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UNITARITY CORRECTIONS FROM THE HIGH ENERGY QCD EFFECTIVE ACTION*

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We investigate the derivation of reggeon transition vertices from Lipatov's gauge invariant effective action for high energy processes in QCD. In particular we address the question of longitudinal integrations in order to reduce the vertices into the required purely transverse form. We explicitly derive the BFKL-kernel and verify vanishing of the 2-to-3 reggeon transition vertex. First results on the derivation of the 2-to-4 reggeon transition vertex are discussed.

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1. Introduction

In 1995 an effective action [1, 2] for QCD scattering processes at high center of mass energies has been proposed by Lipatov which describes the interaction of reggeized gluons with quark-(ψ) and gluon-(v_μ) fields local in rapidity. The action is formulated for the Quasi-Multi-Regge-Kinematics (QMRK), where quark and gluon fields build clusters localized in rapidity, while these clusters themselves are ordered strongly in rapidity w.r.t. each other. Their overall rapidity is bounded from above and below by the rapidities of the scattering particles which are taken to propagate along opposite light-cone directions n^+ and n^-. Interaction between the clusters is mediated by the reggeon fields $A_\pm$. The Lagrangian of the effective action is given by

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}}(v_\mu, \psi) + (A_-(v_\mu) - A_-)\partial^2 A_+ + (A_+(v_\mu) - A_+)\partial^2 A_-$$

with

$$A_\pm(v) = v_\pm D_\pm^{-1}\partial_\pm = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + g^2 v_\pm \frac{1}{\partial_\pm} v_\pm \frac{1}{\partial_\pm} v_\pm - \dots \quad (1)$$

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Light-cone components are defined by $k^{\pm} \equiv n^{\pm} \cdot k$. To obtain quantities of interest from the action, it is necessary to perform integrations over light-cone degrees of freedom, which in particular requires to find the right prescription for the poles in $\partial_{\pm}$. Formally these poles are harmless as the action is given for central rapidities, but to obtain reliable results, a solid understanding of these poles and the integration over light-cone components in general is crucial. In order to do so, we first attempt to rederive some well-known results from the action. In the following we present some first results.

2. The gluon trajectory

We start with the gluon trajectory. From the Feynman rules of the effective action [2] one obtains

$$2g^2 N_c \int \frac{dk^+ dk^- d^2\boldsymbol{k}}{(2\pi)^4} \frac{\boldsymbol{q}^2}{k^+} \frac{\boldsymbol{q}^2}{k^-} \frac{-i}{k^+k^- - \boldsymbol{k}^2 + i\epsilon} \frac{-i}{k^+k^- - (\boldsymbol{k}-\boldsymbol{q})^2 + i\epsilon} . \qquad (2)$$

Comparing this with the corresponding one-loop QCD amplitude in the Regge limit, it appears that the poles in k^+ and k^- should be interpreted as a principal value *i.e.* $1/k^- = 1/2(1/(k^- + i\epsilon) + 1/(k^- - i\epsilon))$. Performing the integral over k^- by closing the contour at infinity one finds

$$\left(-i2\boldsymbol{q}^2\right) \int \frac{dk^+}{|k^+|} \beta(\boldsymbol{q}), \qquad \beta(\boldsymbol{q}) = g^2 \frac{N_c}{2} \int \frac{d^2\boldsymbol{k}}{(2\pi)^3} \frac{-\boldsymbol{q}^2}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2} , \qquad (3)$$

where the k^+ integral will produce the logarithm in s.

3. The 2-to-2 transition kernel

Next we come to the 2-to-2 reggeon transition kernel (see Fig. 1) which is the real part of the BFKL-kernel. From the effective action one obtains:

$$\begin{aligned} i8g^2 \int \frac{dl^+ dk^-}{2\pi} \Bigg[& \frac{f^{a_1 b_1 c} f^{c b_2 a_2}}{-l^+k^- - (\boldsymbol{k}-\boldsymbol{l})^2 + i\epsilon} \left(-\boldsymbol{q}^2 - \frac{(\boldsymbol{l}-\boldsymbol{q})^2 \boldsymbol{k}^2}{l^+k^-} - \frac{(\boldsymbol{k}-\boldsymbol{q})^2 \boldsymbol{l}^2}{l^+k^-} \right) \\ + & \frac{f^{a_2 b_1 c} f^{c b_2 a_1}}{l^+k^- - (\boldsymbol{q}-\boldsymbol{k}-\boldsymbol{l})^2 + i\epsilon} \left(-\boldsymbol{q}^2 + \frac{\boldsymbol{l}^2 \boldsymbol{k}^2}{l^+k^-} + \frac{(\boldsymbol{k}-\boldsymbol{q})^2 (\boldsymbol{l}-\boldsymbol{q})^2}{l^+k^-} \right) \Bigg] . \end{aligned} \qquad (4)$$

Attempting to perform the integral over k^- by closing the contour at infinity, one faces the problem that the term proportional to $\boldsymbol{q}^2$ is logarithmically divergent. However, if one symmetrizes in the color labels b_1 and b_2, the singularity cancels and the contour can be closed. Together with the remaining terms (where poles in light-cone momenta are interpreted as in Sec. 2) one

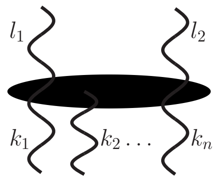

Fig. 1. Transition of 2 to n reggeons. For all transition kernels, momenta and color labels from above will be denoted by l and a respectively, while those from below by k and b respectively, with $\boldsymbol{q} = \sum_i \boldsymbol{k}_i = \sum_j \boldsymbol{l}_j$, and $\boldsymbol{q}^2 = -t$ being the momentum transfer.

obtains the real (connected) part of the BFKL-Kernel. In the anti-symmetric case however, the singularity does not cancel. Instead (defining $\mu = -l^+k^-$) we face the following integral

$$\int \frac{dl^+}{|l^+|} \int \frac{d\mu}{2\pi} \left(\frac{1}{\mu - (\boldsymbol{k}-\boldsymbol{l})^2 + i\epsilon} - \frac{1}{-\mu - (\boldsymbol{q}-\boldsymbol{k}-\boldsymbol{l})^2 + i\epsilon} \right) . \tag{5}$$

As the effective theory is formulated for central rapidities, the integration over μ can be understood as $\int d\mu := \lim_{M\to\infty} \int_{-M}^{M} d\mu$ which leads to vanishing of (5). This might appear surprising from the first sight, as usually the BFKL-Kernel with anti-symmetric color configuration is associated with reggeization of the gluon. However, in the effective theory reggeization is taken into account by resuming contributions like (3), and therefore vanishing of (5) is needed in order to avoid double counting.

4. The 2-to-3 transition

Due to signature conservation, the 2-to-3 transition has to vanish. For the connected part of the kernel one obtains for a particular ordering of color and momenta indices

$$\begin{aligned} &2^5 g^3 f^{a_1b_1c_1} f^{c_1b_2c_2} f^{c_2b_3a_2} \int \frac{dl^+}{l^+} \int \frac{d\mu_1 d\mu_3}{(2\pi)^3} \Bigg[\Bigg(-\boldsymbol{q}^2 + \frac{(\boldsymbol{q}-\boldsymbol{l})^2(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{-\mu_3} \\ &+ \frac{\boldsymbol{l}^2(\boldsymbol{k}_2+\boldsymbol{k}_3)^2}{\mu_1} - \frac{\boldsymbol{l}^2\boldsymbol{k}_2{}^2(\boldsymbol{q}-\boldsymbol{l})^2}{\mu_1(-\mu_3)} \Bigg) \frac{1}{(\mu_1-(\boldsymbol{l}-\boldsymbol{k}_1)^2+i\epsilon)(-\mu_3-(\boldsymbol{l}-\boldsymbol{k}_1-\boldsymbol{k}_2)^2+i\epsilon)} \\ &+ \frac{\boldsymbol{l}^2\boldsymbol{k}_3{}^2}{\mu_1(-\mu_3)} \frac{1}{\mu_1-(\boldsymbol{l}-\boldsymbol{k}_1)^2+i\epsilon} + \frac{(\boldsymbol{q}-\boldsymbol{l})^2\boldsymbol{k}_1{}^2}{\mu_1(-\mu_3)} \frac{1}{-\mu_3-(\boldsymbol{l}-\boldsymbol{k}_1-\boldsymbol{k}_2)^2+i\epsilon} \Bigg] , \end{aligned} \tag{6}$$

where $\mu_i = -l^+k_i^-$, $i = 1,2,3$ and $\mu_1 + \mu_2 + \mu_3 = 0$. The above expression needs to be supplemented by expressions which arise due to all permutations of the indices 1,2,3. Applying the above argument we define (the formally divergent) integral over l^+ by $\int dl^+ := \lim_{\Lambda\to\infty} \int_{-\Lambda}^{\Lambda} dl^+$ which puts (6) to zero. A similar results holds for the disconnected pieces.

5. The 2-to-4 transition vertex

The 2-to-4 transition is allowed by signature conservation. A building block of the complete 2-to-4 kernel, namely the Reggeon-Particle-2 Reggeons vertex, was derived from the effective action and brought into a purely transverse form in [4]. The complete unintegrated 2-to-4 vertex can be derived from (1), but the expression is too lengthy to be reproduced here. In contrast to (6), the l^+ integral is replaced by $\int dl^+/|l^+|$ and therefore the integral does not vanish by the previous argument, but leads to a logarithm in s. The integrals over the variables $\mu_i = -l^+ k_i^-$, $i = 1, 2, 3, 4$ can be performed using the principal value pole prescription, but the obtained result does not reproduce correctly all parts of the 2-to-4 vertex as derived in [3]. Therefore the pole prescription in terms of principal values needs to be generalized in a way that it reproduces all energy discontinuities of the 2-to-4 reggeon vertex. This is work in progress.

6. Conclusions

Using the principal value pole prescription we rederived the gluon trajectory and reproduced the 2-to-2 kernel in the symmetric color configuration. The anti-symmetric color configuration was shown to vanish if one takes into account that the effective action is formulated for central rapidities. By the same argument the 2-to-3 transition vanishes. To integrate out the longitudinal degrees of freedom of the 2-to-4 transition kernel, it seems to be necessary to generalize the principal value pole prescription to reproduce correctly all energy discontinuities.

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