# A TEST OF THE BFKL RESUMMATION AT ILC* 

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We consider the exclusive production of $\rho^{0}$ meson pairs in $\gamma^{*} \gamma^{*}$ scattering in the Regge limit of QCD as a probe of BFKL resummation effects and we show the feasibility of the measurement of this process at the ILC.

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## 1. Collinear and $k_{t}$ factorizations of the process

In the high-energy (Regge) limit, when the cm energy $s_{\gamma^{*} \gamma^{*}}$ is much larger than all other scales of the process, large logarithms of $s_{\gamma^{*} \gamma^{*}}$ emerge and are resummed by the BFKL equation [1]. It describes a $t$-channel hard pomeron exchange, made of a gluonic effective ladder and carrying the quantum numbers of the vacuum. The highly virtual photons collision is a very clean process to probe the BFKL effects since it provides small transverse size objects ( $q \bar{q}$ color dipoles) which overcome the theoretical problem arising from diffusion of the transverse momenta in the BFKL equation, at least in non asymptotical $s_{\gamma^{*} \gamma^{*}}$. We can select events with comparable photon virtualities to avoid the partonic evolution of DGLAP [2] type. Several studies [3]

[^0]have been performed at the level of the $\gamma^{*} \gamma^{*}$ total cross-section and $J / \Psi$ meson pairs production in $\gamma \gamma$ collisions. Here we focus on the exclusive process $\gamma_{\mathrm{L}, \mathrm{T}}^{*}\left(q_{1}\right) \gamma_{\mathrm{L}, \mathrm{T}}^{*}\left(q_{2}\right) \rightarrow \rho_{\mathrm{L}}^{0}\left(k_{1}\right) \rho_{\mathrm{L}}^{0}\left(k_{2}\right)$ (see Fig. 1) through $e^{+} e^{-} \rightarrow e^{+} e^{-} \rho_{\mathrm{L}}^{0} \rho_{\mathrm{L}}^{0}$ with double tagged outgoing leptons. The $k_{t}$-factorization of the scattering amplitude, valid at high energy, has the form of a convolution in the transverse momentum $\underline{k}$ space between the two impact factors, corresponding to the transition of $\gamma_{\mathrm{L}, \mathrm{T}}^{*}\left(q_{i}\right) \rightarrow \rho_{\mathrm{L}}^{0}\left(k_{i}\right)$ via the $t$-channel exchange of two reggeized gluons (with momenta $\underline{k}$ and $\underline{r}-\underline{k}$ ). The virtualities $\left(Q_{i}^{2}=-q_{i}^{2}\right)$ of the photons supply the hard scale which justifies the use of perturbation theory to compute in the collinear factorization the hard part of each impact factor, convoluted with the soft part (encoding the hadronization into the final states $\rho$ mesons) which is given by the corresponding leading twist distribution amplitude (DA) [4].


Fig. 1.

## 2. Non-forward Born order cross-section

We display in Fig. 2 the non-forward Born order cross-sections as a function of the momentum transfer $t$ for the different $\gamma^{*}$ polarizations, having performed analytically the integrations over $\underline{k}$ (using conformal transformations to reduce the number of massless propagators) and numerically the integration over the accessible phase space [5]. We then obtained the corresponding cross-section of the process $e^{+} e^{-} \rightarrow e^{+} e^{-} \rho_{\mathrm{L}}^{0} \rho_{\mathrm{L}}^{0}$ in the planned experimental conditions of the International Linear Collider (ILC). We focus on the LDC detector project and we use the potential of the very forward region accessible through the electromagnetic calorimeter BeamCal. Following the requirements of Regge kinematics, we fix the value of $s_{\gamma^{*}} \gamma^{*}$ on the gluon exchange dominance over the quark exchange contribution calculated in [6]. With the foreseen energy of the collider $\sqrt{s}=500 \mathrm{GeV}$ and nominal
integrated luminosity of $125 \mathrm{fb}^{-1}$, this will yield around $4 \times 10^{3}$ events per year, depending on the theoretical assumptions we have made (scale dependence of $\alpha_{s}$, value of the parameter that controls the Regge kinematics and expansion of DAs).


Fig. 2. $e^{+} e^{-} \rightarrow e^{+} e^{-} \rho_{\mathrm{L}}^{0} \rho_{\mathrm{L}}^{0}$ cross-sections.

## 3. Forward differential cross-section with BFKL evolution

The results obtained at Born approximation can be considered as the starting point for evaluation of the cross-section for $\rho^{0}$ mesons pairs production with complete BFKL evolution taken into account. We first evaluate BFKL evolution in the leading logarithms approximation (LL) which dramatically enhances (by several orders of magnitude) the cross-section (and also the theoretical uncertainties coming mainly from the definition of the rapidity and the scale dependence of $\alpha_{\mathrm{s}}$ ) when increasing $\sqrt{s}$, because of the large value of the LL BFKL Pomeron intercept.

The next-to-leading logarithms (NLL) BFKL evolution will widely reduce both this enhancement and uncertainties, which is essential to make precise predictions. The full NLL cross-section [7], with both impact factors and BFKL kernel computed in the NLL accuracy, can even be lower at moderate values of $s_{\gamma^{*} \gamma^{*}}$ than its Born order approximation. We use the renormalization group improved BFKL kernel [8] (convoluted with LL impact factors) to estimate the NLL differential cross-section of $\gamma^{*} \gamma^{*} \rightarrow \rho_{L}^{0} \rho_{L}^{0}$, which gives a good agreement with the full NLL evolution obtained in [7], as we can see in Fig. 3. In the approach of Ref. [9], we must find the solutions (the NLL Pomeron intercept and the anomalous dimension) of a set of two coupled equations (coming from the saddle point approximation and the


Fig. 3. Cross-sections at $t=t_{\text {min }}$ for $\gamma^{*} \gamma^{*} \rightarrow \rho_{\mathrm{L}}^{0} \rho_{\mathrm{L}}^{0}$ with full NLL BFKL evolution (upper curve, black) [7] and (this work) collinear improved BFKL evolution (lower curve, red) for $Q_{1}=Q_{2}=2 \mathrm{GeV}$ and three quark flavors.
residue of the integral over $\omega$, the Mellin moment of $s_{\gamma^{*} \gamma^{*}}$ ). Although this approach uses a fixed strong coupling, we reconstruct in $\omega_{\mathrm{s}}$ and $\gamma_{\mathrm{s}}$ a scale dependence by fitting with polynomials of $Q_{i}$ a large range of solutions obtained for various values of $\alpha_{\mathrm{s}}\left(\sqrt{Q_{1} Q_{2}}\right)$. Our results are now much less sensitive to the various theoretical asumptions than the ones obtained at LL accuracy. Having integrated over the accessible phase space of this reaction at ILC, we compare in Fig. 4 the curves at Born order (green) with the


Fig. 4. Cross-sections at $t=t_{\text {min }}$ for $e^{+} e^{-} \rightarrow e^{+} e^{-} \rho_{\mathrm{L}}^{0} \rho_{\mathrm{L}}^{0}$ with collinear improved BFKL evolution (upper curve, red) and at Born order (lower curve, green).
(red) one obtained after collinear improved BFKL resummation. The experimental cut imposed by the resolution of the electromagnetic calorimeter BeamCal is responsible for the fall of the cross-sections with $\sqrt{s}$ increasing from 500 GeV . This NLL evolution gives an enhancement of the Born approximation by a factor 4.5 , which allows us to definitively conclude of the measurability of the BFKL evolution for this process at ILC. We finally mention that increasing the collider energy from 500 GeV to 1 TeV will probably lead to a transition between the linear and the saturated regime ( $Q_{\text {sat }} \sim 1.4 \mathrm{GeV}$ for $\left.\sqrt{s}=1 \mathrm{TeV}\right)$.

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