

VANISHING DIMENSION FIVE PROTON
DECAY OPERATORS IN SU(5) SUSY GUT

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We propose a framework of SU(5) supersymmetric grand unified theory with the minimal particle contents, which does not contain dimension five proton decay operators. The suitable fermion mass hierarchy can be reproduced by higher dimensional operators of an adjoint Higgs field which breaks SU(5) gauge symmetry.

PACS numbers: 12.60.Jv, 12.10.-g, 12.10.Kt

1. Introduction

Supersymmetric (SUSY) grand unified theories (GUTs) [1] have been regarded as the most agreeable candidates beyond the standard model for a long time, because they can realize the gauge coupling unification as well as show a natural solution of the hierarchy problem. They can also explain the electroweak symmetry breaking by the so-called the radiative breaking scenario [2]. The proton decay is a crucial prediction of GUTs [3], which has not been observed in the experiments yet [4]. From the current experimental bound on the proton decay, it was claimed that the minimal SU(5) SUSY GUT has already been excluded [5,6]. However, as pointed out in *e.g.* Refs. [7,8], this claim should only be applied to the minimal scenario which is also not favourable in the sense that it cannot reproduce the correct mass spectrum of down type quarks and charged leptons. Notice that we must analyse the Yukawa interactions with the coloured Higgs fields carefully. In

this paper we propose one possibility which does not contain dimension five proton decay operators in the SU(5) SUSY GUT framework with the minimal particle content. We must introduce the GUT scale (M_{GUT}) where the SU(5) gauge symmetry is broken by the vacuum expectation value (VEV) of an adjoint representation (**24**) Higgs field Σ . Since the model at the GUT scale is the effective theory of the fundamental one which realized at the Planck scale, it should contain the higher dimensional operators which are suppressed by the Planck scale $(M_{\text{GUT}}/M_{\text{Pl}})^n$ (n : positive integer) [8–10]. These terms can be the origin of a fermion mass hierarchy of three generations as well as the one between top and bottom quarks. The realistic mass spectrum of the down type quarks and the charged leptons can be reproduced by these terms. We take the following setup:

1. We assume only the top Yukawa coupling exists at the tree level to reproduce the hierarchy between top and bottom quarks. The other Yukawa couplings are induced by the *n-th order* higher dimensional terms, $(M_{\text{GUT}}/M_{\text{Pl}})^n$. We regard this factor $(M_{\text{GUT}}/M_{\text{Pl}})^n$ as the origin of the mass hierarchy.
2. The Yukawa couplings of the bottom quark and the tau lepton are induced by *the first order* terms $(M_{\text{GUT}}/M_{\text{Pl}})$, the mass of the strange quark and the muon are reproduced by *the second order* terms $(M_{\text{GUT}}/M_{\text{Pl}})^2$, and the down quark and the electron masses are provided by *the third order* terms $(M_{\text{GUT}}/M_{\text{Pl}})^3$. The masses of the up type quarks might be appropriately reproduced in a similar way.
3. We should pay attention that some of the terms $(\langle \Sigma \rangle / M_{\text{Pl}})^n$ are regarded as *m-th order* (m : positive integer fulfilling $m < n$) terms effectively due to their coefficients.
4. In order to keep the perturbativity, each entry of the higher order terms should not take an extreme large value.
5. We require that the couplings of the operators associated with the proton decay process completely vanish and the realistic fermion masses are realized.

We try to determine the couplings of the higher dimensional terms by the bottom-up approach with two requirements — the realistic Yukawa couplings of quarks and leptons and no dimension five proton decay operators.

2. Model

The superpotential for the Yukawa sector is represented as the series expansion according to the power of adjoint Higgs fields, such as

$$W_{\text{Yukawa}} = W_0 + W_1 + W_2 + W_3 + W_4 + \cdots. \quad (1)$$

The zeroth order part W_0 is the same as that of the minimal SU(5) model,

$$W_0 = \frac{1}{4} \epsilon_{abcde} Y_1^{ij} \mathbf{10}_i^{ab} \mathbf{10}_j^{cd} H^e + \sqrt{2} Y_2^{ij} \bar{H}_a \mathbf{10}_i^{ab} \mathbf{5}_{jb}^*, \quad (2)$$

where i, j are the indices for generations and $a, b, \dots$ are the ones for the SU(5) indices. The chiral superfield $\mathbf{10}$ contains the right-handed up-type quark u_R^c , the left-handed quark doublet Q , and the right-handed charged lepton e_R^c . The right-handed down-type quark d_R^c and the lepton doublet L belong to the superfield $\mathbf{5}^*$. The Higgs fields H and $\bar{H}$ ($\mathbf{5}$ and $\mathbf{5}^*$) include the coloured Higgs triplets (H_C , $\bar{H}_C$) and the Higgs doublets, respectively. As shown in our setup, $Y_2 = 0$ is assumed in Eq. (2), which is the origin of the hierarchy between the top and the bottom masses. The *first order* part W_1 is expressed as [8]¹

$$W_1 = \frac{\epsilon_{abcde}}{4} \left(f_1^{ij} \mathbf{10}_i^{ab} \mathbf{10}_j^{cd} \frac{\Sigma_f^e}{M_{\text{Pl}}} H^f + f_2^{ij} \mathbf{10}_i^{ab} \mathbf{10}_j^{cd} H^d \frac{\Sigma_f^e}{M_{\text{Pl}}} \right) + \sqrt{2} \left(h_1^{ij} \bar{H}_a \frac{\Sigma_b^a}{M_{\text{Pl}}} \mathbf{10}_i^{bc} \mathbf{5}_{jc}^* + h_2^{ij} \bar{H}_a \mathbf{10}_i^{ab} \frac{\Sigma_b^c}{M_{\text{Pl}}} \mathbf{5}_{jc}^* \right), \quad (3)$$

where Σ takes VEV of $\langle \Sigma \rangle = \text{diag}(2, 2, 2, -3, -3)\sigma$ which breaks the SU(5) gauge group into $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$. The value of σ is the scale of M_{GUT} . For the *second* and *third order* superpotential, we only show the

¹ There are other types of higher dimensional terms which do not depend on the adjoint Higgs field, such as $(1/M_{\text{Pl}})\epsilon_{abcd}\mathbf{10}_i^{ab}\mathbf{10}_j^{cd}\mathbf{10}_k^{ef}\mathbf{5}_{lf}^*$ which are dangerous for the nucleon decay. Here, we do not investigate such terms for simplicity.

down-type quark and the charged lepton sector. They are represented as [9]

$$W_2 = \sqrt{2} \left(h_3^{ij} \bar{H}_a \mathbf{10}_i^{ab} \mathbf{5}_{jb}^* \frac{(\Sigma\Sigma)}{M_{\text{Pl}}^2} + h_4^{ij} \bar{H}_a \frac{(\Sigma\Sigma)_b^a}{M_{\text{Pl}}^2} \mathbf{10}_i^{bc} \mathbf{5}_{jc}^* \right. \\ \left. + h_5^{ij} \bar{H}_a \mathbf{10}_i^{ab} \frac{(\Sigma\Sigma)_b^c}{M_{\text{Pl}}^2} \mathbf{5}_{jc}^* + h_6^{ij} \bar{H}_a \frac{\Sigma_b^a}{M_{\text{Pl}}} \mathbf{10}_i^{bc} \frac{\Sigma_c^d}{M_{\text{Pl}}} \mathbf{5}_{jd}^* \right), \quad (4)$$

$$W_3 = \sqrt{2} \left(h_7^{ij} \bar{H}_a \frac{\Sigma_b^a}{M_{\text{Pl}}} \mathbf{10}_i^{bc} \mathbf{5}_{jc}^* \frac{(\Sigma\Sigma)}{M_{\text{Pl}}^2} \right. \\ + h_8^{ij} \bar{H}_a \mathbf{10}_i^{ab} \frac{\Sigma_b^c}{M_{\text{Pl}}} \mathbf{5}_{jc}^* \frac{(\Sigma\Sigma)}{M_{\text{Pl}}^2} + h_9^{ij} \bar{H}_a \mathbf{10}_i^{ab} \mathbf{5}_{jb}^* \frac{(\Sigma\Sigma\Sigma)}{M_{\text{Pl}}^3} \\ + h_{10}^{ij} \bar{H}_a \frac{(\Sigma\Sigma\Sigma)_b^a}{M_{\text{Pl}}^3} \mathbf{10}_i^{bc} \mathbf{5}_{jc}^* + h_{11}^{ij} \bar{H}_a \mathbf{10}_i^{ab} \frac{(\Sigma\Sigma\Sigma)_b^c}{M_{\text{Pl}}^3} \mathbf{5}_{jc}^* \\ \left. + h_{12}^{ij} \bar{H}_a \frac{(\Sigma\Sigma)_b^a}{M_{\text{Pl}}^2} \mathbf{10}_i^{bc} \frac{\Sigma_c^d}{M_{\text{Pl}}} \mathbf{5}_{jd}^* + h_{13}^{ij} \bar{H}_a \frac{\Sigma_b^a}{M_{\text{Pl}}} \mathbf{10}_i^{bc} \frac{(\Sigma\Sigma)_c^d}{M_{\text{Pl}}^2} \mathbf{5}_{jd}^* \right), \quad (5)$$

where $(\Sigma \dots)$ $((\Sigma \dots)_b^a)$ denotes a singlet (an adjoint) by contracting the SU(5) indices of $\Sigma \dots$. The *fourth order* superpotential suggests

$$W_4 = \sqrt{2} \left(h_{14}^{ij} \bar{H}_a \mathbf{10}_i^{ab} \mathbf{5}_b^* \frac{(\Sigma\Sigma)(\Sigma\Sigma)}{M_{\text{Pl}}^4} \right. \\ + h_{15}^{ij} \bar{H}_a \mathbf{10}_i^{ab} \mathbf{5}_b^* \frac{(\Sigma\Sigma\Sigma\Sigma)}{M_{\text{Pl}}^4} + h_{16}^{ij} \bar{H}_a \frac{\Sigma_b^a}{M_{\text{Pl}}} \mathbf{10}_i^{bc} \mathbf{5}_c^* \frac{(\Sigma\Sigma\Sigma)}{M_{\text{Pl}}^3} \\ + h_{17}^{ij} \bar{H}_a \mathbf{10}_i^{ab} \frac{\Sigma_b^c}{M_{\text{Pl}}} \mathbf{5}_c^* \frac{(\Sigma\Sigma\Sigma)}{M_{\text{Pl}}^3} + h_{18}^{ij} \bar{H}_a \frac{(\Sigma\Sigma)_b^a}{M_{\text{Pl}}^2} \mathbf{10}_i^{bc} \mathbf{5}_c^* \frac{(\Sigma\Sigma)}{M_{\text{Pl}}^2} \\ + h_{19}^{ij} \bar{H}_a \mathbf{10}_i^{ab} \frac{(\Sigma\Sigma)_b^c}{M_{\text{Pl}}^2} \mathbf{5}_c^* \frac{(\Sigma\Sigma)}{M_{\text{Pl}}^2} + h_{20}^{ij} \bar{H}_a \frac{\Sigma_b^a}{M_{\text{Pl}}} \mathbf{10}_i^{bc} \frac{\Sigma_c^d}{M_{\text{Pl}}} \mathbf{5}_d^* \frac{(\Sigma\Sigma)}{M_{\text{Pl}}^2} \\ + h_{21}^{ij} \bar{H}_a \frac{(\Sigma\Sigma\Sigma\Sigma)_b^a}{M_{\text{Pl}}^4} \mathbf{10}_i^{bc} \mathbf{5}_c^* + h_{22}^{ij} \bar{H}_a \mathbf{10}_i^{ab} \frac{(\Sigma\Sigma\Sigma\Sigma)_b^c}{M_{\text{Pl}}^4} \mathbf{5}_c^* \\ + h_{23}^{ij} \bar{H}_a \frac{(\Sigma\Sigma)_b^a}{M_{\text{Pl}}^2} \mathbf{10}_i^{bc} \frac{(\Sigma\Sigma)_c^d}{M_{\text{Pl}}^2} \mathbf{5}_d^* + h_{24}^{ij} \bar{H}_a \frac{(\Sigma\Sigma\Sigma)_b^a}{M_{\text{Pl}}^3} \mathbf{10}_i^{bc} \frac{\Sigma_c^d}{M_{\text{Pl}}} \mathbf{5}_d^* \\ \left. + h_{25}^{ij} \bar{H}_a \frac{\Sigma_b^a}{M_{\text{Pl}}} \mathbf{10}_i^{bc} \frac{(\Sigma\Sigma\Sigma)_c^d}{M_{\text{Pl}}^3} \mathbf{5}_d^* \right). \quad (6)$$

Each matrix element h^{ij} is assumed to have an $\mathcal{O}(1)$ coefficient, and the mass hierarchy is produced by the suppression factors $\sigma/M_{\text{Pl}} \equiv 1/a^2$.

Decomposing the superpotential Eqs. (3)–(6) into its component fields, we obtain the Yukawa couplings of down-type quarks and charged leptons as

$$Y_d = Y' - \frac{3}{a}h'_1 + \frac{2}{a}h'_2 + \frac{9}{a^2}h'_4 + \frac{4}{a^2}h'_5 - \frac{6}{a^2}h'_6 - \frac{27}{a^3}h_{10} + \frac{8}{a^3}h_{11} + \frac{18}{a^3}h_{12} - \frac{12}{a^3}h_{13} + \cdots, \quad (7)$$

$$Y_e = Y' - \frac{3}{a}h'_1 - \frac{3}{a}h'_2 + \frac{9}{a^2}h'_4 + \frac{9}{a^2}h'_5 + \frac{9}{a^2}h'_6 - \frac{27}{a^3}h_{10} - \frac{27}{a^3}h_{11} - \frac{27}{a^3}h_{12} - \frac{27}{a^3}h_{13} + \cdots, \quad (8)$$

where the matrices with a prime symbol are defined as

$$Y' \equiv Y_2 + \frac{30}{a^2}h_3 - \frac{30}{a^3}h_9 + \frac{900}{a^4}h_{14} + \frac{210}{a^4}h_{15}, \quad (9)$$

$$h'_{1,2} \equiv h_{1,2} + \frac{30}{a^2}h_{7,8} - \frac{30}{a^3}h_{16,17}, \quad (10)$$

$$h'_{4,5,6} \equiv h_{4,5,6} + \frac{30}{a^2}h_{18,19,20}. \quad (11)$$

On the other hand, the couplings Y_{ql} and Y_{ud} which are associated with the interactions $Q_i \epsilon L_j \tilde{H}_C$ and $u_{Ri}^c d_{Rj}^c \tilde{H}_C$, respectively, are given by

$$Y_{ql} = Y' + \frac{2}{a}h'_1 - \frac{3}{a}h'_2 + \frac{4}{a^2}h'_4 + \frac{9}{a^2}h'_5 - \frac{6}{a^2}h'_6 + \frac{8}{a^3}h_{10} - \frac{27}{a^3}h_{11} - \frac{12}{a^3}h_{12} + \frac{18}{a^3}h_{13} + \cdots, \quad (12)$$

$$Y_{ud} = Y' + \frac{2}{a}h'_1 + \frac{2}{a}h'_2 + \frac{4}{a^2}h'_4 + \frac{4}{a^2}h'_5 + \frac{4}{a^2}h'_6 + \frac{8}{a^3}h_{10} + \frac{8}{a^3}h_{11} + \frac{8}{a^3}h_{12} + \frac{8}{a^3}h_{13} + \cdots. \quad (13)$$

The terms in which the adjoint Higgs fields are contracted by themselves have larger pre-factors than the others. Such terms should be practically

² The authors of Ref. [8] have also introduced higher dimensional terms but up to the first order of $1/a$. In this case, there is an inevitable relation between the Yukawa matrices as $Y_{ql} - Y_{ud} = Y_e - Y_d \neq 0$, where the dimension five operators cannot completely vanish and the suppression factor $1/a$ has nothing to do with the fermion mass hierarchy. On the other hand, the author of Ref. [9] has tried to reproduce the so called Georgi–Jarlskog texture [13] by introducing the terms up to the third order of $1/a$.

regarded as lower order contribution. For example, we should regard h_3 as *the first order* term like h_1 and h_2 . The terms of h_{7-9} and h_{14} are referred as *the second order* terms such as h_{4-6} , and those of h_{15-20} should belong to *the third order* terms such as h_{10-13} .

The value of σ is related to the mass of the coloured Higgs triplet M_C and the GUT scale. The GUT scale is represented as $(M_V^2 M_\Sigma)^{1/3}$, where M_V stands for mass of X and Y bosons (SU(5) breaking gauge bosons) and M_Σ for the mass of Σ . The magnitudes of these mass parameters are strictly constrained from the gauge coupling unification condition [6, 11]. However, the value of σ itself can be larger than the GUT scale $(M_V^2 M_\Sigma)^{1/3} \simeq 2.0 \times 10^{16}$ GeV by taking the Higgs couplings among H , $\bar{H}$, and Σ to be small. Therefore, we can take $a = \mathcal{O}(10 \sim 100)$, where dimension six proton decay operators are suppressed enough.

3. Examples

Let us now illustrate a concrete example for the Yukawa couplings which reproduces not only the realistic fermion mass spectrum but also the completely vanishing dimension five proton decay operators. We deal with the case where only the diagonal entries of the higher order terms are significant to simplify the examples. Through the following examples, we will see that the determination of each entry keeping the perturbativity is not so trivial.

The coefficients of the dimension five proton decay operators are denoted

$$C_{5L}^{ijkl} \equiv Y_{ql}^{ij} Y_{qq}^{kl}, \quad C_{5R}^{ijkl} \equiv Y_{ud}^{ij} Y_{eu}^{kl}, \quad (14)$$

where Y_{qq} and Y_{eu} are the couplings of interactions of the coloured Higgs field coming from the $\mathbf{10}_i \mathbf{10}_j H$ type terms. Since the Yukawa coupling for the top quark is included in Y_{qq} and Y_{eu} and it is too large to be vanished by the higher order terms of $\mathbf{10}_i \mathbf{10}_j H$, here we take a possibility that Y_{ql} and Y_{us} are vanished. Avoiding unreliable large couplings in h_i 's, the texture of h_i 's for the third generation is uniquely determined except for $\Delta y_3 \equiv y_\tau - y_b$ which is the difference between the Yukawa couplings of the tau lepton and the bottom quark. It should be small at the GUT scale and we assume that it is provided by *second order* terms. From Eqs. (7)–(13), the third generation Yukawa components are induced as

$$(h_6)_{33} = a^2 \Delta y_3 / 25, \quad (15)$$

$$(h_5)_{33} = a (h'_2)_{33} + 2a^2 \Delta y_3 / 25, \quad (16)$$

$$(h_4)_{33} = a (h'_1)_{33} + a^2 y_b / 5 + 2a^2 \Delta y_3 / 25, \quad (17)$$

$$(h'_3)_{33} = -2a^2 (y_b + \Delta y_3) / 75 - a (h'_1 + h'_2)_{33} / 5, \quad (18)$$

up to the *second order*. Here, $h'_3 \equiv h_3 - h_9/a + 30h_{14}/a^2$. In order to avoid $\mathcal{O}(a^2 y_b)$ terms in $(h_5)_{33}$ and $(h_4)_{33}$, we must take

$$(h_1)_{33} = -ay_b/5, \quad (h_2)_{33} = 0. \quad (19)$$

Then, the value of $(h_3)_{33}$ is determined up to order $\mathcal{O}(a^2 y_b)$ as

$$(h_3)_{33} = a^2 y_b/75. \quad (20)$$

This term should be regarded as the *first order* term because of the large pre-factor $1/75$, in which the value of $(h_3)_{33}$ itself is kept of $\mathcal{O}(1)$. Now all the third generation components of h_i 's are determined. The couplings for the second generation are also determined in a similar way. Those for the first generation are not uniquely determined since there are large degrees of freedom in the *third order* terms.

Summarising above discussions, the *first order* terms are uniquely determined as

$$\begin{aligned} h_1 &= -\frac{a}{5} \text{diag}(0, 0, y_b), \quad h_2 = \mathbf{0}, \\ h_3 &= \frac{a^2}{75} \text{diag}(0, 0, y_b). \end{aligned} \quad (21)$$

The *second order* terms are also determined almost automatically as

$$\begin{aligned} h_4 &= \frac{a^2}{75} \text{diag}(0, 9y_s + y_\mu, \Delta y_3), \\ h_5 &= \frac{a^2}{75} \text{diag}(0, -6y_s + y_\mu, \Delta y_3), \\ h_6 &= \frac{a^2}{25} \text{diag}(0, -y_s + y_\mu, \Delta y_3), \\ h_7 &= h_8 = -\frac{a^3}{450} \text{diag}(0, y_\mu, \Delta y_3), \\ h_9 &= h_{14} = \mathbf{0}. \end{aligned} \quad (22)$$

For the *third order* terms, there are various choices, and one example is

$$\begin{aligned} h_{10} &= h_{16-25} = \mathbf{0}, \\ h_{11} &= \frac{a^3}{25} \text{diag}(-y_d + 2y_e/7, 0, 0), \\ h_{12} &= \frac{a^3}{25} \text{diag}(3y_d/2 - 2y_e/3, 0, 0), \\ h_{13} &= \frac{a^3}{25} \text{diag}(-y_d/2 - y_e/3, 0, 0), \\ h_{15} &= \frac{4a^4}{3675} \text{diag}(y_e, 0, 0). \end{aligned} \quad (23)$$

It is worth noting that the accurate Yukawa couplings of the down-sector quarks and charged leptons are obtained as

$$Y_d = \text{diag}(y_d, y_s, y_b), \quad Y_e = \text{diag}(y_e, y_\mu, y_\tau), \quad (24)$$

as well as the couplings of coloured Higgs triplet vanish as

$$Y_{ql} = Y_{ud} = \mathbf{0}. \quad (25)$$

Once these Yukawa interactions are realized at the GUT scale, any dimension five proton decay process will not appear even if the renormalisation group equation (RGE) effects are taken into account.

In Table I, we present the magnitudes of Yukawa couplings at the GUT scale by using the results in Ref. [12]. The applicability of the perturbation

TABLE I

Examples of the matrix elements of h_i 's at the GUT scale which reproduce the realistic fermion mass spectrum and vanish the proton decay operators. Here, we take $a = 55$ and $\tan \beta = 10$. We take the diagonal basis of the down-type quark and the charged lepton mass matrices. The flavour mixing is imposed into the up-sector Yukawa couplings.

	Order		Example 1	Example 2
W_1	1st	h_1	$(0, 0, -0.64)$	
		h_2	$\mathbf{0}$	
W_2	1st	h_3	$(0, 0, 2.3)$	
		h_4	$(0, 0.72, 0.40)$	
		h_5	$(0, -0.21, 0.40)$	
		h_6	$(0, 0.29, 1.2)$	
W_3	2nd	h_7	$(0, -1.5, -3.6)$	
		h_8	$(0, -1.5, -3.6)$	
		h_9	$\mathbf{0}$	
		h_{10}	$\mathbf{0}$	$\mathbf{0}$
	3rd	h_{11}	$(-0.48, 0, 0)$	$\mathbf{0}$
		h_{12}	$(0.68, 0, 0)$	$\mathbf{0}$
		h_{13}	$(-0.29, 0, 0)$	$\mathbf{0}$
W_4	2nd	h_{14}	$\mathbf{0}$	
	3rd	h_{15}	$(0.19, 0, 0)$	$(-0.65, 0, 0)$
		h_{16}	$\mathbf{0}$	$\mathbf{0}$
		h_{17}	$\mathbf{0}$	$\mathbf{0}$
		h_{18}	$\mathbf{0}$	$(3.3, 0, 0)$
		h_{19}	$\mathbf{0}$	$(-1.4, 0, 0)$
		h_{20}	$\mathbf{0}$	$(-0.71, 0, 0)$

(couplings $\lesssim \sqrt{4\pi}$) is satisfied for all components. Notice that it can be satisfied even in the rather large $\tan\beta$ ($\tan\beta \sim 10$) region. Ordinal SU(5) GUT models, *e.g.* with decoupling SUSY breaking spectrum, should have a small $\tan\beta$ of order 1 [8].

The Yukawa couplings of the up-type quarks can also be derived from higher dimensional terms. However, it is hard to imagine that the coefficients of dimension five proton decay operator is vanished due to the null coefficients of Y_{qq} and Y_{eu} because of the large top Yukawa coupling. We do not investigate this possibility, and consequently, the various choices are left in terms of the couplings of the higher dimensional terms for the $10_i 10_j H$ type interaction. As one example, here we suppose the simple superpotential,

$$W = \frac{\epsilon_{abcde}}{4} \mathbf{10}_i^{ab} \mathbf{10}_j^{cd} H^e \left\{ f_c^{ij} \frac{(\Sigma\Sigma)}{M_{\text{Pl}}^2} + f_u^{ij} \frac{(\Sigma\Sigma\Sigma\Sigma)}{M_{\text{Pl}}^4} \right\}, \quad (26)$$

we obtain the Yukawa matrices

$$Y_u = Y_1 + \frac{30}{a^2} f_c + \frac{210}{a^4} f_u. \quad (27)$$

In the basis Y_u should be given by $Y_u = U_{\text{CKM}}^\dagger Y_u^{\text{diag}}$. This example gives

$$(Y_1)_{33} = 0.75, \quad (f_c)_{22} = 0.18, \quad (f_u)_{11} = 0.26,$$

for Y_u^{diag} with the same values of a and $\tan\beta$ in Table I.

4. Discussion and summary

Some comments are in order. The first is about the coefficients of the proton decay operators. They must include at least one first generation quark superfield. Therefore, it is not necessary to eliminate all components of the coloured Higgs Yukawa couplings as in Eq. (25). In fact we can realize such Yukawa matrices which have more choices than the examples shown above. However, in this case the RGE effect must be taken into account to estimate the proton decay rate, since the second and third generation components of Y_{ql} and Y_{ud} are transmitted into the first generation components through the generation mixings. The RGE analysis shows that the large entry of the second and third generation components could be destructive³. This means that RGE effects will break the proton stability even if all the

³ In Ref. [8], the authors adopted the ingenious texture which they referred as *the consistent model 1*. This model has the non-zero first generation components, however, they avoided the large RGE effects by vanishing second and third generations' components in Y_{ql} . This scenario is effective only in the case of small $\tan\beta$ such as $\tan\beta = \mathcal{O}(1)$.

first generation components of Y_{ql} and Y_{ud} are zero. If Y_{ql} and Y_{ud} do not include the first generation components at the nucleon mass scale (not at the GUT scale), the dimension five proton decay processes will disappear as pointed out in Ref. [7]. It is an interesting possibility. However, there must be a reason why the scale is not the GUT scale but the nucleon mass scale.

The second comment is about the contribution from the sub-leading effects. When the soft SUSY breaking tri-linear scalar interactions of the coloured Higgs

$$-\mathcal{L}_{\text{soft}} \supset A_{ql}^{ij} \tilde{Q}_i \epsilon \tilde{L}_j \bar{H}_C + A_{ud}^{ij} \tilde{u}_{Ri}^* \tilde{d}_{Rj}^* \bar{H}_C + \text{H.c.}, \quad (28)$$

are introduced, $A_{ql} Y_{qq}$ and $A_{ud} Y_{eu}$ can contribute to the proton decay processes. We have neglected these A -term contributions in the above discussions, which can be justified in the minimal supergravity context. It is because these terms are generally proportional to the corresponding Yukawa couplings. Therefore, when they do not exist at the GUT scale, there will be no contribution at the low energy scale even if we take account into the RGE effects.

Finally, we would like to make some comments on the relation between the former works. The use of the higher dimension terms in GUT models has already proposed and examined by some works [8–10]. In the case which most of papers dealt with, only first (or second) order terms are introduced although there is no reason to omit the further higher terms. In such a case, since the hierarchical factor $M_{\text{GUT}}/M_{\text{Pl}}$ does not contribute to the hierarchy in the Yukawa matrices, the coefficients of higher order terms should be finely tuned to realize the realistic fermion masses and vanishing the proton decay, *i.e.*, range of the adjustment becomes *order*. We include the more higher order terms corresponding to the hierarchy in the Yukawa matrices. Therefore, the adjustment of the coefficients are mild, *i.e.*, the coefficients could take not so small values (order one) and also not so large values (keeping the perturbativity). Actually, when many higher order terms are introduced, there are many choices to reproduce the fermion masses without proton decay. However, it is not so trivial to realize them keeping mild entries to all the coefficients.

We have tried to reproduce the suitable fermion mass hierarchy as well as to suppress the proton decay in the SU(5) SUSY GUT framework with the minimal field contents. The realistic fermion mass spectrum can be realized simultaneously with vanishing dimension five proton decay processes. From these requirements, we try to determine the couplings of the higher dimensional terms by the bottom-up approach.

The authors would like to thank N. Okada and M. Kakizaki for useful discussions and are grateful to A. Merle for reading the manuscript carefully. N.H. is supported by the Alexander von Humboldt Foundation.

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