CHAOS CONTROL OF THE AUTONOMOUS VAN DER POL MATHIEU EQUATION FOR DUST-CHARGE FLUCTUATION IN DUSTY PLASMA USING BACK-STEPPING CONTROL

S.O. Kareem[†], U.E. Vincent[‡], J.A. Laoye, T.A. Akinola

Nonlinear and Statistical Physics Research Group
Department of Physics, Faculty of Science
Olabisi Onabanjo University, P.M.B. 2002, Ago-Iwoye, Nigeria

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This paper investigated chaos control of four-dimensional autonomous van der Pol Mathieu (vdPM) system that describes dust-charge fluctuation in dusty plasma. A recursive backstepping scheme was employed to design a single control input that effectively controlled the undesirable unstable behaviour of the vdPM system. Both theoretical analysis and numerical simulations were presented to illustrate the effectiveness of the proposed control scheme.

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1. Introduction

For over a decade, dynamic chaos theory has been deeply studied and applied to many fields extensively, such as optical system, biology, secure communication and plasma, amongst others [1–7]. The interest in the dynamic chaos theory is hinged on the well demonstrated applications of chaos including the explanation of many physical processes such as transition from laminar to turbulent fluid, multi-photon infrared absorption, microwave excitation, ionization of Rydberg atoms, and more specifically dust-charge fluctuation in dusty plasma [6, 7]. At the same time in many situations, chaos is an undesirable phenomenon which often leads to violent vibrations; irregular operations in mechanical systems; breaking materials and so on.

† semiukareem@gmail.com
‡ Present address: Department of Statistical Physics and Nonlinear Dynamics, Institute of Theoretical Physics, Technical University of Clausthal, Arnold-Sommer Str.6, 38678 Clausthal-Zellerfeld, Germany, u.vincent@tu-clausthal.de

Thus, from a practical point of view, it is often desired to convert and control the system dynamics with minimal effort suitably so that whenever chaotic motion is physically harmful it can be changed to a desired periodic or fixed point attractor. In this direction, increasing attention is being paid to this aspect since the first classical work of Ott, Grebogi and Yorke appeared in 1990 [8].

Basically, the chaos control problem can be formulated as follows. For a given chaotic system, a control mechanism is designed, which forces the system to maintain a desired dynamical behavior even when intrinsically chaotic. The designed control is added to an isolated chaotic system in form of a time dependent input function(s) (see for instance Refs. [2, 8–10]). There has been intensive research activities aimed at achieving this goal in the last few years and a large variety of controllers has been introduced such as nonlinear state-feedback controller [11–13], the sliding mode theory [14], feedback and non-feedback methods, which are based on the Lyapunov direct method and Routh-Hurwitz criteria [15] and backstepping recursive nonlinear controller [16–20]. Among these methods, the backstepping based controller is less explored [20]. In particular, backstepping based controller can guarantee global stability, tracking and transient performance for a broad class of nonlinear systems [16,21,23], because of its robustness. This method in its various forms has been employed in a number of works on control and synchronization of chaotic systems [20–26], including the RCL-shunted Josephson junctions [20] as well as the inertial ratchet [25].

The problem of controlling the chaotic states of plasma, however, has received little attention, even though the onset and control of chaos are of the greatest importance for the plasma turbulence phenomenon and related fluctuations including transport. In plasma, many turbulent phenomena have been observed and these are very troublesome phenomena which could have harmful consequences. In general, chaos in high density, hot and magnetized plasma such as fusion-oriented plasmas will evolve into fully developed turbulence and lead to anomalous transport. Thus, the role of turbulence in fusion-oriented plasmas motivates the special interest in chaos control [29]. A few attempts has been made to address this problem from different perspectives, including experimental approach (See for example Refs. [6, 7, 29, 30]). In this present paper, we present a recursive backstepping control algorithm that effectively controls the chaotic behavior in a four-dimensional (4D) autonomous van der Pol-Mathieu (vdPM) equation describing the dynamics of dust-charge fluctuation in dusty plasma [27, 28]. The rest of this paper is organized as follows: In Section 2, the vdPM system is described and we present the backstepping design in Section 3. Numerical simulations are performed in Section 4; while we conclude the paper in Section 5.

2. The van der Pol Mathieu equation

Recently, Bora *et al.* [27] obtained a set of four-dimensional autonomous vdPM system which modeled the evolution of simplified dusty plasma with dust-charge fluctuation via Gram–Schmid orthogonalization procedure. The vdPM system can be described as a set of four first order differential equations which may be written as

$$\begin{aligned} \dot{x} &= y\,, \\ \dot{y} &= (\alpha - \beta x^2)y - \omega_d^2 x(1 - \varepsilon\lambda u)\,, \\ \dot{u} &= u(1 - u^2 - v^2) - \frac{2\pi v}{T}\,, \\ \dot{v} &= v(1 - u^2 - v^2) + \frac{2\pi u}{T}\,, \end{aligned} \tag{1}$$

where $u(t)$ and $v(t)$ are defined to have stable and unique solutions,

$$u(t) = \cos(2\pi t/T)\,, \qquad v(t) = \sin(2\pi t/T)\,,$$

for the initial values $[u(0), v(0)] = [1, 0]$. The vdPM system has been found to exhibit chaotic behaviour over a wide range of relevant system parameters via the period-doubling bifurcation cascades (For detailed discussion of the

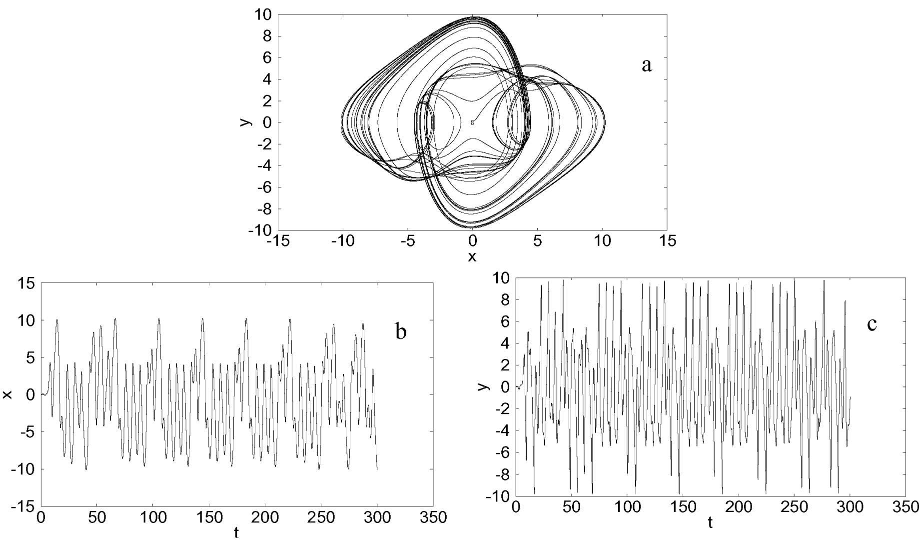

Fig. 1. Chaotic dynamics of the vdPM system in the uncontrolled state; (a) phase portrait, (b) time series of the x variable and (c) time series of the y variable. The parameters of the system are $\omega_{\mathrm{d}} = \lambda = 1$, $\alpha = 0.07$, $\varepsilon = 3.41$, $\beta = 0.1$ and $T = 6.5$.

dynamics of the dusty plasma, including the transition to chaos, see for example Refs. [28, 29]). Specifically, for the following parameter space: $\omega = \lambda = 1$, $\alpha = 0.007$, $\varepsilon = 3.41$, $\beta = 0.1$ and $T = 6.5$ a chaotic attractor has been found. The chaotic attractor and time evolution corresponding to this chaotic behavior is depicted in Fig. 1. In what follows, we present a systematic approach based on recursive backstepping nonlinear control scheme to enable the stabilization of this chaotic motion to a stable state.

3. Design of backstepping control

In order to achieve our aim, a time-dependent input control function $p(t)$ is added to system (1), which gives the following system:

$$\begin{aligned}
\dot{x} &= y\,,\\
\dot{y} &= (\alpha - \beta x^2)y - \omega_d^2 x(1 - \varepsilon\lambda u)\,,\\
\dot{u} &= u(1 - u^2 - v^2) - \frac{2\pi v}{T}\,,\\
\dot{v} &= v(1 - u^2 - v^2) - \frac{2\pi v}{T} + p(t)\,.
\end{aligned} \tag{2}$$

The following error states are then defined:

$$\begin{aligned}
e_1 &= x - x_{\mathrm{D}}\,,\\
e_2 &= y - y_{\mathrm{D}}\,,\\
e_3 &= u - u_{\mathrm{D}}\,,\\
e_4 &= v - v_{\mathrm{D}}\,,
\end{aligned} \tag{3}$$

where $e_i (i = 1, 2, 3, 4)$ are feedbacks representing the differences between the non-periodic and the desired periodic states of the system. This control problem arises from the simplified model of the dusty plasma. Hence, we let $p(t)$ be as simple as possible, letting $x_{\mathrm{D}} = 0$ be the references point, while the subsequent desired variables are recursively defined in terms of the preceding error states with appropriate feedback gains, such that

$$\begin{aligned}
x_{\mathrm{D}} &= 0\,,\\
y_{\mathrm{D}} &= c_1 e_1\,,\\
u_{\mathrm{D}} &= c_2 e_1 + c_3 e_2\,,\\
v_{\mathrm{D}} &= c_4 e_1 + c_5 e_2 + c_6 e_3\,,
\end{aligned} \tag{4}$$

where the c_i's $(i = 1, 2, 3, 4, 5, 6)$ are arbitrary feedback gains that would be chosen later to ensure the desired periodic state of the system is attained. The recursive technique (4) is quite powerful in practical implementation.

For instance, recursive feedback inputs similar to Fig. 2 of Ref. [29] could be realized in a block electronic circuit. Using Eq. (3) in (2), we obtain the error dynamic equations:

$$\begin{aligned}
\dot{e}_1 &= e_2 + c_1 e_1 \,, \\
\dot{e}_2 &= (\alpha - \beta x^2)(e_2 + c_1 e_1) - \omega_d^2[1 - \varepsilon\lambda(e_3 + c_2 e_1 + c_3 e_2)] - c_1 \dot{e}_1 \,, \\
\dot{e}_3 &= (e_3 + c_2 e_1 + c_3 e_2)\left[1 - (e_3 + c_2 e_1 + c_3 e_2)^2\right. \\
&\quad \left. -(e_4 + c_4 e_1 + c_5 e_2 + c_6 e_3)^2\right] \\
&\quad -2\pi(e_4 + c_4 e_1 + c_5 e_2 + c_6 e_3)/T - c_2 \dot{e}_1 - c_3 \dot{e}_2 \,, \\
\dot{e}_4 &= (e_4 + c_4 e_1 + c_5 e_2 + c_6 e_3)\left[1 - (e_3 + c_2 e_1 + c_3 e_2)^2\right. \\
&\quad \left. -(e_4 + c_4 e_1 + c_5 e_2 + c_6 e_3)^2\right] \\
&\quad +2\pi(e_3 + c_2 e_1 + c_3 e_2)/T - c_4 \dot{e}_1 - c_5 \dot{e}_2 - c_6 \dot{e}_3 + p(t) \,.
\end{aligned} \tag{5}$$

Since the c_i' s $= 0$ are arbitrary control gains, it is convenient without loss of generality to set c_i's $(i = 1, 2, 3, 5, 6)$ and $c_4 = 1$; and with this choice, Eq. (5) reduces to:

$$\begin{aligned}
\dot{e}_1 &= e_2 \,, \\
\dot{e}_2 &= e_2(\alpha - \beta x^2) - \omega_d^2 e_1(1 - \varepsilon\lambda e_3) \,, \\
\dot{e}_3 &= e_3\left[1 - e_3^2 - (e_4 + e_1)^2\right] - 2\pi(e_4 + e_1)/T \,, \\
\dot{e}_4 &= (e_4 + e_1)\left[1 - e_3^2 - (e_4 + e)^2\right] + 2\pi e_3/T - e_2 + p(t) \,.
\end{aligned} \tag{6}$$

In the absence of the control, the error dynamics system (6) would generally have equilibrium at (0,0,0). Thus, if an appropriate controller $p(t)$ is chosen such that the equilibrium remains unchanged, theoretically, the control problem reduces to that of achieving asymptotic stabilization of the zero solutions of system (6). To achieve this, consider the following Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{n} k_i e_i^2 \,, \qquad (n = 4) \tag{7}$$

whose time derivative is written as

$$V = \sum_{i=1}^{n} k_i \dot{e}_i \,, \qquad (n = 4) \,. \tag{8}$$

Substituting $\dot{e}_i (i = 1, 2, 3, 4)$ using Eq. (6) and letting $k_i (i = 1, 2, 3) = 0$; $k_4 = 1$ we have

$$\dot{v} = -\dot{e}_4^2 \,. \tag{9}$$

Indeed, $\dot{v}$ is negative definite and thus satisfies the Lasalle–Yoshizawa stability criteria [16]; with the controller $p(t)$ chosen as

$$p(t) = e_2 - \frac{2\pi e_3}{T} - (e_4 + e_1)\left[1 - e_3^2 - (e_4 + e_1)^2\right] \tag{10}$$

which forces the system to exhibit stable periodic orbit and hence the control problem is solved. It is important to note that control input $p(t)$ depends on the feedback terms, implying that for practical implemented, the control block could be quite simple.

4. Numerical simulations

In this section, the nonlinear Eqs. (1) and (2) are integrated numerically by using the fourth order Runge–Kutta integration algorithm. The parameter values were fixed at $\omega_d = \lambda = 1$, $\alpha = 0.07$, $\varepsilon = 3.41$, $\beta = 0.1$ and $T = 6.5$ such that the system is simulated in its chaotic state. With this parameter values, the chaotic attractor shown in Fig. 1 is fully recovered. In Fig. 2, the designed controller $p(t)$ is activated at $t = 50$ to allow the for the transient behavior of the system. It is clear that as soon as the controller is

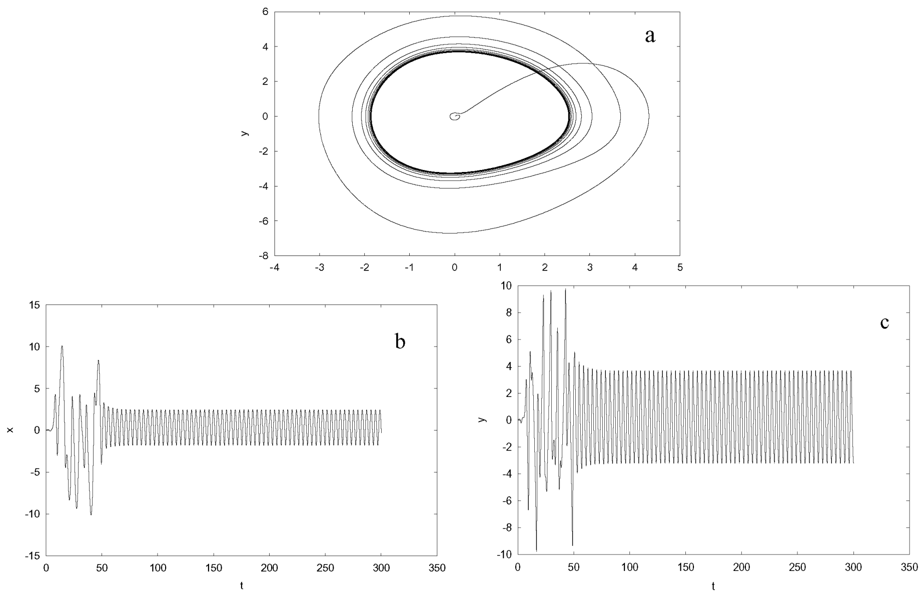

Fig. 2. Dynamics of the vdPM system in controlled state when $p(t)$ has been activated; (a) the controlled phase portrait, (b) time series of the x variable and (c) time series of the y variable.

applied, the system is driven to a stable state with regular vibration, thus preventing it from entering the turbulent. This validates the effectiveness of the proposed control method.

5. Conclusion

This study has investigated the control of chaos found in a simplified plasma-dust grain system with temporally varying dust charge, which is described by a 4D autonomous vdPM equation. Our result shows that the unstable trajectories exhibited by the vdPM chaotic system have been effectively pinned to a regular stable state by the proposed recursive backstepping nonlinear controller. Our theoretical analysis and the numerical results are in perfect agreement. Although many turbulent phenomenon in plasma remains yet unexplained, the present results would complement existing approaches for chaos control in plasmas and more importantly, shed more light on the analysis and understanding of various plasma control processes.

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