obviates this theorem.

The matter of an independent cross-check of the standard backward evolution algorithm for the parton shower itself [40] is also under study with the results of Refs. [41, 42]. Staszek's group are actively involved in this development.

There are many more issues for which we do not have space to list here: they are all under study. All of the necessary theoretical formalism is at hand — this underscores the need to support exact results for higher order calculations, cross checks, tests, *etc.*, to prove 1% precision for LHC luminosity processes for example. We can not emphasize this too much.

5. Extension to QGR

The exactness of the YFS re-arrangement means that we can apply the same resummation algebra to quantum gravity [43–46]. We find that the scalar propagator for mass m resums in quantum gravity to

$$i\Delta'_F(k)|_{\text{resummed}} = \frac{ie^{B''_g(k)}}{(k^2 - m^2 - \Sigma'_s + i\epsilon)} \tag{9}$$

for ($\Delta = k^2 - m^2$)

$$\begin{aligned} B''_g(k) &= -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\epsilon} \frac{1}{(\ell^2 + 2\ell k + \Delta + i\epsilon)^2} \\ &= \frac{\kappa^2 |k^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2 + |k^2|}\right), \end{aligned} \tag{10}$$

where the latter form holds for the UV regime, so that (9) falls faster than any power of $|k^2|$. An analogous result [43] holds for $m = 0$. We also note that, as Σ'_s starts in $\mathcal{O}(\kappa^2)$, we may drop it in calculating one-loop effects. It follows that when the respective analogs of (9) are used, one-loop corrections are finite. In fact, it can be shown that the use of our resumed propagators renders all quantum gravity loops UV finite [43–46]. We have called this representation of the quantum theory of general relativity resummed quantum gravity (RQG). Its phenomenology is under study: we show in Refs. [46] that the final state of Hawking radiation [47] leads to Planck scale cosmic rays, *etc.*

6. Future

All of the developments extend to higher energy and/or higher precision at lower energies down to 1GeV: at the B-Factory, the KK MC is already in wide use [48]; at the Φ-factories there are cross checks [49] using KK MC with the distributions of the program PHOKHARA [50], *etc.* For higher energies in e^+e^- annihilation, YFSWW, KoralW, BHWIDE, BHLUMI and KK MC are all in play. For example, the ILC luminosity requirement [51] is 0.01%.

We show in Table I what the extension of BHLUMI from version 4.04 to version 5.0 for 0.011% would involve (the references in the table can be found in Refs. [52, 53]). We have already explained in Ref. [52] what this achievement would involve and how long in time it would take, about 3 years. Again, it is all a question of support. It may be needed by 2025–2030?

From 1987 to 2027, what fun it is! And, we all owe a debt of special thanks to Staszek for his seminal role in it.

TABLE I

Summary of the total (physical + technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for the angular range within 1°–3°, for LEP2 it covers energies up to 176 GeV, and angular range within 1°–3° and 3°–6°, and for ILC the projection is for 3°–6° and energies up to 3 TeV.

	LEP I		LEP II	ILC
Type of correction/error	Past [BW22,BW23]	Present	Present [BW16,BW17]	Future
Missing photonic $\mathcal{O}(\alpha^2)$ [BW24]	0.10%	0.027%	0.04%	0.001%
Missing photonic $\mathcal{O}(\alpha^3)$ [BW25]	0.015%	0.015%	0.03%	0.0011%
Vacuum polarization [BW26,BW27]	0.04%	0.04%	0.10%	0.0096%
Light pairs [BW19,BW20]	0.03%	0.03%	0.05%	0.005%
Z-exchange [BW28]	0.015%	0.015%	0.0%	0.001%
Total	0.11%	0.061%	0.122%	0.011%

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