

WOUNDED QUARK–DIQUARK MODEL PREDICTIONS FOR HEAVY ION COLLISIONS AT THE LHC

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The ratios of particle densities in lead–lead and proton–lead collisions to particle density in proton–proton collision in the central rapidity region at the LHC energy are predicted on the basis of wounded quark–diquark model.

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1. Introduction

The wounded quark–diquark model [1, 2] proved to be rather successful in description of particles production from nuclear targets. Assuming that high energy interactions of nucleons are dominated by independent interactions of its two constituents, a quark and a diquark, it was possible to describe pp , dAu , $CuCu$ and $AuAu$ multiplicity data collected at the RHIC collider [3]. It indicates that in all hadronic collisions the early stage of the particle production process can be understood as a simple superposition of contributions from hadronic constituents. As explained in [2] this does not preclude further collective evolution of the system that is obviously present [4, 5].

Encouraged by these results we present here quantitative predictions of the wounded quark–diquark model for the particle density ratios $R_{AB} = N_{AB}/N_{pp}$ in the central rapidity region of $PbPb$ and pPb collisions at the LHC energy $\sqrt{s} = 5500$ GeV¹.

Our main conclusion is that the model provides rather precise predictions for the nuclear collisions at LHC energies. This should allow its effective test when the data are available.

¹ The density of particles produced in pp collision at the LHC energy cannot be predicted in the present approach.

In the next section the prediction of the wounded quark–diquark model for particle density in the central rapidity region in PbPb collision is presented. In Section 3 we discuss the consequences of the model for mid-rapidity density in p Pb collisions. Our conclusions are listed in the last section where also some comments are included.

2. PbPb collision

The relation between particle production in nucleon–nucleon and symmetric nucleus–nucleus collisions implied by the wounded quark–diquark model is given by [1]

$$R_{AA} \equiv \frac{N_{\text{PbPb}}(y)}{N_{pp}(y)} = \frac{w_{\text{PbPb}}^{(q+d)}}{2w_p^{(q+d)}}, \quad (1)$$

where the r.h.s. of this equation is independent of rapidity y^2 . $N_{\text{PbPb}}(y)$ and $N_{pp}(y)$ are the particle densities in PbPb and pp collisions, respectively. $w_p^{(q+d)}$ is the average number of wounded constituents in a single pp collision (per one proton). Mean number of wounded quarks and diquarks in both colliding nuclei $w_{\text{PbPb}}^{(q+d)}$ at a given impact parameter b is given by (mass number $A = 208$) [6]

$$w_{\text{PbPb}}^{(q+d)}(b) = \frac{2A}{\sigma_{\text{PbPb}}(b)} \int T(b-s) \left\{ 2 - [1 - p_q G(s)]^A - [1 - p_d G(s)]^A \right\} d^2s, \quad (2)$$

with $G(s)$ defined as

$$G(s) = \int d^2s' \sigma_{\text{in}}(s-s') T(s'), \quad (3)$$

where $T(s)$ is the nuclear thickness function $T(s) = \int dz \rho(\sqrt{s^2 + z^2})$ (normalized to unity). Here and in the following for the nuclear density ρ we take the standard Woods–Saxon formula with the nuclear radius $R_{\text{Pb}} = 6.5$ fm and the skin depth $d = 0.54$ fm. $\sigma_{\text{PbPb}}(b)$ is the inelastic differential PbPb cross section³. Finally, p_q and p_d are the probabilities for a quark and a diquark to interact in a single pp collision, respectively.

We assume the differential inelastic pp cross section $\sigma_{\text{in}}(s)$ (probability for inelastic pp collision at a given impact parameter s) to be in a simple Gaussian form⁴

² Provided we are far enough from the fragmentation regions, where contributions from cascade and unwounded constituents are expected [2].

³ $\sigma_{\text{PbPb}}(b) = 1$, except at very large impact parameters ($b > 14$ fm) which are of no interest.

⁴ We believe that $\sigma_{\text{in}}(0) = 1$ is very close to reality. At ISR energies $\sigma_{\text{in}}(0) = 0.92$ [7].

$$\sigma_{\text{in}}(s) = e^{-s^2/\varkappa^2}, \quad (4)$$

where $\varkappa^2 = \sigma_{\text{in}}/\pi$ and σ_{in} is the total inelastic pp cross section $\sigma_{\text{in}} = \int \sigma_{\text{in}}(s) d^2s$.

The multiplicity data are usually presented *versus* the number of wounded nucleons [8]

$$w_{\text{PbPb}}^{(n)}(b) = \frac{2A}{\sigma_{\text{PbPb}}(b)} \int T(b-s) \left\{ 1 - [1 - G(s)]^A \right\} d^2s. \quad (5)$$

This completes all necessary formulas.

To obtain $w_p^{(q+d)}$ we followed exactly the procedure proposed at lower energies, where we extracted this number [1] by studying differential elastic pp scattering cross section data. Indeed, assuming a nucleon to be composed of a quark and a diquark, it was possible to describe the small momentum transfer, $|t| < 3 \text{ GeV}^2$, elastic pp and πp scattering cross section data with a very high precision [9]. In the present case we studied the small t elastic $p\bar{p}$ scattering data at the Tevatron energy giving $w_p^{(q+d)} = 1.24 \pm 0.01$. Considering many different predictions regarding elastic pp scattering at 14 000 GeV [10] we obtained $w_p^{(q+d)} = 1.28 \pm 0.02$. Thus in our calculations at $\sqrt{s} = 5500 \text{ GeV}$ for the average number of wounded quarks and diquarks in a single pp collision, per one colliding proton, we take

$$w_p^{(q+d)} = 1.26 \pm 0.02. \quad (6)$$

This number is the dominant uncertainty of our approach. The detailed discussion of this problem, however, is beyond the scope of this investigation.

Since the total inelastic pp cross section σ_{in} is not known at $\sqrt{s} = 5500 \text{ GeV}$ we performed our calculations for three different inelastic cross sections $\sigma_{\text{in}} = 60, 67$ and 75 mb . We noticed that at a given number of wounded nucleons we obtain practically the same number of wounded constituents for each value of σ_{in} . This observation allows for predictions at the LHC energy, which are practically independent of the value of σ_{in} .

The calculated numbers for $\sigma_{\text{in}} = 60 \text{ mb}$ are presented in Table I, where following [9], we assumed $p_q = p_d/2 = w_p^{(q+d)}/3^5$.

Dividing $w_{\text{PbPb}}^{(q+d)}$ by $2w_p^{(q+d)}$ we obtain our prediction for the ratio R_{AA} (1) shown in Fig. 1. For comparison the prediction of the wounded nucleon model [8] is also shown.

⁵ We also checked different choices, ranging from $p_d = p_q$ to $p_d = 2p_q$. We observe that the relation $w_{\text{PbPb}}^{(q+d)}$ *versus* $w_{\text{PbPb}}^{(n)}$ is not changed.

TABLE I

Mean number of wounded quarks and diquarks $w_{\text{PbPb}}^{(q+d)}$ and wounded nucleons $w_{\text{PbPb}}^{(n)}$ in PbPb collision as a function of the impact parameter b .

b [fm]	$w_{\text{PbPb}}^{(n)}$	$w_{\text{PbPb}}^{(q+d)}$	b [fm]	$w_{\text{PbPb}}^{(n)}$	$w_{\text{PbPb}}^{(q+d)}$
0	409.2	793.3	8	179.9	315.2
1	405.8	783.4	9	139	238.4
2	394.3	753.5	10	101.6	169.6
3	373.3	704.2	11	68.8	111.1
4	343.2	638.9	12	42.1	65.2
5	306.4	562.8	13	22.4	33.1
6	265.4	480.7	14	9.9	14
7	222.5	397	15	3.6	4.9

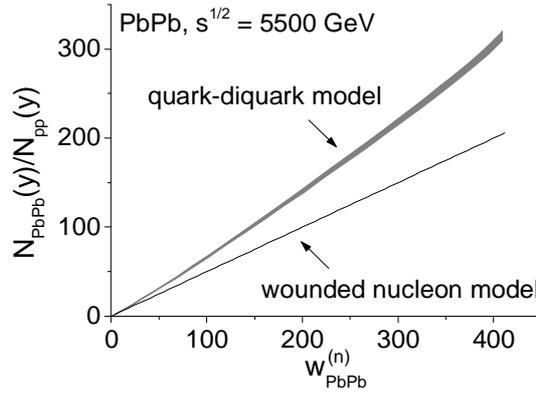


Fig. 1. Wounded quark–diquark model prediction for the multiplicity ratio of particles produced in PbPb collision to those produced in pp collision at any rapidity y . The grey band reflects the uncertainty in the value of $w_p^{(q+d)}$. The prediction of the wounded nucleon model is also shown.

3. pPb collision

At the vanishing c.m. rapidity we have the following relation between particle production in pp and pA collisions [1]

$$R_{pA} \equiv \frac{N_{pPb}(y=0)}{N_{pp}(y=0)} = \frac{w_{pPb}^{(q+d)}}{2w_p^{(q+d)}}, \quad (7)$$

where the average number of wounded quarks and diquarks in pPb collision $w_{pPb}^{(q+d)}$ at a fixed impact parameter b is given by

$$w_{p\text{Pb}}^{(q+d)}(b) = \frac{AG(b)w_p^{(q+d)}}{1 - [1 - G(b)]^A} + \frac{2 - [1 - p_q G(b)]^A - [1 - p_d G(b)]^A}{1 - [1 - G(b)]^A}. \quad (8)$$

The first term gives the number of wounded constituents in the target (Pb nucleus). Indeed, it is the number of wounded nucleons in the target times the number of wounded constituents in a single pp collision. The second term gives the number of wounded constituents in the projectile that underwent many inelastic collisions. The derivation of this term is presented in the appendix.

Mean number of wounded nucleons at a given impact parameter b is given by

$$w_{p\text{Pb}}^{(n)}(b) = \frac{AG(b)}{1 - [1 - G(b)]^A} + 1, \quad (9)$$

where the first term gives the number of wounded nucleons in the target, plus one wounded nucleon being the projectile itself.

Again, we performed the calculations for three different inelastic cross sections $\sigma_{\text{in}} = 60, 67$ and 75 mb. At a given impact parameter b we obtain significantly different numbers of wounded nucleons and wounded constituents, however, when we plot $w_{p\text{Pb}}^{(q+d)}$ versus $w_{p\text{Pb}}^{(n)}$ the three curves almost exactly follow each other. Similarly to the previous case of PbPb collision, this observation allows for predictions at the LHC energy which are independent of the value of σ_{in} . The obtained numbers for $\sigma_{\text{in}} = 75 \text{ mb}^6$ and $p_q = p_d/2 = w_p^{(q+d)}/3$ are presented in Table II.

TABLE II

Mean number of wounded quarks and diquarks $w_{p\text{Pb}}^{(q+d)}$ and wounded nucleons $w_{p\text{Pb}}^{(n)}$ in $p\text{Pb}$ collision as a function of the impact parameter b .

b [fm]	$w_{p\text{Pb}}^{(n)}$	$w_{p\text{Pb}}^{(q+d)}$	b [fm]	$w_{p\text{Pb}}^{(n)}$	$w_{p\text{Pb}}^{(q+d)}$
0	17.05	22.23	6	6.78	9.2
1	16.83	21.95	7	4.01	5.49
2	16.15	21.08	8	2.6	3.47
3	14.9	19.51	9	2.15	2.76
4	12.92	17.01	10	2.03	2.57
5	10.1	13.44	11	2.01	2.53

⁶ This time we take the largest number. Maximal number of wounded nucleons noticeably depends on σ_{in} . The relation $w_{p\text{Pb}}^{(q+d)}$ versus $w_{p\text{Pb}}^{(n)}$ hardly depends on it, however.

Dividing $w_{p\text{Pb}}^{(q+d)}$ by $2w_p^{(q+d)}$ we obtain our prediction for the ratio R_{pA} (7) presented in Fig. 2. The maximal number of wounded nucleons is 14 and 17 for $\sigma_{\text{in}} = 60$ and 75 mb, respectively. For comparison we also show the prediction of the wounded nucleon model.

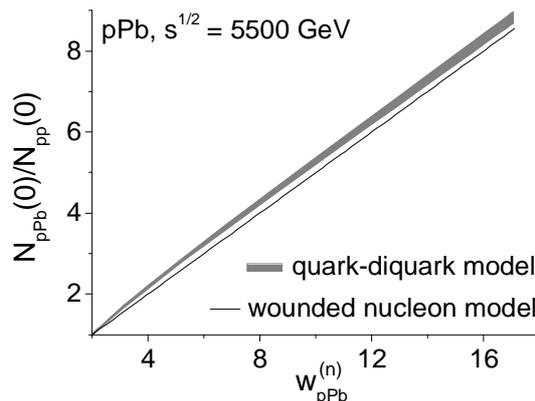


Fig. 2. Wounded quark–diquark model prediction for the multiplicity ratio at mid-rapidity of particles produced in $p\text{Pb}$ collision to those produced in pp collision. The maximal number of wounded nucleons is 14 and 17 for $\sigma_{\text{in}} = 60$ and 75 mb, respectively. The grey band reflects the uncertainty in the value of $w_p^{(q+d)}$. The prediction of the wounded nucleon model is also shown.

It is not surprising that the wounded quark–diquark model prediction is rather close to the line predicted by the wounded nucleon model. Indeed, comparing both scenarios the only difference is the projectile that undergoes many inelastic collisions producing slightly more particles [2].

4. Conclusions and comments

Our conclusions can be formulated as follows.

- (i) Encouraged by a very good agreement of the wounded quark–diquark model with the RHIC pp , $d\text{Au}$, CuCu and AuAu data, we evaluated particle densities in the central rapidity region in PbPb and $p\text{Pb}$ collisions at the LHC energy $\sqrt{s} = 5500$ GeV.
- (ii) In our approach the particle density in PbPb (at the central rapidity region) and $p\text{Pb}$ (at mid-rapidity) is proportional to the density of particles produced in an elementary pp collision. Since the pp particle density is presently unknown and it cannot be calculated in the present approach we only give the ratios $R_{AB} = N_{AB}/N_{pp}$.

- (iii) The dominant uncertainty of our calculation is the number of wounded quarks and diquarks in a single pp collision at $\sqrt{s} = 5500$ GeV which we estimated to be 1.26 ± 0.02 .
- (iv) Since the total inelastic pp cross section is not known at $\sqrt{s} = 5500$ GeV we performed our calculations for three different inelastic cross sections $\sigma_{\text{in}} = 60, 67$ and 75 mb. The functional relation between number of wounded quarks and diquarks and number of wounded nucleons practically does not depend on the value of σ_{in} . This observation allowed for predictions at the LHC energy, which are independent of the value of σ_{in} .

Following comments are in order.

(a) Our prediction regarded the multiplicity density ratio R_{AA} can be also applied to the total multiplicities measured for central PbPb collisions. For such centralities additional contributions from cascade and unwounded constituents seem to be less important [2].

(b) We found previously that the 200 GeV RHIC data in the range $|y| < 3.7$ can be solely described by the contribution from the wounded constituents. Beyond this region unwounded constituents and cascade seem to appear [2]. Assuming that these additional contributions begin at y proportional to the rapidity beam Y , it allows us to estimate that at $\sqrt{s} = 5500$ GeV the ratio R_{AA} should be independent of y in the approximate range $|y| < 6$.

(c) In principle our predictions could be applied to any energy provided σ_{in} remains in the range from 60 mb to 75 mb. The only difference is the number of wounded quarks and diquarks in a single pp collision. For instance at $\sqrt{s} = 14000$ GeV we estimate $w_p^{(q+d)}$ to be 1.28 ± 0.02 .

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Appendix A

Wounded constituents in the projectile

The average number of wounded quarks and diquarks in a nucleon that underwent exactly k inelastic collisions is given by

$$w_k = 1 - (1 - p_q)^k + 1 - (1 - p_d)^k, \quad (\text{A.1})$$

where p_q and p_d are the probabilities for a quark and a diquark to interact in a single pp collision, respectively.

The probability that the nucleon at a given impact parameter b underwent exactly k inelastic collisions is given by a standard formula

$$P_k(b) = \frac{1}{1 - [1 - G(b)]^A} \binom{A}{k} [G(b)]^k [1 - G(b)]^{A-k}, \quad (\text{A.2})$$

where we assume that at least one inelastic collision takes place.

Thus, the number of wounded constituents in a nucleon that passed through the nucleus of mass number A at a given impact parameter b is

$$\sum_{k=1}^A w_k P_k(b) = \frac{2 - [1 - p_q G(b)]^A - [1 - p_d G(b)]^A}{1 - [1 - G(b)]^A}, \quad (\text{A.3})$$

i.e. the second term in Eq. (8).

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