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IDENTIFICATION OF STELLAR SPECTRA USING METHODS OF STATISTICAL SPECTROSCOPY*

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The aim of this paper is to present a new method of classification of the stellar spectra. The parameters characterizing the spectra are moments of intensity distributions. Statistical Theory of Spectra is used as a tool for the classification of the stellar spectra. The method of using distribution moments has been already proposed by the present authors as new molecular descriptors in the theory of molecular similarity. In this paper the method is implemented in astrophysics. Moments of intensity distributions have different values for every spectral type. We present diagrams based on the knowledge of the moments in which all types of stars can be recognized. Distribution moments obtained from the experimental spectra can be helpful to recognize easily the spectral type.

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1. Introduction

Spectra of stars are determined by the physical conditions in their atmospheres and are the richest sources of information about stellar physical properties. The spectral classification is a rough and most common approach to the recovery and exploitation of this information. In the classification system the stars with similar spectra are grouped together in ordered boxes. Each classification box, determined by its standard star, corresponds to a unique spectral type of the system. Thus, the spectral type (box) gives

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a concise description of both the spectrum morphology and the corresponding physical properties of the stars in the box. In principle, classification is an assignment of a star to a given classification box by the comparison (the degree of similarity) of its spectrum to the spectrum of a box standard star. The boxes are characterized by one or more sets of labels depending on classification dimensions. The older classification systems were based on visual inspection of the spectrograms, usually in a low or moderate dispersion. In such an approach, the classification depends not only on the used spectral range but also on spectral resolution. Modern, more objective classification systems are based on quantitative measures of the chosen classification criteria, usually relative intensities of some chosen spectral lines, bands and blends. The most commonly used Morgan–Keenan classification system (MK system) employs to characterize their boxes two labels: the spectral and the luminosity class. They are related, respectively, to the stellar surface temperature and to the gravity. The MK system is fully empirical. It is determined by criteria defined by directly observable features of the spectrum (in the photographic range). These criteria (in the form of relative intensities of chosen spectral lines (blends)) were subject to many revisions aimed at establishing direct relations between them and the physical parameters of the star, *e.g.*, the temperature and the surface gravity [1, 2]. The HR diagram is a graphical representation of the MK system.

In general, the more spectral features of a star is taken into account, the more precise is a classification scheme. This suggests a statistical approach to the classification procedure. The resulting classification scheme depends on the number of used criteria and on the number of parameters describing properties of stars.

In the present paper we propose a new method of classification based on the use of moments of intensity distribution of stellar spectra. The idea of using the new parameters characterizing stellar spectra comes from the statistical theory of spectra. The origins of statistical spectroscopy may be traced back to the thirties [3]. For many years statistical spectroscopy was mainly used in the nuclear physics, where not exactly known character of the interparticle interactions motivated using the language of statistics [4]. Later, the concepts of statistical theory of spectra has been applied in many areas of physics. The motivations for the introduction of a statistical description in various areas of physics were different. The first statistical studies of atomic spectra were performed by Rosenzweig and Porter [5]. In this case the authors tried to create a global description of the detailed features of the spectra. They studied the "repulsion of the energy levels" in complex atomic spectra. Since then, the methods of statistical spectroscopy were applied in atomic and molecular physics in order to avoid detailed calculations, to reduce the computing time, or in order to notice some new global features of the systems. Let us just mention methods of determining envelopes of the molecular electronic bands [6], or statistical studies on properties of spectra of the Heisenberg Hamiltonian [7]. In all these considerations the basic quantities which have been calculated are moments of different types of distributions.

Recently, the authors of the present paper introduced the theory to the studies on molecular similarity. Using the language of molecular similarity [8–11], new descriptors (moments of intensity distributions) have been introduced [12–14]. The new descriptors have been tested using spectra of nitriles and of amides [15]. It has been demonstrated that the moments of the intensity distributions for amides are different that the ones for nitriles. The application of the statistical moments in the theory of similarity seems to be very attractive.

In this paper, we extend this concept to astrophysics. We propose moments of stellar intensity distributions as parameters that determine the spectral type. We implement the theory using model stellar spectra. On the basis of the presented results, it is easy to determine the spectral type of the experimental spectra. We show that several lowest moments are sufficient to such kind of studies. The first moment alone may describe several different spectra (similar only in the sense of the mean value of the distribution). The higher-order moments can supply the information which may be considered as corrections to the standard HR diagram. In particular, they allow us to distinguish between the spectral subtypes.

2. Theory

In this paper new parameters characterizing the stellar spectra are defined. They are derived using methods of statistical spectroscopy. In this approach spectra are considered as statistical intensity distributions. The distributions are characterized by their moments. For a discrete distribution I_ν, the n-th moment is defined as:

$$M_n = \mathcal{N} \sum_i I_{\nu_i} \nu_i^n \,, \tag{1}$$

where I_{ν_i} is the intensity of the i-th line, ν_i is the corresponding frequency and

$$\mathcal{N} = \frac{1}{\sum\limits_i I_{\nu_i}} \,, \qquad n = 0, 1, 2, \ldots \tag{2}$$

Due to the normalization constant $\mathcal{N}$, the moments are independent of the units of the intensities. The same effect can be obtained if the intensities rather than the moments are properly normalized.

Convenient characteristics of the distributions may also be derived from properly scaled distribution moments. The moments which are normalized to the mean value equal to zero ($M_1' = 0$) are referred to as the *centered moments*. Thus, the n-th centered moment is defined as

$$M_n' = \mathcal{N} \sum_i I_{\nu_i} (\nu_i - M_1)^n \,. \tag{3}$$

Both M_n and M_n' (for which the frequency is scaled by a shift: $\nu_i \rightarrow \nu_i - M_1$) are expressed in units of the n-th powers of ν.

The moments for which, additionally, the variance is equal to 1 ($M_1'' = 0$, $M_2'' = 1$) are defined as

$$M_n'' = \mathcal{N} \sum_i I_{\nu_i} \left[\frac{(\nu_i - M_1)}{\sqrt{M_2 - (M_1)^2}} \right]^n \,. \tag{4}$$

The frequency ν_i in the scaled moments, defined in Eq. (4), has been transformed to a dimensionless quantity $(\nu_i - M_1)/\sqrt{M_2 - (M_1)^2}$. Therefore, M_n'' are dimensionless.

We propose a set of the intensity distribution moments as new estimators of the stellar spectra. A very clear meaning has the first moment, M_1. It describes the mean value of the distribution. The second centered moment, M_2', is the variance, that gives the width of the distribution. The skewness coefficient, M_3'', characterizes the asymmetry of the spectrum. The kurtosis coefficient, M_4'', is connected to the excess of the distribution.

3. Results and discussion

The method is presented for model stellar spectra [16]. Figs. 1–7 show moments of intensity distributions for 48 model spectra as functions of frequency. Fig. 1 presents M_1. The horizontal axis labels the spectra. The numbers correspond to the following types:

(1) O56If	(2) O5V	(3) O7B0V	(4) O7B1III	(5) O8I	(6) B1I
(7) B34V	(8) B35I	(9) B5III	(10) B6V	(11) B8I	(12) B9III
(13) A03I	(14) A13V	(15) A3III	(16) A57V	(17) A6F0III	(18) A79I
(19) A7IV	(20) A8V	(21) A9F0V	(22) F03I	(23) F3IV	(24) F47III
(25) F67V	(26) F7I	(27) F89IV	(28) F89V	(29) G04III	(30) G12V
(31) G2IV	(32) G56III	(33) G68V	(34) G8IV	(35) G8K0III	(36) G9K0V
(37) K2III	(38) K4III	(39) K4V	(40) K5V	(41) K7III	(42) M01III
(43) M2V	(44) M3III	(45) M4III	(46) M5III	(47) M6III	(48) M7SIII

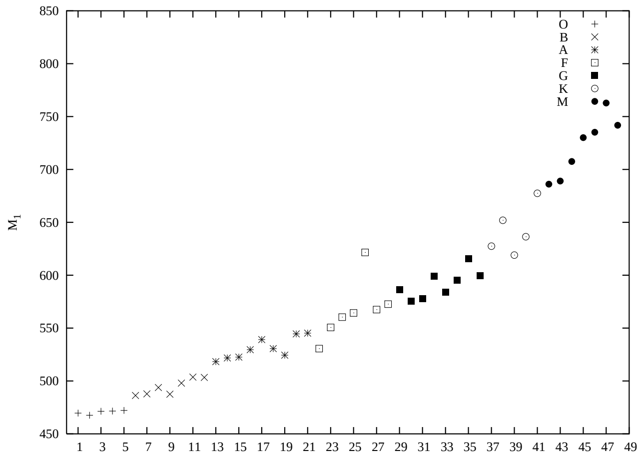

Fig. 1. M_1 for particular spectral types.

This figure gives us a standard classification (HR diagram). In many cases we have degeneracies for M_1. For example the first moment is the same for the spectra labeled by 1, 2, 3. Therefore, a single moment (M_1) is not sufficient to distinguish between the spectra. Using the language of quantum mechanics, the degeneracy vanishes for higher order moments. Even the second moments (Fig. 2) are different for the cases 1, 2, 3. In this sense, higher order moments can be recognized as the corrections to HR diagram. Figs. 2–7 show moment-based classification of stellar spectra. As it is seen, particular spectral types concentrate in different regions of the plots. The plot M_1–M_2' (Fig. 2) has a shape of a parabola. The two wings of the

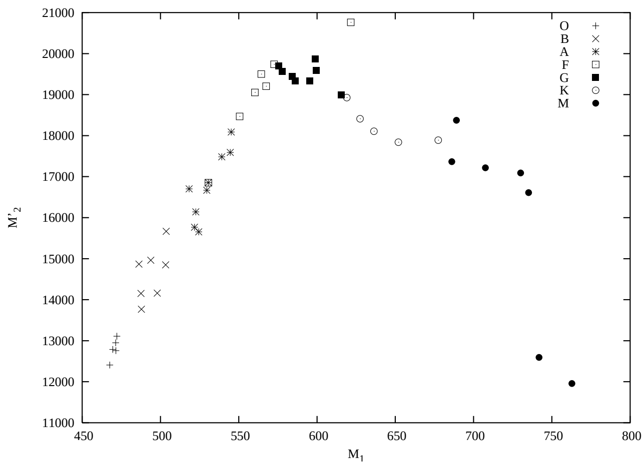

Fig. 2. Moment-based classification of stellar spectra: M_1 *versus* M_2'.

parabola correspond to different spectral types (for example M_2' is the same for A and M types). This kind of degeneracy suggests that also higher order moments should be considered for a proper classification of all spectral types or subtypes. The degeneracy vanishes if we take into account for example the third moment that is different for all stellar types (Fig. 3).

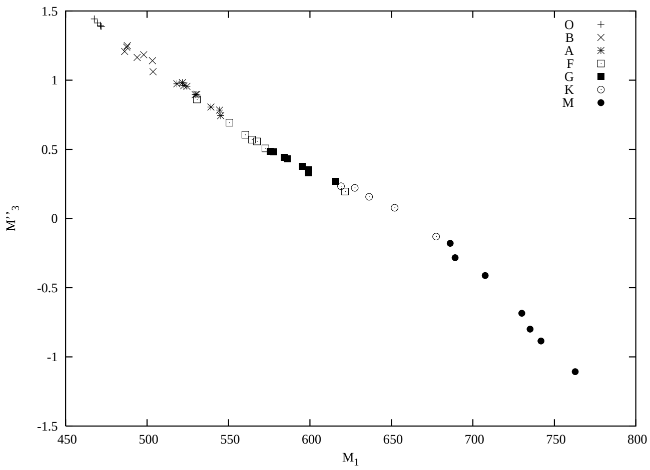

Fig. 3. Moment-based classification of stellar spectra: M_1 *versus* M_3''.

Similar shape as M_1–M_2' has M_1–M_4'' moment-based classification. This observation may suggest some kind of correlations between the second and the fourth moment (Fig. 2 and Fig. 4). Probably the same similarity relations between pairs of spectra using M_2' and M_4'' can be obtained. This

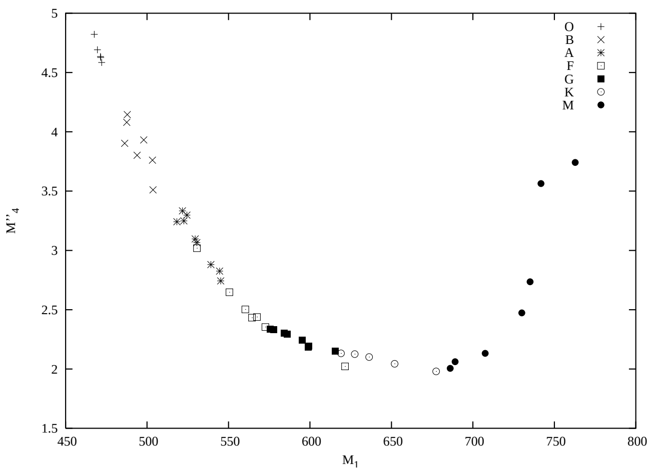

Fig. 4. Moment-based classification of stellar spectra: M_1 *versus* M_4''.

correlation is clearly seen in Fig. 6: There is a negative linear correlation between M_2' and M_4'' (M_2' increases while M_4'' decreases). This correlation is reflected in M_3''–M_2' and M_3''–M_4'' dependencies (Figs. 5 and 7, respectively). In both cases the dependence is parabolic though the parabolas, one relative to the other, are inverted.

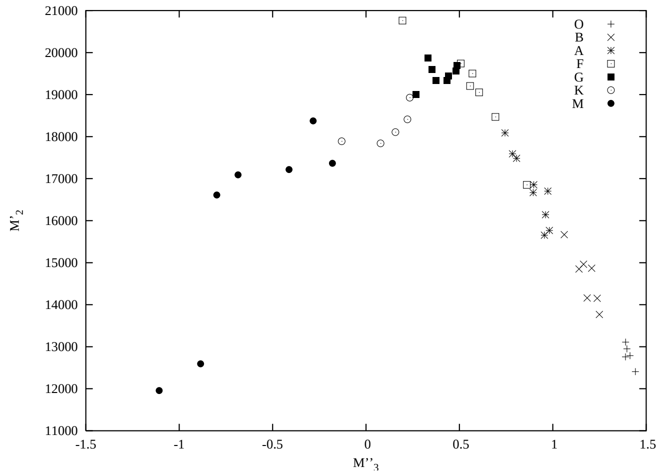

Fig. 5. Moment-based classification of stellar spectra: M_3'' *versus* M_2'.

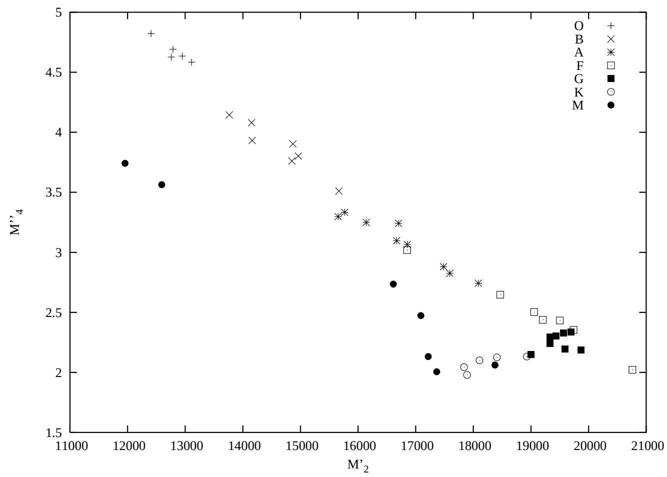

Fig. 6. Moment-based classification of stellar spectra: M_2' *versus* M_4''.

We observe also the negative correlation between the first and the third moment seen in Fig. 3 (M_1 decreases and the M_3'' increases). It means that a shift of the mean value of the distribution (M_1) is connected with a change

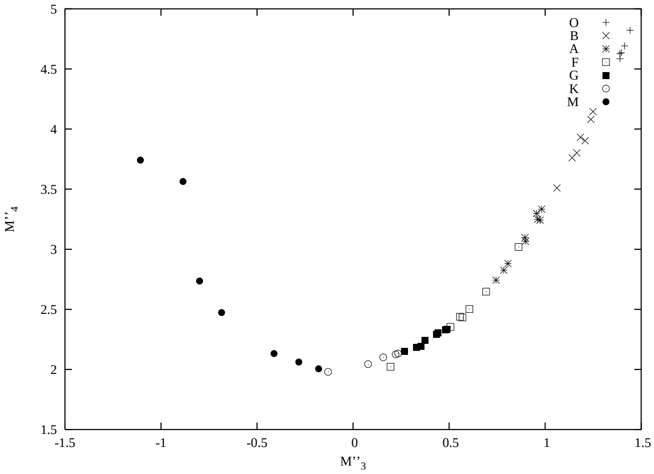

Fig. 7. Moment-based classification of stellar spectra: M_3'' *versus* M_4''.

of the asymmetry coefficient (M_3''). However, the studies on correlations are not relevant for the presented approach. In this work we are interested in a set of moments for a single spectrum that can give a good characteristics of spectra obtained experimentally. We observe that moments are similar for particular types (they cluster). For example, M types spectra are in the left bottom part of the Fig. 5 (M_3''–M_2' moment-based classification). However, the moments for particular subtypes are different (similar but not the same).

4. Conclusions

The standard HR diagram has been derived from the intensity distribution moments. By using the higher-order moments the degeneracy, present in the HR diagram, has been removed. In this sense, the contributions from the higher-order moments can be recognized as corrections to the first order approximation, *i.e.* to the HR diagram. Then, by a simple analysis of the intensity distribution moments one can distinguish between the spectral subtypes and a set of several lowest moments may be used as a composite label identifying a star. In this context, a catalogue containing several lowest moments for each subtype may be very useful. Summarizing, the intensity distribution moments appear to be reliable and simple parameters characterizing spectral types of stars.

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