

COLLINEAR IMPROVEMENT OF THE BFKL KERNEL IN THE ELECTROPRODUCTION OF TWO LIGHT VECTOR MESONS*

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We test the performance of a RG-improved kernel in the determination of the amplitude of a physical process, the electroproduction of two light vector mesons, in the BFKL approach at the next-to-leading approximation (NLA). We find that a smooth behavior of the amplitude with the center-of-mass energy can be achieved, setting the renormalization and energy scales appearing in the subleading terms to values much closer to the kinematical scales of the process than in approaches based on unimproved kernels.

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1. Introduction

It is well known that the NLA corrections to the BFKL [1] Green's function turn out to be very large, this being a signal of the bad behavior of the BFKL series. In order to "cure" the resulting instability, more convergent kernels have been introduced, including terms generated by renormalization group (RG), or collinear, analysis [2]. They are based on the ω -shift method, ω being the variable Mellin-conjugated to the squared center-of-mass energy s . In Ref. [3] this original approach has been revisited and an approximation to the original ω -shift has been performed, leading to an explicit expression for the RG-improved NLA kernel. It would be quite interesting to test the RG-improvement of the kernel in the calculation of a full physical amplitude. A test-field for this comparison can be provided

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by the physical process $\gamma^*\gamma^* \rightarrow VV$, where γ^* represents a virtual photon and V a light neutral vector meson (ρ^0, ω, ϕ). The amplitude of this reaction¹ has been calculated in Ref. [4] through the convolution of the (unimproved) BFKL Green's function with the $\gamma^* \rightarrow V$ impact factors, calculated in Ref. [6]. For this amplitude a smooth behavior in s could be achieved by "optimizing" the choice of the energy scale s_0 and of the renormalization scale μ_R , which appear in the subleading terms. The optimal values of the two energy parameters turned out to be quite far from the kinematical scales of the reaction, probably because they mimic the unknown next-to-NLA corrections, which should be large and of opposite sign respect to the NLA in order to preserve the renorm- and energy scale invariance of the exact amplitude. If this explanation is correct and if the RG-improvement of the kernel catches the essential dynamics from subleading orders, then, by the use of an RG-improved kernel, one should get more "natural" values for the optimal choices of the energy scales and, of course, results consistent with the previous determinations.

2. The NLA amplitude with the RG-improved Green's function: numerical results

We consider the production of two light vector mesons ($V = \rho^0, \omega, \phi$) in the collision of two virtual photons $\gamma^*(Q_1) \gamma^*(Q_2) \rightarrow V(p_1) V(p_2)$. The action of the modified BFKL kernel on his leading eigenfunctions is (the details of all the analytical calculations can be found in Ref. [7]):

$$\begin{aligned} \hat{K}|\gamma\rangle = & \bar{\alpha}_s(\mu_R)\chi(\gamma)|\gamma\rangle + \bar{\alpha}_s^2(\mu_R) \left(\chi^{(1)}(\gamma) + \frac{\beta_0}{4N_c}\chi(\gamma)\ln(\mu_R^2) \right) |\gamma\rangle \\ & + \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(\gamma) \left(-\frac{\partial}{\partial\gamma} \right) |\gamma\rangle + \chi_{\text{RG}}(\gamma)|\gamma\rangle, \end{aligned} \quad (1)$$

where the first term represents the action of LLA kernel, the second and the third ones stand for the diagonal and the non-diagonal parts of the NLA BFKL kernel [4] and

$$\begin{aligned} \chi_{\text{RG}}(\gamma) = & 2\text{Re} \left\{ \sum_{m=0}^{\infty} \left[\left(\sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^n n! (n+1)!} \frac{(\bar{\alpha}_s + a\bar{\alpha}_s^2)^{n+1}}{(\gamma+m-b\bar{\alpha}_s)^{2n+1}} \right) \right. \right. \\ & \left. \left. - \frac{\bar{\alpha}_s}{\gamma+m} - \bar{\alpha}_s^2 \left(\frac{a}{\gamma+m} + \frac{b}{(\gamma+m)^2} - \frac{1}{2(\gamma+m)^3} \right) \right] \right\} \end{aligned} \quad (2)$$

is the solution of the ω -shift equation obtained in [3], with

$$a = \frac{5}{12} \frac{\beta_0}{N_c} - \frac{13}{36} \frac{n_f}{N_c^3} - \frac{55}{36}, \quad b = -\frac{1}{8} \frac{\beta_0}{N_c} - \frac{n_f}{6N_c^3} - \frac{11}{12}. \quad (3)$$

¹ The same process has been analyzed, with different approaches, also in [5].

We present our numerical results for the dependence in s of the BFKL amplitude calculated for the process under study, using both the “exponentiated” and the “series” representations [4], equivalent within NLA accuracy. Following Ref. [4], we will adopt the principle of minimal sensitivity (PMS) [8] requiring, for each value of s , the minimal sensitivity of the predictions to the change of both the renormalization and the energy scales, μ_R and s_0 .

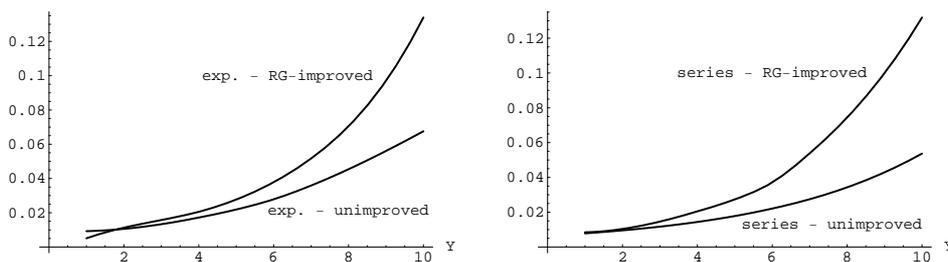


Fig. 1. $\text{Im}_s(\mathcal{A})Q^2/(sD_1D_2)$ as a function of Y at $Q^2=24\text{ GeV}^2$ and $n_f=5$ in the “exponentiated” (left-hand side) and “series” (right-hand side) representation with and without RG-improvement of the kernel.

2.1. Symmetric kinematics

We consider here the $Q_1 = Q_2 \equiv Q$ kinematics, *i.e.* the “pure” BFKL regime, with $Q^2 = 24\text{ GeV}^2$ and $n_f = 5$. We set $\ln(s/s_0) = Y - Y_0$, where $Y = \ln(s/Q^2)$ and $Y_0 = \ln(s_0/Q^2)$ and we have looked for the optimal value for the scales μ_R and Y_0 . We have found that for both representations the amplitude is always quite stable under variation of the scales and exhibits generally only one stationary point. We choose as optimal values of the parameters those corresponding to this stationary point. For the “exponentiated” representation the optimal values turned out to be typically $\mu_R \simeq 3Q$ and $Y_0 \simeq 2$ while for the “series” representation we have found $\mu_R \simeq 3Q$ and $Y_0 \simeq 3$. In comparison with Ref. [4], where the optimal choices were typically $Y_0 \simeq 2$ and $\mu_R \simeq 10Q$, we can see that there is a remarkable move towards “naturalness”. In Fig. 1 we show the results for the (imaginary part of the) “improved” amplitude in the two representations compared with the result obtained in Ref. [4]. Looking at the first plot, the curves are in good agreement at the lower energies, the deviation increasing for large values of Y . This is consistent with having a larger asymptotic intercept when the RG-improvements are taken into account. Moreover, when the condition $\bar{\alpha}_s(\mu_R)Y \sim 1$ is satisfied ($Y \sim 6$) the discrepancy is not so pronounced. In the case of the “series” representation (Fig. 1, second plot) the situation is similar to the previous one, but the deviation between the curves appears to be more marked here. We observe that both the curves for the amplitude

with RG-improvement fall almost on top of each other. This is a further indication of a better stability, induced by the RG-improvement.

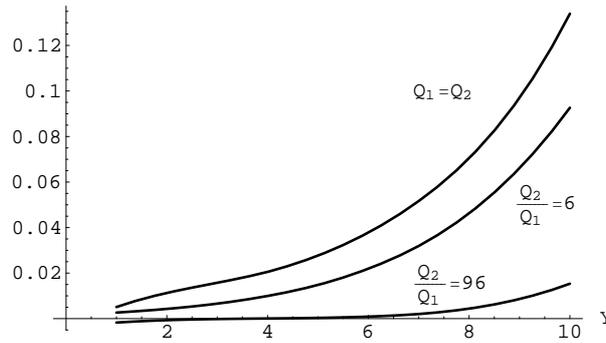


Fig. 2. $\text{Im}_s(\mathcal{A})Q_1Q_2/(sD_1D_2)$ as a function of Y for photons with strongly ordered virtualities ($Q_2/Q_1 = 6$ and $Q_2/Q_1 = 96$, with $Q_1Q_2 = 24 \text{ GeV}^2$), in comparison with the case of photons with equal virtualities ($Q_1^2 = Q_2^2 = 24 \text{ GeV}^2$).

2.2. Asymmetric kinematics

When the virtualities of the photons are strongly ordered, we enter the “DGLAP” regime, where RG-effects should come heavily into the game. In this regime, previous attempts to numerically determine the amplitude using unimproved kernels were unsuccessful due to severe instabilities [9]. We have found here that these instabilities disappear if, instead, the RG-improved kernel is used. In the numerical analysis to follow, we consider two choices for the virtualities of the photons, $Q_1 = 2 \text{ GeV}$, $Q_2 = 12 \text{ GeV}$ and $Q_1 = 0.5 \text{ GeV}$, $Q_2 = 48 \text{ GeV}$, so that $Q_1Q_2 = Q^2 = 24 \text{ GeV}^2$ in both cases, and used the “exponentiated” representation. We define $Y = \ln(s/Q_1Q_2)$ and $Y_0 = \ln(s_0/Q_1Q_2)$. For the first choice of virtualities, we find that for each Y value the amplitude is still quite stable under variation of the energy parameters and the optimal values are $\mu_R \simeq 4\sqrt{Q_1Q_2}$ and $Y_0 \simeq 2$, almost independently of Y . The same holds for the second choice of virtualities, with the only difference that now the optimal values depend strongly on Y . As an example, for $Y = 6$, when $\bar{\alpha}_s(\mu_R)Y \sim 1$, the optimal μ_R is $\simeq 3\sqrt{Q_1Q_2}$, but $Y_0 = 7$. This large value for Y_0 should not be surprising: if we use Q_2^2 as normalization scale in Y_0 instead of Q_1Q_2 , the optimal value lowers down ~ 2.5 , which looks more “natural”. In Fig. 2 we plot the amplitude for the two choices of photons’ virtualities we have considered, together with the amplitude for $Q_1 = Q_2 = \sqrt{24} \text{ GeV}$. The amplitude becomes smaller and smaller when Q_2/Q_1 increases, as it must be expected due to the presence of the factor $\cos(\nu \log(Q_2^2/Q_1^2))$ [7] in the integration over ν .

3. Conclusions

We have applied a RG-improved kernel to determine the amplitude for the forward transition from two virtual photons to two light vector mesons in the Regge limit of QCD with next-to-leading order accuracy. The result obtained is independent on the energy scale s_0 , and on the renormalization scale μ_R within the next-to-leading approximation. Using two different representations of the amplitude, we have performed a numerical analysis both in the kinematics of equal and strongly ordered photons' virtualities. An optimization procedure, based on the principle of minimal sensitivity, has led to results stable in the considered energy interval, which allow to predict the energy behavior of the forward amplitude. The important finding is that the optimal choices of s_0 and μ_R are much closer to the kinematical scales of the problem than in previous determinations based on unimproved kernels. This leads us to conclude that the extra-terms in the BFKL kernel coming from RG-improvement, which are subleading to the NLA, catch an important fraction of the dynamics at higher orders. Moreover, the use of the improved kernel has allowed to obtain the energy behavior of the forward amplitude in the case of strongly ordered photons' virtualities, which turned out to be inaccessible to previous attempts using unimproved kernels.

REFERENCES

- [1] V.S. Fadin, E.A. Kuraev, L.N. Lipatov, *Phys. Lett.* **B60**, 50 (1975); E.A. Kuraev, L.N. Lipatov, V.S. Fadin, *Zh. Eksp. Teor. Fiz.* **71**, 840 (1976) [*Sov. Phys. JETP* **44**, 443 (1976)]; **72**, 377 (1977) [**45**, 199 (1977)]; Ya.Ya. Balitskii, L.N. Lipatov, *Sov. J. Nucl. Phys.* **28**, 822 (1978).
- [2] G.P. Salam, *J. High Energy Phys.* **9807**, 019 (1998).
- [3] A. Sabio Vera, *Nucl. Phys.* **B722**, 65 (2005).
- [4] D.Yu. Ivanov, A. Papa, *Nucl. Phys.* **B732**, 183 (2006); D.Yu. Ivanov, A. Papa, *Eur. Phys. J.* **C49**, 947 (2007).
- [5] B. Pire, L. Szymanowski *Eur. Phys. J.* **C44**, 545 (2005); R. Enberg, B. Pire, S. Wallon, *Eur. Phys. J.* **C45**, 759 (2006), Erratum: **C51**, 1015 (2007); B. Pire, M. Segond, L. Szymanowski *Phys. Lett.* **B639**, 642 (2006).
- [6] D.Yu. Ivanov, M.I. Kotsky, A. Papa, *Eur. Phys. J.* **C38**, 195 (2004).
- [7] F. Caporale, A. Papa, A. Sabio Vera, *Eur. Phys. J.* **C53**, 525 (2008).
- [8] P.M. Stevenson, *Phys. Lett.* **B100** (1981) 61; *Phys. Rev.* **D23**, 2916 (1981).
- [9] D.Yu. Ivanov, private communication.