

STUDY OF THE NUCLEON SPIN-DEPENDENT
STRUCTURE FUNCTION g_1 . A COMPARISON WITH
RECENT HERMES AND COMPASS DATA

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Predictions for the spin dependent structure function g_1 of the nucleon are presented. We use an unified approach incorporating the LO DGLAP evolution and the resummation of double logarithmic terms $\ln^2(x)$. We show, that the singular input parametrisation as $x \rightarrow 0$ can be a substitute of the $\ln^2(x)$ resummation. An impact of the ‘more running’ coupling is discussed. We determine the contribution to the Bjorken sum rule solving the evolution equation for the truncated moment of g_1^{NS} . A comparison with the re-analysed HERMES and COMPASS data is given.

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1. Introduction

Experimental data confirm (at least for $Q^2 > 1 \text{ GeV}^2$) the theoretical predictions of an increase of the nucleon structure functions at small values of the Bjorken x . The low- x behaviour of both spin averaged and spin dependent structure functions is controlled by the double logarithmic terms $(\alpha_s \ln^2(x))^n$ [1–3]. In an unpolarised case, this singular PQCD behaviour is however overridden by the leading Regge contribution present in the input parametrisation [4]. The situation is quite different in the spin-dependent case, where the double logarithmic effects are very important. The resummation of the $\ln^2(x)$ terms at low- x goes beyond the standard LO and NLO PQCD evolution of the parton densities. Double logarithmic contributions become essential for $x \sim 0.01$, where there is little experimental data. Determination of the sum rules and the nucleon spin decomposition among partons requires knowledge of the structure functions over the entire

region of the variable $x \in (0; 1)$. Therefore the small- x behaviour of the spin dependent parton distributions is a topic of the intensive theoretical investigations. Standard approach describing structure functions is based on the DGLAP- Q^2 evolution equation via two-step convolution: of the initial parton densities and splitting functions and then of the evolved parton distributions and the coefficient functions. Because there is no way to calculate the initial parton densities, which have a nonperturbative origin, they must be put ‘by hand’. Different parametrisations of the initial gluon and quark densities known in literature *e.g.* [5, 6] are singular when $x \rightarrow 0$. This choice enables to study DIS phenomena within DGLAP approach not only for the large- x region but for the small- x one as well. Singular terms $\sim x^{-\lambda}$ can be a substitute of the double logarithmic $\ln^2(x)$ resummation, which is absent in the standard DGLAP scenario. This problem has been widely discussed and argued in [7]. Also important is the problem of the α_s dependence of the QCD evolution. Following [8] we take into account the running coupling effects not via $\alpha_s(Q^2)$ but with use of the more running $\alpha_s(Q^2/x)$. This approach is better justified at small values of x , whereas for large $x \sim 1$ leads to the usual DGLAP coupling $\alpha_s(Q^2)$. It seems to be reasonable to study an impact of the double logarithmic and running coupling effects on theoretical predictions for spin structure functions.

In this paper we present the unified approach, in which the familiar Q^2 evolution is extended by the $\ln^2(x)$ resummation. In our analysis we use so-called unintegrated parton distributions and solve the combined LO DGLAP + $\ln^2(x)$ evolution equation with help of the Chebyshev polynomial technique. We take into account the ‘very running’ coupling effects at small- x and discuss the role, they play. We also show that the singular input parametrisation of the parton distributions can be some kind of substitution for the double logarithmic terms, missing in the standard DGLAP approximation. Our theoretical predictions for the spin dependent structure function g_1 are compared with recently re-analysed HERMES and COMPASS data.

The content of this paper is as follows. In Section 2 we recall the unified approach incorporating DGLAP evolution of structure functions and the double logarithmic $\alpha_s^n \ln^{2n}(x)$ terms, which are essential in the small- x region. Section 3 is devoted to the impact of running coupling effects on the g_1 results in the small- x region. In Section 4 we show that the singular initial parton densities $\sim x^{-\lambda}$ can mimic the resummation of double logarithmic terms $\ln^2(x)$. Using this fact, in Section 5 we solve the evolution equation for the truncated moments themselves and obtain contribution to the Bjorken sum rule. We also present numerical predictions for the structure function g_1 and compare them to re-analysed HERMES and COMPASS data. Finally, in Section 6 we summarise our results.

2. Unified $\ln^2(x)$ +LO DGLAP approach

The structure functions of the nucleon can be expressed in terms of the parton distributions. These depend on two kinematic variables: the Bjorken x and $Q^2 = -q^2$ with q being the four-momentum transfer in the deep-inelastic lepton–nucleon scattering (DIS). The scaling variable is defined as $x = Q^2/(2pq)$, where p is the nucleon four-momentum. The strong interactions between quarks and gluons cause the changes in the parton densities. For medium and large- x , the evolution with Q^2 of the parton distributions is well described by the standard Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations [9–12]. This approach which effectively sums up the leading $\ln(Q^2)$ terms is, however, incomplete at small- x , where another large logarithm — $\ln(1/x)$ becomes essential and which leading powers $\alpha_s^n \ln^{2n}(x)$ needs to be resummed. The double logarithmic terms $\ln^2(x)$ come from the ladder diagrams with quark and gluon exchanges along the chain. Treating both potentially large logarithms $\ln(Q^2)$ and $\ln(1/x)$ on equal footing, the authors of [14–16] obtained equations which incorporate DGLAP evolution and $\ln^2(x)$ terms as well. The double logarithmic effects go beyond the standard LO and even NLO Q^2 evolution of the spin dependent parton distributions and significantly modify the Regge pole model expectations for the structure functions. Theoretical analyses of the small- x behaviour of the polarised structure functions [17] predict that resummation of the double logarithmic terms $(\alpha_s \ln^2(x))^n$ leads to the singular form as $x \rightarrow 0$:

$$g_1^{\text{NS,S}}(x, Q^2) \sim x^{-\lambda_{\text{NS,S}}}, \quad (2.1)$$

where $\lambda_{\text{NS}} \approx 0.4$, $\lambda_{\text{S}} \approx 0.8$ and $g_1^{\text{NS,S}}$ denotes nonsinglet or singlet part of the polarised structure function of the proton. For larger but still low $x \in (10^{-5}; 10^{-2})$, g_1 is less steep with the slope $\lambda \approx 0.2$ – 0.3 for the nonsinglet part [1]. This power-like behaviour $x^{-\lambda}$ remains significantly steeper than the DGLAP solution in absence of the singular input parametrisations of parton densities

$$g_1^{\text{DGLAP}}(x \rightarrow 0) \sim \exp \sqrt{\ln(1/x) \ln \ln(Q^2/\Lambda_{\text{QCD}}^2)}. \quad (2.2)$$

The unified equation, which includes the LO DGLAP evolution and $\ln^2(x)$ terms resummation reads:

$$f(x, Q^2) = f_0(x) + \underbrace{\int_{Q_0^2}^{Q^2} \frac{dk'^2}{k'^2} \frac{\alpha_s}{2\pi} \Delta P \otimes f(x, k'^2)}_{\text{DGLAP}}$$

$$\begin{aligned}
& + \underbrace{\frac{4}{3} \int_x^1 \frac{dz}{z} \int_{Q^2}^{Q^2/z} \frac{dk'^2}{k'^2} \frac{\alpha_s}{2\pi} f\left(\frac{x}{z}, k'^2\right)}_{\text{LN}^2(X) \text{ LADDER}} \\
& + \underbrace{\text{Bremsstrahlung corrections}}_{\text{LN}^2(X) \text{ NONLADDER}}, \tag{2.3}
\end{aligned}$$

where \otimes abbreviates a Mellin convolution over x

$$(\Delta P \otimes f)(x, Q^2) = \int_x^1 \frac{dy}{y} \Delta P\left(\frac{x}{y}\right) f(y, Q^2), \tag{2.4}$$

ΔP denote the polarised version of the splitting function P and f is the un-integrated distribution, related to the ordinary polarised parton distribution $\Delta p(x, Q^2)$ via

$$f(x, Q^2) = \frac{\partial \Delta p(x, Q^2)}{\partial \ln(Q^2)}. \tag{2.5}$$

The double logarithmic terms come from ladder-type graphs as well as from the nonladder ones which represent radiative corrections [2, 18–20]. In a case of the nonsinglet polarised structure functions the contribution of nonladder diagrams is negligible. However, for the singlet spin dependent structure functions, besides the ladder graphs, one has to include Bremsstrahlung corrections [3], which are important. The full evolution equations for non-singlet and singlet unintegrated parton distributions within DGLAP+ $\ln^2 x$ approach have been presented in [14, 15]. This forms the basis of our analysis in the next section, where we discuss the modified running coupling effects at small- x .

3. Running coupling α_s in the small- x region

DGLAP formalism uses the following prescription for the running coupling (in the lowest order):

$$\alpha_s = \alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f) \ln \frac{Q^2}{\Lambda_{\text{QCD}}^2}}, \tag{3.1}$$

where N_f is the number of active quark flavours and $\Lambda_{\text{QCD}} \approx 200$ MeV is the QCD cut-off parameter. It has, however, been argued that in the small- x region Eq. (3.1) should be rearranged into the following form [8]:

$$\alpha_s = \alpha_s(Q^2/z), \tag{3.2}$$

with z being the longitudinal momentum fraction of a parent parton, carried by a next generation parton. In this way α_s becomes ‘very running’ *i.e.* runs in each ladder rung depending on the gluon virtuality. This prescription of α_s , widely discussed also in [21, 22], has been used *e.g.* in [1, 23] within double logarithmic effect $\ln^2(x)$ resummation. Here, we study an impact of the different α_s parametrisation on the polarised parton densities. Because

$$\alpha_s(Q^2/z) \leq \alpha_s(Q^2), \quad (3.3)$$

in the case of the ‘very running’ α_s (3.2), the growth of the parton distributions in the low- x region is damped. A scale of the damping for nonsinglet and singlet (gluons) distributions is shown in Figs. 1–2, where we plot the ratio

$$R = \frac{\Delta p(\alpha_s(Q^2))}{\Delta p(\alpha_s(Q^2/z))} \quad (3.4)$$

as a function of x . Here, Δp denotes the nonsinglet (valence) Δq^{NS} and the gluon ΔG distribution function, respectively. One can see, that the difference becomes essential at $x \sim 0.01$ and the impact of the running coupling effects for the singlet case is larger than for the nonsinglet one. Double logarithmic resummation additionally amplifies the split between results in comparison to the pure DGLAP approach. At very small $x = 10^{-5}$ we find the ratio (3.4) about 2 for the nonsinglet polarised distribution and above 6 for the polarised gluons. Our estimations of R (3.4) show that for the small val-

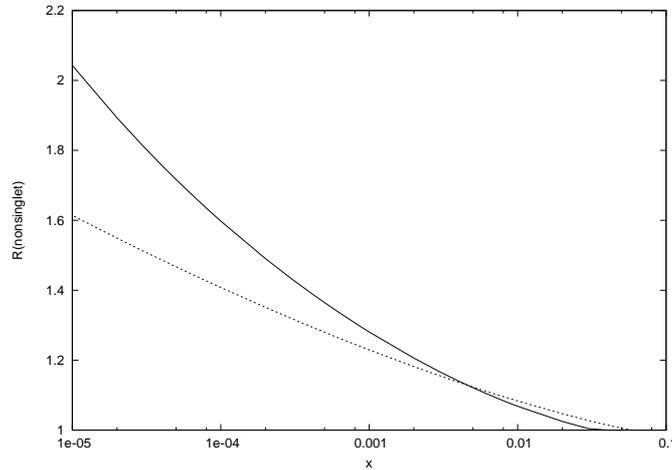


Fig. 1. The ratio (3.4) for the polarised nonsinglet quark distribution $\Delta q^{\text{NS}} = \Delta u - \Delta d$ as a function of x at $Q^2 = 10 \text{ GeV}^2$. Solid: unified DGLAP+ $\ln^2(x)$ approach, dotted: DGLAP alone.

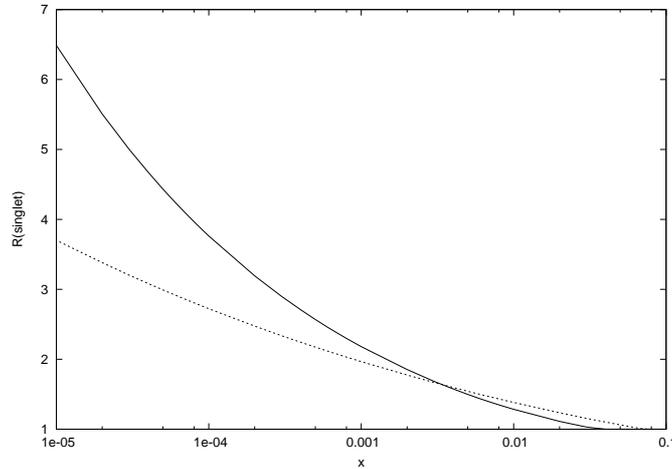


Fig. 2. The ratio (3.4) for the polarised gluon distribution ΔG as a function of x at $Q^2 = 10 \text{ GeV}^2$. Solid: unified DGLAP+ $\ln^2(x)$ approach, dotted: DGLAP alone.

ues of Bjorken parameter $x \leq 10^{-2}$ the coupling $\alpha_s(Q^2)$ should be replaced by $\alpha_s(Q^2/z)$. In standard DGLAP analysis, where rather large- x region is considered, this modification converts into $\alpha_s(Q^2)$ ($z \sim 1$). Parametrisation of the coupling α_s is not the only crucial point in the low- x analysis of structure functions. Another problem are initial parton distributions at low $Q_0^2 \sim 1 \text{ GeV}^2$, which enter into the evolution equations. The behaviour of the quark and gluon distributions at very small- x is mainly generated by the double logarithmic $\ln^2(x)$ effects. Therefore singular as $x \rightarrow 0$ inputs $\sim x^{-a_1}$ seem to be needless in PQCD analysis, unless one does not consider $\ln^2(x)$ terms. Within standard DGLAP approach, parametrisations in a form $\sim x^{-a_1}$ can be regarded as a substitute of the missing double logarithmic effects resummation. In the next section we discuss this problem in detail.

4. Singular input parametrisations as an ersatz of the double logarithmic terms $\ln^2(x)$ resummation

According to the philosophy of DGLAP approach, structure functions of the nucleon are a convolution of the coefficient functions and the evolved parton distributions. In this formalism, the polarised structure function $g_1(x, Q^2)$ for the proton is given by [13]

$$g_1^p(x, Q^2) = \frac{1}{2} \langle e^2 \rangle \left[C_{\text{NS}} \otimes \Delta q^{\text{NS}}(x, Q^2) + C_S \otimes \Delta q^{\text{S}}(x, Q^2) + 2N_f C_G \otimes \Delta G(x, Q^2) \right], \quad (4.1)$$

where

$$\langle e^2 \rangle = \frac{1}{N_f} \sum_{i=1}^{N_f} e_i^2. \quad (4.2)$$

Here, e_i denotes the electric charge of the i quark-flavour, Δq^{NS} , Δq^{S} , ΔG are, respectively, the nonsinglet and singlet quark and the gluon polarised densities (helicity distributions). The coefficient functions C_i are computed to a given order in α_s . PQCD evolution equations for the quarks and gluons distribution functions need the nonperturbative input quantities at some initial scale Q_0^2 . These input parametrisations, fitted to the experimental data, together with the suitable PQCD framework provide a satisfactory agreement of theory and measurements. The standard theoretical investigation of deep-inelastic scattering structure functions based on the DGLAP approach concerned originally the region of large- x and large- Q^2 . In this way the parton evolution with respect to Q^2 is taken into account, whereas the evolution with respect to Bjorken x is neglected. In the small- x region, logarithms of x become also important and therefore must be taken into account. Assuming singular as $x \rightarrow 0$ initial parton distributions $\sim x^{-a_1}$ ($a_1 > 0$), one can obtain within standard DGLAP approach a substitute of the double logarithmic $\ln^2(x)$ resummation, which is essential at low $x \ll 1$. However, if we take into account the double logarithmic terms via the suitable kernel of the evolution equations, we do not need to use the ‘artificial support’ in a form of the singular initial parametrisations. An impact of the input parton distributions on the final (after evolution) results is large. This is shown in Fig. 3, where we plot the LO DGLAP evolution from $Q_0^2 = 1 \text{ GeV}^2$ to $Q^2 = 10 \text{ GeV}^2$ of the nonsinglet polarised structure function Δq^{NS} . We test different input parametrisations of the general form:

$$\Delta q^{\text{NS}}(x, Q_0^2) \sim x^{-a_1} (1-x)^{a_2}. \quad (4.3)$$

There is no doubt, that the small- x behaviour of the parton densities is dominated just by the x^{-a_1} term, which survives the QCD evolution when $a_1 > 0$. Hence appropriate choice of the initial conditions must be consistent with used theoretical treatment. Thus there are two possible scenarios. Either we consider the unified evolution equations with two parts of the kernel: the standard DGLAP one and the other one — generating $\ln^2(x)$ terms. Then the input distributions are assumed to be nonsingular as $x \rightarrow 0$. In this case the small- x behaviour of the structure functions is totally governed by the evolution. Or we use the pure DGLAP analysis together with the singular parametrisations, which mimic the missing at low- x resummation of the leading logarithms. In Fig. 4 we plot the logarithm of the polarised nonsinglet and gluon distributions evaluated at $Q^2 = 10 \text{ GeV}^2$ within unified DGLAP+ $\ln^2(x)$ approach as a function of $\ln(1/x)$. We can estimate

the effective slopes of the presented curves $\lambda(x, Q^2)$, defined as:

$$\lambda_p(x, Q^2) = \frac{\partial \ln [\Delta p(x, Q^2)]}{\partial \ln (\frac{1}{x})}. \quad (4.4)$$

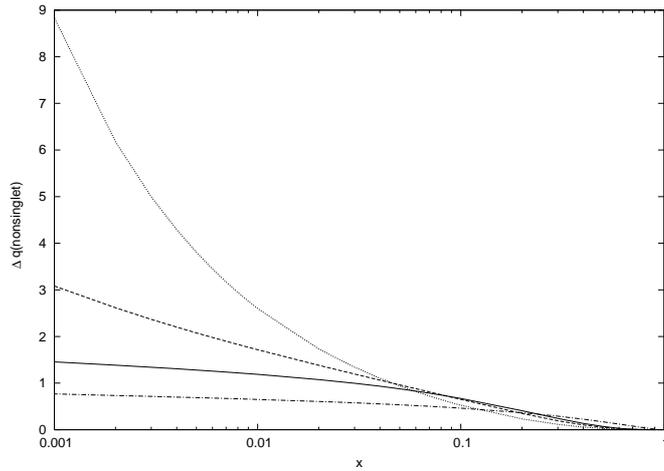


Fig. 3. The LO DGLAP evolution from $Q_0^2 = 1 \text{ GeV}^2$ to $Q^2 = 10 \text{ GeV}^2$ of the nonsinglet polarised structure function Δq^{NS} as a function of x . Different input parametrisations (4.3). Solid: $a_1 = 0, a_2 = 3$; dashed: $a_1 = 0.2, a_2 = 3$; dashed-dotted: $a_1 = 0, a_2 = 1$; dotted: $a_1 = 0.5, a_2 = 3$.

Here, Δp denotes again, respectively, the nonsinglet quark and gluon helicity distributions ($\Delta q^{\text{NS}}, \Delta G$). From the plots we find, namely, $\lambda_{\text{NS}} \approx 0.2$ and $\lambda_G \approx 0.6$. Hence the ‘ersatz’ input $\sim x^{-\lambda}$, which is able to reproduce the double logarithmic $\ln^2(x)$ resummation in the small- x region $x \in (10^{-4}; 10^{-2})$ should have a form:

$$\Delta q^{\text{NS}} \sim x^{-0.2} \quad (4.5)$$

for the nonsinglet part and

$$\Delta G \sim x^{-0.6} \quad (4.6)$$

for the gluons, respectively. As one can see, the behaviours (4.5), (4.6) are less steep than their asymptotic limits as $x \rightarrow 0$:

$$\Delta q^{\text{NS}}(x \rightarrow 0) \sim g_1^{\text{NS}}(x \rightarrow 0) \sim x^{-0.4} \quad (4.7)$$

and

$$\Delta G(x \rightarrow 0) \sim g_1^{\text{S}}(x \rightarrow 0) \sim x^{-0.8}. \quad (4.8)$$

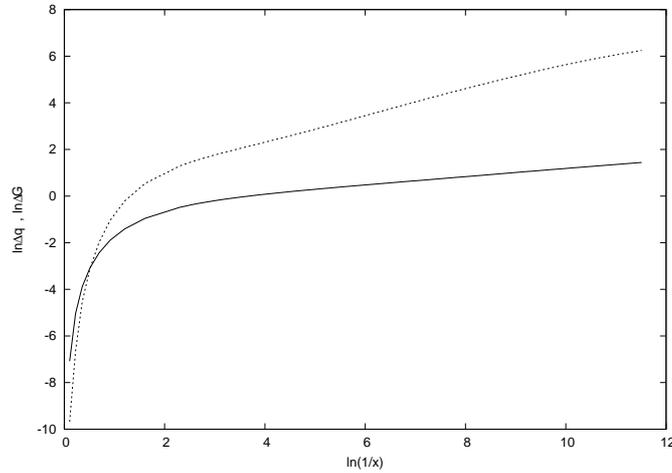


Fig. 4. The logarithm of the polarised quark nonsinglet (solid) and gluon (dotted) distributions evaluated at $Q^2 = 10 \text{ GeV}^2$ within unified DGLAP+ $\ln^2(x)$ approach as a function of $\ln(1/x)$. An illustration of the slope λ , defined in (4.4).

The results (4.7), (4.8) were obtained in [15] via estimation of the anomalous dimensions and also in [17] — within IREE (infrared evolution equation) formalism.

In conclusion, the power-like behaviour $x^{-\lambda}$ of the quark and gluon polarised distribution functions, generated by the resummation of the double logarithmic terms $\ln^2(x)$, can be also obtained via the singular factors in the initial parton distributions. Finally, let us shortly discuss the possible values of a_1 in the leading term of the initial parton densities. The choice of the value of a_1 in the input parametrisations (4.3), which controls the singular small- x behaviour, depends on the evolution length ($Q^2 - Q_0^2$). If one assumes a very low input scale $Q_0^2 \lesssim 1 \text{ GeV}^2$, then already the smaller value of $a_1 = 0.2$ in the nonsinglet case can ‘mimic’ the $\ln^2(x)$ effects. In contrast, for longer $Q_0^2 \approx 4 \text{ GeV}^2$, what denotes the shorter evolution, one should use more singular input with $a_1 \approx 0.4$ for the nonsinglet case. Similar (or even more singular) input parametrisations of the spin-dependent parton distributions have been assumed *e.g.* in [5, 6, 32]. In the next section we shall compare our theoretical predictions based on either the unified DGLAP+ $\ln^2(x)$ approach or the DGLAP analysis alone together with the singular inputs, with experimental data.

5. Comparison with experimental data

In this section we shall present our results obtained using the unified DGLAP+ $\ln^2(x)$ approach (2.3) with ‘very running’ α_s (3.2). We shall also apply an alternative scenario, described in the previous section, in which the pure DGLAP analysis is accompanied by the singular input parton densities at the low scale $Q_0^2 = 1 \text{ GeV}^2$. In this latter approach we shall compute *i.e.* the truncated Mellin moments of structure functions using directly the evolution equations for truncated moments, derived recently in [24]. Let us recall now some basic formulas concerning this approach.

The evolution equations for the truncated moments of the parton densities have the form:

$$\frac{d\bar{q}_n(x_0, Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} (P' \otimes \bar{q}_n)(x_0, Q^2), \quad (5.1)$$

$$P'(n, z) = z^n P(z), \quad (5.2)$$

where $P(z)$ is the well-known splitting function from the DGLAP equation. $\bar{q}_n(x_0, Q^2)$ denotes the n th Mellin moment of the distribution function $q(x, Q^2)$ truncated at x_0 :

$$\bar{q}_n(x_0, Q^2) = \int_{x_0}^1 dx x^{n-1} q(x, Q^2). \quad (5.3)$$

This formula is obviously valid also in the spin-dependent case where one replaces q by Δq , \bar{q}_n by $\Delta\bar{q}_n$ and P by ΔP *i.e.* the unpolarised quantities by their ‘polarised’ versions. It is particularly interesting to note that the evolution equation for the n th truncated moment has the same form as that for the parton density function itself with the modified splitting function P' (5.2). The truncated moments approach refers directly to the physical values — moments (rather than to the parton distributions), what enables one to use a wide range of deep-inelastic scattering data in terms of smaller number of parameters. In this way, no assumptions on the shape of parton distributions are needed. Using the evolution equations for the truncated moments one can also avoid uncertainties from the unmeasurable very small $x \rightarrow 0$ region. Eq. (5.1) does not account for the double logarithmic $\ln^2(x)$ terms resummation, what can be mimicked by the appropriate input, as it was described in the previous section. This is the motivation that we use the equations for the truncated moments in the studies presented here.

In order to compare theoretical predictions with experimental data over the kinematic range explored one should generalize the results to the small- $Q^2 < 1 \text{ GeV}^2$ region. Thus, one can use the prescription introduced in [25] and applied in the studies [14, 15, 26, 27], valid for arbitrary Q^2 :

$$Q^2 \rightarrow Q^2 + Q_0^2, \quad (5.4)$$

and

$$x \rightarrow \bar{x} = \frac{(Q^2 + Q_0^2)}{(2pq)}. \quad (5.5)$$

After this rearrangement the structure function g_1 can be extrapolated to the low- Q^2 region (for fixed $2pq$) including the point $Q^2 = 0$, although perturbative Q^2 -power and higher twist corrections may also play a role in this region [28]. Taking into account the small- Q^2 corrections is particularly important when one studies recent COMPASS measurements obtained for very small $4 \times 10^{-5} < x < 2.5 \times 10^{-2}$ at simultaneously very low $Q^2 \ll 1 \text{ GeV}^2$ [29]. We use in our analysis input parametrisations of polarised parton densities at the initial scale $Q_0^2 = 1 \text{ GeV}^2$ in a simple general form:

$$\Delta q(x, Q_0^2) = \eta x^{-a_1} (1-x)^{a_2}, \quad (5.6)$$

where η is a normalization factor. The exponent $-a_1$ controls the behaviour of Δq in the small- x region and the factor $(1-x)^{a_2}$ ensures the vanishing of the parton density as $x \rightarrow 1$. The singular part x^{-a_1} , where $a_1 > 0$, can mimic the resummation of the leading logarithms $\ln^2 x$.

Figs. 5–8 and Table I contain our numerical results. Fig. 5 shows the spin dependent structure function for proton g_1^p

$$g_1^p(x, Q_0^2) = \frac{1}{2} \langle e^2 \rangle [\Delta q^S(x, Q^2) + \Delta q^{\text{NS}}(x, Q^2)] \quad (5.7)$$

as a function of x , compared with HERMES data [30]. Here, $\langle e^2 \rangle$ is given by (4.2). We obtain our results solving the unified evolution Eqs. (2.3),(2.5) with ‘flat’ parametrisations $\sim (1-x)^{a_2}$ of the parton densities. We present plots for different values of the parameter a_2 : 3, 2, 1 and for a negative and positive parametrisation of gluons. In Fig. 6 we plot g_1^{N}

$$g_1^{\text{N}} = \frac{1}{2} (g_1^p + g_1^n) = \frac{g_1^d}{1 - \frac{3}{2}\omega_D} \quad (5.8)$$

as a function of x together with COMPASS data [31]. Here, g_1^p , g_1^n and g_1^d denotes the polarised structure function of proton, neutron and deuteron, respectively, and $\omega_D \approx 0.05$ is the D-state admixture to the deuteron wave function. Results are shown for different contributions of gluons to the proton’s spin at initial scale Q_0^2 , namely $\Delta G(Q_0^2) = -0.25, 0, 0.25, 0.5, 0.75$, where

$$\Delta G(Q^2) = \int_0^1 dx \Delta G(x, Q^2). \quad (5.9)$$

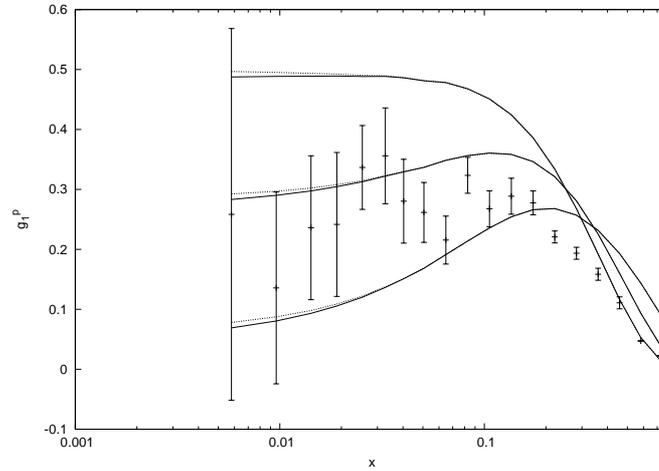


Fig. 5. The spin dependent structure function for proton g_1^p versus x , compared with HERMES data. Q^2 is the measured mean value $\langle Q^2 \rangle$ at each x . Plots for different input parametrisations of the valence quarks $\sim (1-x)^{a_2}$ from up to bottom at $x = 0.01$: $a_2 = 3$, $a_2 = 2$, $a_2 = 1$. Solid (dotted) line corresponds to the positive (negative) solutions for polarised gluons ΔG . Error bars represent the statistical uncertainties.

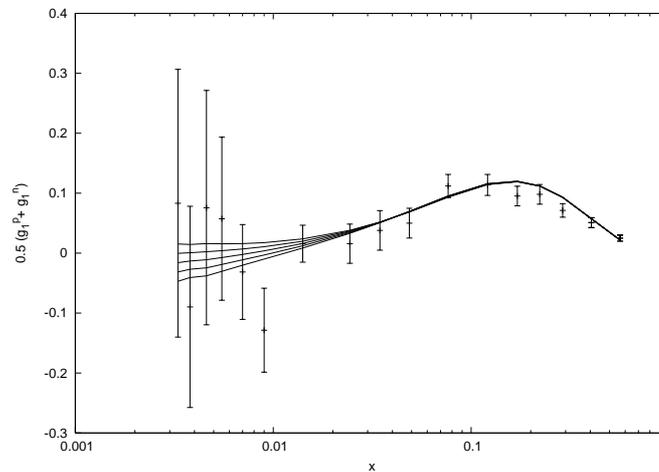


Fig. 6. $g_1^N = (g_1^p + g_1^n)/2$ versus x , compared with COMPASS data. Q^2 is the measured mean value $\langle Q^2 \rangle$ at each x . Results shown for five different contributions of gluons ΔG to the proton's spin: (from up to bottom) -0.25 , 0 , 0.25 , 0.5 , 0.75 . Error bars represent the statistical uncertainties.

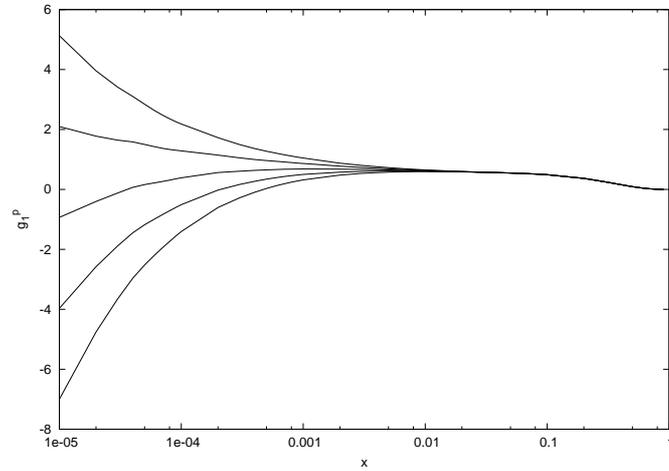


Fig. 7. The polarised structure function g_1 for the proton *versus* x at $Q^2 = 10$ GeV^2 . Results shown for five different contributions of gluons ΔG to the proton's spin: (from up to bottom) $-0.25, 0, 0.25, 0.5, 0.75$.

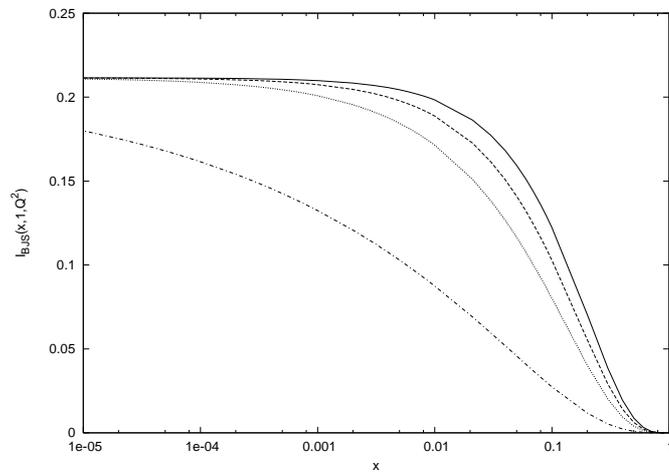


Fig. 8. Integral of the spin dependent nonsinglet structure function $g_1^{\text{NS}} = g_1^p - g_1^n$ over the range $10^{-5} \leq x \leq 1$ as a function of the low- x limit of integration. $Q^2 = 10$ GeV^2 . The comparison for different a_1 in the input parametrisation $\sim x^{-a_1}(1-x)^3$ at $Q_0^2 = 1$ GeV^2 . Plots (from up to bottom): $a_1 = 0, 0.2, 0.4, 0.8$.

In Table I we collect the integrals of g_1^N and g_1^{NS} over the range of x from COMPASS and HERMES experiments.

TABLE I

Comparison of integrals of $g_1^N = (g_1^p + g_1^n)/2$ and g_1^{NS} with COMPASS (C) and HERMES (H) data. g_1^N results for both gluon scenarios: ${}^1\Delta G < 0$ and ${}^2\Delta G > 0$ are shown. For g_1^{NS} the result incorporating \ln^2 resummation^a is compared with LO DGLAP solutions for singular input parametrisations $\sim x^{-a_1}(1-x)^3$ with a_1 : ^b0.1 ÷ 0.2, ^c0.4.

	Q^2	x_1	x_2	$\int_{x_1}^{x_2} dx g_1$	Experiment
N	10	0.004	0.7	0.04904 ¹	0.051 C
				0.04871 ²	± 0.003 (stat.) ± 0.005 (syst.)
NS	5	0.021	0.9	0.1766 ^a	0.1479 H
				0.1718 ^b	± 0.0055 (stat.) ± 0.0142 (syst.)
				0.1486 ^c	± 0.0055 (par.) ± 0.0049 (evol.)

Our results for the function g_1^N as well as its first moment are in a very good agreement with the experimental COMPASS data. There is certain discrepancy between our predictions and HERMES data. This is particularly visible for the contribution to the Bjorken sum rule

$$I_{\text{BJS}}(x_1, x_2, Q^2) = \int_{x_1}^{x_2} dx g_1^{\text{NS}}(x, Q^2) \int_{x_1}^{x_2} dx [g_1^p(x, Q^2) - g_1^n(x, Q^2)] \quad (5.10)$$

shown in Table I. Some ansatz (input parametrisation) must be adopted in two degrees of evolution. Values of g_1 measurements in the two or three Q^2 bins for each x must be evolved to their mean Q^2 and then averaged. Also, the evaluation of the first moment of the structure function g_1 requires the evolution of all measurements to a common Q^2 . In HERMES analysis this is done by using a fitted parametrisation [5], which increases as $x \rightarrow 0$: $g_1^{\text{NS}} \sim x^{-0.8}$. COMPASS Group have used several fits [5, 6, 32] which have been averaged. The discrepancy between our results and HERMES data reflects the fact that the fit used by HERMES collaboration is significantly different from ours (5.6) with $a_1 = 0.0$. Note also that very close to the HERMES value for I_{BJS} is our result obtained within LO DGLAP approach with use of the singular input parametrisation $g_1^{\text{NS}}(Q_0^2) \sim x^{-0.4}$. This makes

the contribution from the small- x region $0 < x < 0.021$ more significant — at level of 30% of the total BJS compared to our estimation based on the unified DGLAP+ $\ln^2(x)$ theoretical analysis, which gives about 17%. Furthermore, it can be seen from Table I that LO DGLAP evolution with the appropriate input $\sim x^{-a_1}$ ($a_1 > 0$) for a given region of x can reproduce the result of the DGLAP+ $\ln^2(x)$ approach. In this way, a suitably chosen initial parton density can compensate missing low- x effects in QCD analysis.

We would like also to pay special attention to the evolution equation for truncated moments of the parton distributions. Fig. 8 illustrates the truncated contribution to the Bjorken sum as a function of the truncation point x . Solving the equation for moments (5.1), (5.2) we test different input parametrisations and find the small- x contribution $I_{\text{BJS}}(0, 0.01, 10)$ (5.10) being between about 6% for the flat input $\sim (1-x)^3$ and about 60% for the very steep one $\sim x^{-0.8}(1-x)^3$. The problem of the low- x part of the Bjorken sum we have also discussed in [23, 33].

Finally, let us discuss the dependence of the polarised nucleon structure functions on the gluon distribution Δg . From Figs. 5–6 and Table I one can see that the predictions for g_1^p and $g_1^N \sim g_1^d$ (5.8) in the available experimentally x -region ($x > 0.003$) are compatible with the data independently of the assumed gluon function. Large experimental uncertainties for low- x do not allow one to discriminate between different, in particular positive and negative polarised gluon densities. In Fig. 7 we compare the proton structure function g_1^p at $Q^2 = 10$ GeV for different fractions of the nucleon spin carried by gluons at the initial scale Q_0^2 . Note, that g_1^p essentially depends on the gluon distribution only for very low- x — not before $x \approx 0.01$. Our parametrisations of ΔG (5.9) reflect the latest experimental determinations of the gluon polarisation at COMPASS [34], RHIC [35] and STAR [36]. The shape of $\Delta G(x, Q^2)$ is poorly known and the present experimental data support both positive and negative distributions, resulting in small $|\Delta G| \approx 0.2$ to 0.3 (COMPASS) or large $\Delta G = -0.56 \pm 2.16$ (RHIC), $\Delta G = -0.45$ to 0.7 (STAR). It is possible that a significant contribution to ΔG comes from low- x . A knowledge of the small- x behaviour of $\Delta G(x, Q^2)$ would provide a constraint on the shape and the sign of the gluon component. We hope future measurements at RHIC over a wide range of x and Q^2 will enable precise determination of the gluon contribution to the nucleon spin.

6. Conclusions

In this paper we have presented results for the spin structure function g_1 of the nucleon together with comparison with latest HERMES and COMPASS data. We have applied an approach that combines LO DGLAP Q^2 evolution with the resummation of the double logarithmic terms $\ln^2(x)$. This unified framework goes beyond the standard LO and NLO PQCD evolution

of the parton densities and becomes essential for $x \lesssim 0.01$. In our analysis, we have focused on the taking into account the ‘very running’ coupling effects. For the small- x region the more justified is the use of $\alpha_s = \alpha_s(Q^2/z)$ instead of $\alpha_s = \alpha_s(Q^2)$, with z being the longitudinal momentum fraction of a parent parton, carried by a next generation. In this way α_s becomes ‘very running’ *i.e.* runs in each ladder rung depending on the gluon virtuality. We have shown, that the impact of these running coupling effects becomes important at $x \lesssim 0.01$ and significantly damp the results. The decreasing factor at very small $x = 10^{-5}$ can be about 1/2 for the nonsinglet and 1/6 for the singlet (gluon) distribution function in comparison to the standard $\alpha_s(Q^2)$ prescription.

In order to calculate the first moment of g_1 over the available experimentally x region, we have solved the direct evolution equations for truncated moments of the parton densities. In this approach we have utilized the fact that the resummation of the double logarithmic terms, missing in the standard DGLAP approximation, can be mimicked by singular input parametrisation of the parton distributions. The truncated moments approach refers to the physical values — moments (rather than to the parton distributions), what in future analyses could enable one to use a wide range of deep-inelastic scattering data in terms of smaller number of parameters. In this way, no assumptions on the shape of parton distributions are needed.

Our theoretical predictions for the polarised structure function g_1 and its first moment for the deuteron are in a very good agreement with COMPASS data. There is certain discrepancy between our predictions and HERMES data, particularly visible for the contribution to the Bjorken sum. This reflects the fact that the fit used by HERMES collaboration is significantly different from ours. It must be emphasized, that the final (after evolution) results strongly depend on the input parton distributions assumed.

Finally, let us discuss the dependence of the polarised nucleon structure functions on the gluon distribution Δg . Large experimental uncertainties in the low- x region do not allow one to discriminate between different polarised gluon densities. The shape of $\Delta G(x, Q^2)$ is poorly known and the present experimental data support both positive and negative gluon distributions. A knowledge of the small- x behaviour of the gluon component and possibly a significant contribution to ΔG from this region would enable to resolve the nucleon spin puzzle. This is a challenge for future theoretical and experimental efforts.

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