Vol. 39 (2008) *ACTA PHYSICA POLONICA B* No 8

CONTROLLED QUANTUM TELEPORTATION OF A TWO-QUBIT ARBITRARY STATE

Xiao-Ming Xiu, Li Dong, Ya-Jun Gao, Feng Chi

Department of Physics, Bohai University, Jinzhou 121000, China

(Received April 3, 2008)

A controlled teleportation scheme of a two-qubit state in general form is proposed. The sender performs four-qubit jointly projective measurement and tells the measurement outcome to the receiver. The receiver performs the unitary transformations on his qubits to obtain the original state under the cooperation of the controller.

PACS numbers: 03.67.Hk, 03.65.Ud

Quantum teleportation is a charming process of quantum information, which can make an unknown quantum state from one place to another place with the aid of quantum entanglement. In 1993, Charles Bennett *et al.* [1] first showed that a single qubit unknown state can be teleported via an Einstein–Podolsky–Rosen (EPR) pair [2]. Since then, quantum teleportation is always an interesting topic due to its important applications in quantum computation and quantum communication. A number of theoretical and experimental schemes were presented [3–15].

In recent years, controlled quantum teleportation was studied by some Refs. [16–24], in which the controllers are included and teleportation of an unknown state cannot be completed without the controllers' agreement and cooperation. In these schemes, the quantum channel is the product state of two entangled states when a two-qubit arbitrary state is teleported.

In this paper, a scheme for controlled teleporting a two-qubit arbitrary and unknown state when a five-qubit entangled state is used as quantum channel is proposed.

A two-qubit arbitrary state can be denoted as the following state

$$|\xi\rangle = (a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle)_{a_1,a_2}\,, \tag{1}$$

where the unknown coefficients satisfy the normalized condition, $|a|^2+|b|^2+|c|^2+|d|^2=1$.

The sender (Alice) uses a five-qubit entangled state as quantum channel to realize the controlled quantum teleporation of the above two-qubit arbitrary state,

$$
\begin{aligned}
&|\phi_5\rangle_{A_1,A_2,B_1,B_2,C} = \frac{1}{4\sqrt{2}}(|00000\rangle + |00110\rangle - |01010\rangle + |01100\rangle + |10010\rangle \\
&+ |10100\rangle + |11000\rangle - |11110\rangle + |10001\rangle + |10111\rangle - |11011\rangle \\
&+ |11101\rangle + |00011\rangle + |00101\rangle + |01001\rangle - |01111\rangle)_{A_1,A_2,B_1,B_2,C}\,. \qquad (2)
\end{aligned}
$$

Alice remains qubits (A_1, A_2) and transmits qubits (B_1, B_2) and qubit C to the receiver (Bob) and the controller (Charlie), respectively. Bob and Charlie confirm Alice that they have received qubits (B_1, B_2) and qubit C through classical channel.

The whole system state $|\Psi\rangle$ including the two-qubit state to be teleported and quantum channel of a five-qubit entangled state can be denoted as

$$
\begin{aligned}
|\Psi\rangle \;=\; & \frac{1}{2}|\Pi^1\rangle_{a_1,a_2,A_1,A_2}(a|000\rangle + b|010\rangle + c|100\rangle + d|110\rangle \\
& + a|101\rangle + b|111\rangle + c|001\rangle + d|011\rangle)_{B_1,B_2,C} \\
& + \frac{1}{2}|\Pi^2\rangle_{a_1,a_2,A_1,A_2}(a|010\rangle + b|000\rangle + c|110\rangle + d|100\rangle \\
& + a|111\rangle + b|101\rangle + c|011\rangle + d|001\rangle)_{B_1,B_2,C} \\
& + \frac{1}{2}|\Pi^3\rangle_{a_1,a_2,A_1,A_2}(a|000\rangle - b|010\rangle + c|100\rangle - d|110\rangle \\
& + a|101\rangle - b|111\rangle + c|001\rangle - d|011\rangle)_{B_1,B_2,C} \\
& + \frac{1}{2}|\Pi^4\rangle_{a_1,a_2,A_1,A_2}(-a|010\rangle + b|000\rangle - c|110\rangle + d|100\rangle \\
& - a|111\rangle + b|101\rangle - c|011\rangle + d|001\rangle)_{B_1,B_2,C} \\
& + \frac{1}{2}|\Pi^5\rangle_{a_1,a_2,A_1,A_2}(a|100\rangle + b|110\rangle + c|000\rangle + d|010\rangle \\
& + a|001\rangle + b|011\rangle + c|101\rangle + d|111\rangle)_{B_1,B_2,C} \\
& + \frac{1}{2}|\Pi^6\rangle_{a_1,a_2,A_1,A_2}(a|110\rangle + b|100\rangle + c|010\rangle + d|000\rangle \\
& + a|011\rangle + b|001\rangle + c|111\rangle + d|101\rangle)_{B_1,B_2,C} \\
& + \frac{1}{2}|\Pi^7\rangle_{a_1,a_2,A_1,A_2}(a|100\rangle - b|110\rangle + c|000\rangle - d|010\rangle \\
& + a|001\rangle - b|011\rangle + c|101\rangle - d|111\rangle)_{B_1,B_2,C} \\
& + \frac{1}{2}|\Pi^8\rangle_{a_1,a_2,A_1,A_2}(-a|110\rangle + b|100\rangle - c|010\rangle + d|000\rangle \\
& - a|011\rangle + b|001\rangle - c|111\rangle + d|101\rangle)_{B_1,B_2,C} \\
& + \frac{1}{2}|\Pi^9\rangle_{a_1,a_2,A_1,A_2}(a|000\rangle + b|010\rangle - c|100\rangle - d|110\rangle \\
& + a|101\rangle + b|111\rangle - c|001\rangle - d|011\rangle)_{B_1,B_2,C}
\end{aligned}
$$

$$+\frac{1}{2}\left|\Pi^{10}\right\rangle_{a_1,a_2,A_1,A_2}(a|010\rangle+b|000\rangle-c|110\rangle-d|100\rangle$$
$$+a|111\rangle+b|101\rangle-c|011\rangle-d|001\rangle)_{B_1,B_2,C}$$
$$+\frac{1}{2}\left|\Pi^{11}\right\rangle_{a_1,a_2,A_1,A_2}(a|000\rangle-b|010\rangle-c|100\rangle+d|110\rangle$$
$$+a|101\rangle-b|111\rangle-c|001\rangle+d|011\rangle)_{B_1,B_2,C}$$
$$+\frac{1}{2}\left|\Pi^{12}\right\rangle_{a_1,a_2,A_1,A_2}(-a|010\rangle+b|000\rangle+c|110\rangle-d|100\rangle$$
$$-a|111\rangle+b|101\rangle+c|011\rangle-d|001\rangle)_{B_1,B_2,C}$$
$$+\frac{1}{2}\left|\Pi^{13}\right\rangle_{a_1,a_2,A_1,A_2}(-a|100\rangle-b|110\rangle+c|000\rangle+d|010\rangle$$
$$-a|001\rangle-b|011\rangle+c|101\rangle+d|111\rangle)_{B_1,B_2,C}$$
$$+\frac{1}{2}\left|\Pi^{14}\right\rangle_{a_1,a_2,A_1,A_2}(-a|110\rangle-b|100\rangle+c|010\ \rangle+d|000\rangle$$
$$-a011\rangle-b|001\rangle+c|111\rangle+d|101\rangle)_{B_1,B_2,C}$$
$$+\frac{1}{2}\left|\Pi^{15}\right\rangle_{a_1,a_2,A_1,A_2}(-a|100\rangle+b|110\rangle+c|000\rangle-d|010\rangle$$
$$-a|001\rangle+b|011\rangle+c|101\rangle-d|111\rangle)_{B_1,B_2,C}$$
$$+\frac{1}{2}\left|\Pi^{16}\right\rangle_{a_1,a_2,A_1,A_2}(a|110\rangle-b|100\rangle-c|010\rangle+d|000\rangle$$
$$+a|011\rangle-b|001\rangle-c|111\rangle+d|101\rangle)_{B_1,B_2,C}\,. \tag{3}$$

Alice performs four-qubit jointly projective measurement on qubits (a_1, a_2, A_1, A_2) in the following bases

$$\left|\Pi^1\right\rangle_{a_1,a_2,A_1,A_2}$$
$$=\frac{1}{2\sqrt{2}}(|0000\rangle+|0011\rangle+|0110\rangle-|0101\rangle+|1010\rangle+|1001\rangle+|1100\rangle-|1111\rangle)\,,$$
$$\left|\Pi^2\right\rangle_{a_1,a_2,A_1,A_2}$$
$$=\frac{1}{2\sqrt{2}}(|0100\rangle+|0111\rangle+|0010\rangle-|0001\rangle+|1110\rangle+|1101\rangle+|1000\rangle-|1011\rangle)\,,$$
$$\left|\Pi^3\right\rangle_{a_1,a_2,A_1,A_2}$$
$$=\frac{1}{2\sqrt{2}}(|0000\rangle+|0011\rangle+|0101\rangle-|0110\rangle+|1010\rangle+|1001\rangle+|1111\rangle-|1100\rangle)\,,$$
$$\left|\Pi^4\right\rangle_{a_1,a_2,A_1,A_2}$$
$$=\frac{1}{2\sqrt{2}}(|0100\rangle+|0111\rangle+|0001\rangle-|0010\rangle+|1110\rangle+|1101\rangle+|1011\rangle-|1000\rangle)\,,$$
$$\left|\Pi^5\right\rangle_{a_1,a_2,A_1,A_2}$$
$$=\frac{1}{2\sqrt{2}}(|1000\rangle+|1011\rangle+|1110\rangle-|1101\rangle+|0010\rangle+|0001\rangle+|0100\rangle-|0111\rangle)\,,$$
$$\left|\Pi^6\right\rangle_{a_1,a_2,A_1,A_2}$$
$$=\frac{1}{2\sqrt{2}}(|1100\rangle+|1111\rangle+|1010\rangle-|1001\rangle+|0110\rangle+|0101\rangle+|0000\rangle-|0011\rangle)\,,$$

$$\left|\Pi^{7}\right\rangle_{a_1,a_2,A_1,A_2}$$
$$= \frac{1}{2\sqrt{2}}(|1000\rangle + |1011\rangle + |1101\rangle - |1110\rangle + |0010\rangle + |0001\rangle + |0111\rangle - |0100\rangle)\,,$$
$$\left|\Pi^{8}\right\rangle_{a_1,a_2,A_1,A_2}$$
$$+\frac{1}{2\sqrt{2}}(|1100\rangle + |1111\rangle + |1001\rangle - |1010\rangle + |0110\rangle + |0101\rangle + |0011\rangle - |0000\rangle)\,,$$
$$\left|\Pi^{9}\right\rangle_{a_1,a_2,A_1,A_2}$$
$$= \frac{1}{2\sqrt{2}}(|0000\rangle + |0011\rangle + |0110\rangle - |0101\rangle - |1010\rangle - |1001\rangle + |1111\rangle - |1100\rangle)\,,$$
$$\left|\Pi^{10}\right\rangle_{a_1,a_2,A_1,A_2}$$
$$= \frac{1}{2\sqrt{2}}(|0100\rangle + |0111\rangle + |0010\rangle - |0001\rangle - |1110\rangle - |1101\rangle + |1011\rangle - |1000\rangle)\,,$$
$$\left|\Pi^{11}\right\rangle_{a_1,a_2,A_1,A_2}$$
$$= \frac{1}{2\sqrt{2}}(|0000\rangle + |0011\rangle + |0101\rangle - |0110\rangle - |1010\rangle - |1001\rangle + |1100\rangle - |1111\rangle)\,,$$
$$\left|\Pi^{12}\right\rangle_{a_1,a_2,A_1,A_2}$$
$$= \frac{1}{2\sqrt{2}}(|0100\rangle + |0111\rangle + |0001\rangle - |0010\rangle - |1110\rangle - |1101\rangle + |1000\rangle - |1011\rangle)\,,$$
$$\left|\Pi^{13}\right\rangle_{a_1,a_2,A_1,A_2}$$
$$= \frac{1}{2\sqrt{2}}(|1000\rangle + |1011\rangle + |1110\rangle - |1101\rangle - |0001\rangle - |0010\rangle + |0111\rangle - |0100\rangle)\,,$$
$$\left|\Pi^{14}\right\rangle_{a_1,a_2,A_1,A_2}$$
$$= \frac{1}{2\sqrt{2}}(|1100\rangle + |1111\rangle + |1010\rangle - |1001\rangle - |0110\rangle - |0101\rangle + |0011\rangle - |0000\rangle)\,,$$
$$\left|\Pi^{15}\right\rangle_{a_1,a_2,A_1,A_2}$$
$$= \frac{1}{2\sqrt{2}}(|1000\rangle + |1011\rangle + |1101\rangle - |1110\rangle - |0010\rangle - |0001\rangle + |0100\rangle - |0111\rangle)\,,$$
$$\left|\Pi^{16}\right\rangle_{a_1,a_2,A_1,A_2}$$
$$= \frac{1}{2\sqrt{2}}(|1100\rangle + |1111\rangle + |1001\rangle - |1010\rangle - |0110\rangle - |0101\rangle + |0000\rangle - |0011\rangle)\,.$$

After these measurements, the state of qubits (B_1, B_2, C) can be projected into one of the state denoted as Eq. (3).

If he agrees to help Bob to recover the state to be teleported, Charlie should perform the projective measurement on the basis of $\{|0\rangle, |1\rangle\}$ and inform Bob of the measurement outcome $|0\rangle$ or $|1\rangle$ via the classical channel.

After having received the measurement outcomes from Alice and Charlie, Bob should perform the unitary transformation which can be showed in Table I on the qubits (B_1, B_2) to make the two-qubit arbitrary state reconstruct on them. After Bob's unitary transformation, the teleportation process of the two-qubit arbitrary state from Alice to Bob completes successfully. Obviously, Bob can also teleport a two-qubit arbitrary state to Alice under the charge of Charlie.

TABLE I

Alice's jointly projective measurement outcomes (J.P.M.Os.) on qubits (a_1, a_2, A_1, A_2) and the corresponding unitary transformations (U.Ts.(0)) should be performed by Bob on qubits (B_1, B_2) if the measurement outcome of Charlie is $|0\rangle$, and the unitary transformations (U.Ts.(1)) should be performed by Bob on qubits (B_1, B_2) when the measurement outcome of Charlie is $|1\rangle$.

Alice's J.P.M.Os.	Bob's U.Ts.(0)	Bob's U.Ts.(1)
$\|\Pi^1\rangle_{a_1,a_2,A_1,A_2}$	$I_{B_1}I_{B_2}$	$X_{B_1}I_{B_2}$
$\|\Pi^2\rangle_{a_1,a_2,A_1,A_2}$	$I_{B_1}X_{B_2}$	$X_{B_1}X_{B_2}$
$\|\Pi^3\rangle_{a_1,a_2,A_1,A_2}$	$I_{B_1}Z_{B_2}$	$X_{B_1}Z_{B_2}$
$\|\Pi^4\rangle_{a_1,a_2,A_1,A_2}$	$I_{B_1}XZ_{B_2}$	$X_{B_1}XZ_{B_2}$
$\|\Pi^5\rangle_{a_1,a_2,A_1,A_2}$	$X_{B_1}I_{B_2}$	$I_{B_1}I_{B_2}$
$\|\Pi^6\rangle_{a_1,a_2,A_1,A_2}$	$X_{B_1}X_{B_2}$	$I_{B_1}X_{B_2}$
$\|\Pi^7\rangle_{a_1,a_2,A_1,A_2}$	$X_{B_1}Z_{B_2}$	$I_{B_1}Z_{B_2}$
$\|\Pi^8\rangle_{a_1,a_2,A_1,A_2}$	$X_{B_1}XZ_{B_2}$	$I_{B_1}XZ_{B_2}$
$\|\Pi^9\rangle_{a_1,a_2,A_1,A_2}$	$Z_{B_1}I_{B_2}$	$XZ_{B_1}I_{B_2}$
$\|\Pi^{10}\rangle_{a_1,a_2,A_1,A_2}$	$Z_{B_1}X_{B_2}$	$XZ_{B_1}X_{B_2}$
$\|\Pi^{11}\rangle_{a_1,a_2,A_1,A_2}$	$Z_{B_1}Z_{B_2}$	$XZ_{B_1}Z_{B_2}$
$\|\Pi^{12}\rangle_{a_1,a_2,A_1,A_2}$	$Z_{B_1}XZ_{B_2}$	$XZ_{B_1}XZ_{B_2}$
$\|\Pi^{13}\rangle_{a_1,a_2,A_1,A_2}$	$XZ_{B_1}I_{B_2}$	$Z_{B_1}I_{B_2}$
$\|\Pi^{14}\rangle_{a_1,a_2,A_1,A_2}$	$XZ_{B_1}X_{B_2}$	$Z_{B_1}X_{B_2}$
$\|\Pi^{15}\rangle_{a_1,a_2,A_1,A_2}$	$XZ_{B_1}Z_{B_2}$	$Z_{B_1}Z_{B_2}$
$\|\Pi^{16}\rangle_{a_1,a_2,A_1,A_2}$	$XZ_{B_1}XZ_{B_2}$	$Z_{B_1}XZ_{B_2}$

Further investigation indicates that the teleportation of a two-qubit arbitrary and unknown state can be controlled by four partners at most when the eight-qubit entangled state is utilized as quantum channel. Although the four-qubit jointly projective measurements which are necessary in the scheme are difficult to implement today, we expect that it can be realized in near future due to rapidly developing technology.

This study was supported by the National Natural Science Foundation of China Grant Nos.10647101 and 10704011.

REFERENCES

[1] C.H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, W.K. Wootters, *Phys. Rev. Lett.* **70**, 1895 (1993).

[2] A. Einstein, B. Podolsky, N. Rosen, *Phys. Rev.* **47**, 770 (1935).

[3] D. Bouwmeester, J.W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger *Nature* **390**, 575 (1997).

[4] D. Boschi, S. Branca, F.D. Martini, L. Hardy, S. Popescu, *Phys. Rev. Lett.* **80**, 1121 (1998).

[5] A. Karlsson, M. Bourennane, *Phys. Rev.* **A58**, 4394 (1998).

[6] B.S. Shi, A. Tomita, *Phys. Lett.* **A296**, 161 (2002).

[7] X.M. Xiu, L. Dong, Y.J. Gao, *Commun. Theor. Phys.* **47**, 625 (2007).

[8] W.L. Li, C.F. Li, G.C. Guo, *Phys. Rev.* **A61**, 034301 (2000).

[9] H. Lu, G.C. Guo, *Phys. Lett.* **A276**, 209 (2000).

[10] H.J. Cao, Y.Q. Guo, H.S. Song, *Chin. Phys.* **15**, 915 (2006).

[11] B.S. Shi, Y.K. Jiang, G.C. Guo, *Phys. Lett. A* **268**, 161 (2000).

[12] Y.J. Gu, Y.Z. Zheng, G.C. Guo, *Chin. Phys. Lett.* **18**, 1543 (2001).

[13] H.Y. Dai, P.X. Chen, C.Z. Li, *J. Opt. B: Quantum Semiclass* **6**, 106 (2004).

[14] J.X. Fang, Y.S. Lin, S.Q. Zhu, X.F. Chen, *Phys. Rev.* **A67**, 014305 (2003).

[15] L. Dong, X.M. Xiu, Y.J. Gao, *Chin. Phys.* **15**, 2835 (2006).

[16] F.L. Yan, D. Wang, *Phys. Lett.* **A316**, 297 (2003).

[17] C.P. Yang, S.I. Chu, S. Han, *Phys. Rev.* **A70**, 022329-1 (2004).

[18] F.G. Deng, C.Y. Li, Y.S. Li, H.Y. Zhou, Y. Wang, *Phys. Rev.* **A72**, 022338 (2005).

[19] T. Gao, F.L. Yan, Z.X. Wang, *Chin. Phys.* **14**, 893 (2005).

[20] Z.J. Zhang, Y.M. Liu, Z.X. Man, *Commun. Theor. Phys.* **44**, 847 (2005).

[21] H.C. Yuan, K.G. Qi, *Chin. Phys.* **14**, 1716 (2005).

[22] J. Yang, *Chin. Phys.* **14**, 2149 (2005).

[23] X.M. Xiu, L. Dong, Y.J. Gao, F. Chi, *Commun. Theor. Phys.* **48**, 261 (2007).

[24] X.M. Xiu, L. Dong, Y.J. Gao, F. Chi, *Chin. Phys.* **16**, 2194 (2007).