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LETTERS TO THE EDITOR

A NOTE ON THE LAGRANGIAN DENSITY FOR FLUID SYSTEMS IN GENERAL RELATIVITY II

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A generalization of a previously discussed variational principle for perfect fluids in general relativity is presented. By the addition of two new constraints to the variational principle the extremals describe general motions of a perfect fluid.

Some time ago in this journal we presented a variational principle for perfect fluids in general relativity [1]. This variational principle was not completely general because it applied only to isentropic, irrotational motions of the fluid. The fact that the flow was assumed isentropic was contained in Eq. (I, 3) where it was assumed that the fluid had only one thermodynamic degree of freedom ϱ. The fact that the motion was irrotational is contained in Eqs (I, 6, 7, 8) where if one solves Eq. (6c) one obtains

$$U_i = -c^3(F'(\varrho))^{-1}\lambda_{2,i}. \tag{1}$$

Calculating the rotational angular velocity ω^i [2] one obtains

$$\omega^i = \tfrac{1}{2}\,\eta^{ijkl}U_jU_{k,l} = 0. \tag{2}$$

One can generalize the variational principle so that it describes general motions of a perfect fluid. One needs to introduce two new constraint equations

$$X_{,i}U^i = 0, \tag{3}$$

$$s_{,i}U^i = 0. \tag{4}$$

Here $X(x)$ associates a number with each fluid particle and Eq. (3) requires that this number not change as we move along with the particle. This is called the law of conservation of particle identity and has the consequence that isentropic flow need not be irrotational. This

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same technique is used in classical hydrodynamics [3]. Eq. (4) is just the statement that heat is not exchanged between fluid particles in a perfect fluid therefore the entropy of a given fluid particle is constant. This removes the restriction that the flow must be isentropic. The new Lagrangian density is Eq. (I, 5), L^*, plus the two new constraints represented by Eqs (3, 4)

$$L = L^* + \sqrt{-g}\,\lambda_3 X_{,i}U^i + \sqrt{-g}\,\lambda_4 s_{,i}U^i. \tag{5}$$

Variation of the action associated with L with respect to g_{ik}, ϱ, s, U^i, λ_1, λ_2, λ_3, λ_4 again yields Eqs (I, 7, 8, 10, 11) where $F(\varrho)$ is replaced by $F(\varrho, s)$ and $F'(\varrho)$ is replaced by

$$F'(\varrho, s) = \left(\frac{\partial F}{\partial \varrho}\right)_s. \tag{6}$$

Therefore, the generalized variational principle yields the same field equations, Eqs (I, 10, 19), for a perfect fluid in general relativity except now, P, ε are functions of both ϱ and s. The extremals of our generalized variational principle describe general motions of the perfect fluid. Details will be published elsewhere [4].

REFERENCES

[1] J. Ray, *Acta Phys. Polon.*, **30**, 481 (1966). This paper will be referred to as I hereafter. We shall label the formulas of I as Eq. (I,—).

[2] J. Synge, *Relativity: The General Theory*, North-Holland Pub. Co., Amsterdam 1964, page 172.

[3] C. Lin, in *Liquid Helium*, Proceedings of the International School of Enrico Fermi, course 21, editor G. Careri, Academic Press, New York 1963.

[4] J. Ray, *J. Math. Phys.*, to be published.