ON THE INDEPENDENCE OF THE QUARK MODEL CLASS (A) RELATIONS FOR THE REACTIONS $0^- \frac{1}{2}^+ \to 1^- \frac{3}{2}^+$

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It is shown that class (a) relations following from the additive quark model for the reaction $0^- \frac{1}{2}^+ \to 1^- \frac{3}{2}^+$ are not independent because some of the joint statistical tensors satisfy bilinear kinematical relations.

A large amount of information has recently became available on the angular decay distributions of resonances produced in high energy collisions of hadrons [1, 2].

Some interesting theoretical results on double resonance production have been obtained in the quark model [3, 4], especially the relations between the joint statistical tensors [5] of the decaying resonances. The so-called class (a) quark model relations which follow from the additivity alone [3] are in a very good agreement with the experimental data [1, 3].

In this note we investigate the reaction

$$PB \to VB^*$$

where P and V are pseudoscalar and vector mesons, B stands for a $\frac{1}{2}^+$ baryon and B^* is a $\frac{3}{2}^+$ isobar. We show that there exist *a priori* some bilinear relations between the density matrix elements of the produced resonances and consequently between the statistical tensors.

Our relations enable us to establish that not all class (a) predictions are independent. In fact, it follows from our bilinear relations that the class (a) relations

$$T^{22}_{00} = \frac{1}{2\sqrt{6}} - \frac{1}{\sqrt{2}} T^{02}_{00}, \quad T^{20}_{00} = \sqrt{2}\, T^{02}_{00} \tag{1}$$

imply another class (a) relation between even-even tensors

$$T^{22}_{02} = \frac{1}{\sqrt{2}} T^{02}_{02} \tag{2}$$

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and two class (a) relations between even-odd tensors

$$T^{23}_{13} = 0, \quad T^{23}_{1-3} = 0. \tag{3}$$

Here $T^{J_1J_2}_{M_1M_2}$ are the statistical tensors in the transversity frame [5]. The first index J_1 refers to V and the second J_2 refers to B^*. Furthermore, if relations (1) are satisfied there exist two additional relations between even-odd tensors, namely

$$\sqrt{2}\, T^{23}_{02} - T^{03}_{02} = 0 \tag{4}$$

and

$$3\left(T^{21}_{00} - \frac{1}{\sqrt{2}} T^{01}_{00}\right) + T^{23}_{00} - \frac{1}{\sqrt{2}} T^{03}_{00} = 0. \tag{5}$$

In other words, two class (a) relations

$$T^{23}_{02} = 0, \quad T^{03}_{02} = 0$$

are linearly dependent, and similarly three class (a) relations

$$T^{21}_{00} = \frac{1}{\sqrt{2}} T^{01}_{00}, \quad T^{23}_{02} = 0, \quad T^{03}_{02} = 0$$

are also linearly dependent if only relations (1) are fulfilled. Thus, it is evident that of 16 class (a) relations between even-even and even-odd statistical tensors only 11 are independent.

In order to obtain our bilinear relations we examine the structure of the density matrix $\varrho^{cc'}_{dd'}$ of the VB^* system produced on an unpolarized target. Here the indices c, c' and d, d' stand for spin projections of V and B^* on the normal to the scattering plane.

It follows from the parity conservation in the production process that $\varrho^{cc'}_{dd'} = 0$ unless $c-c'+d-d'$ is even. Using the arguments quite analogous to those of Ademollo, Gatto and Preparata [6], it is easy to show that the density matrix $\varrho^{cc'}_{dd'}$ splits into a direct sum of two submatrices

$$\varrho = \varrho_+ \oplus \varrho_- = \left(\begin{array}{c|c} \varrho_+ & 0 \\ \hline 0 & \varrho_- \end{array}\right).$$

The submatrices $\varrho_\pm$ consist of elements $\varrho^{cc'}_{dd'}$, for which $(-1)^{s_c - c} = \pm(-1)^{s_d - d}$. Each element $\varrho^{cc'}_{dd'}$ of the submatrix ϱ_+ corresponds to an element $\varrho^{-c'-c}_{-d'-d}$ of the submatrix ϱ_-. In our particular case amplitudes with initial baryon transversity equal to $+\frac{1}{2}$ contribute only to ϱ_+ and amplitudes with initial baryon transversity equal to $-\frac{1}{2}$ contribute only to ϱ_-. There is no summation over initial transversities at all.

If we express the density matrix in terms of the statistical tensors, we see immediately that even-even and odd-odd statistical tensors contribute only to the sum $\varrho_+ + \varrho_-$ whereas even-odd and odd-even tensors contribute only to the difference $\varrho_+ - \varrho_-$. Thus, given an element $\varrho^{cc'}_{dd'}$ of ϱ_+ in the form of a linear combination of the statistical tensors, we can obtain an expression for $\varrho^{-c'-c}_{-d'-d}$ of ϱ_- by changing signs of all even-odd and odd-even tensors. This results from the obvious relations

$$\varrho_+ = \tfrac{1}{2}[(\varrho_+ + \varrho_-) + (\varrho_+ - \varrho_-)],$$
$$\varrho_- = \tfrac{1}{2}[(\varrho_+ + \varrho_-) - (\varrho_+ - \varrho_-)].$$

Since the target baryon can have only two different spin projections $\pm\frac{1}{2}$, the density matrix of the target is of rank 2. The essential point in this discussion is that because of this fact the rank of the final density matrix cannot exceed 2. Furthermore,

$$\mathrm{rank}\ (\varrho) = \mathrm{rank}\ (\varrho_+) + \mathrm{rank}\ (\varrho_-)$$

and neither ϱ_+ nor ϱ_- vanishes identically. This implies that each of the submatrices ϱ_+ and ϱ_- is of rank 1. Thus all the 2×2 minors of ϱ_+ and ϱ_- must vanish.

In this way we obtain a set of bilinear relations between the density matrix elements and consequently between joint statistical tensors. Unfortunately only some of the statistical tensors $T^{J_1J_2}_{M_1M_2}$ can be evaluated from the angular decay distributions in our case, namely those with even J_1 and even J_2 or (for some reactions as Y_1^* (1385) production) odd J_2 [7, 4]. It is impossible to measure tensors with odd J_1 in our reaction neither from the three-body decay distributions [8] nor by method proposed by Chung [9] for sequential decays of the boson resonance.

Only four of the above-mentioned relations contain even-even and even-odd tensors alone:

two relations for the submatrix ϱ_+

$$\varrho^{00}_{11}\varrho^{00}_{-3-3} - |\varrho^{00}_{-31}|^2 = 0, \tag{6}$$

$$\varrho^{-+}_{33}\varrho^{-+}_{-1-1} - \varrho^{-+}_{3-1}\varrho^{-+}_{-13} = 0 \tag{7}$$

and two relations for the submatrix ϱ_-

$$\varrho^{00}_{33}\varrho^{00}_{-1-1} - |\varrho^{00}_{-13}|^2 = 0, \tag{8}$$

$$\varrho^{-+}_{11}\varrho^{-+}_{-3-3} - \varrho^{-+}_{1-3}\varrho^{-+}_{-31} = 0. \tag{9}$$

Here we write for the spin indices d, d' twice the spin projections. Thus for instance ϱ^{-+}_{3-1} corresponds to $c = -1$, $c' = 1$, $d = \frac{3}{2}$, $d' = -\frac{1}{2}$.

The relations for statistical tensors equivalent to relations (6) and (7) are:

$$\left(\frac{3}{\sqrt{5}}T^{23}_{00} + T^{22}_{00} - \frac{1}{\sqrt{5}}T^{21}_{00} - T^{20}_{00} - \frac{3}{\sqrt{10}}T^{03}_{00} - \frac{1}{\sqrt{2}}T^{02}_{00} + \frac{1}{\sqrt{10}}T^{01}_{00} + \frac{1}{2\sqrt{6}}\right)\times$$
$$\times\left(\frac{1}{\sqrt{5}}T^{23}_{00} - T^{22}_{00} + \frac{3}{\sqrt{5}}T^{21}_{00} - T^{20}_{00} - \frac{1}{\sqrt{10}}T^{03}_{00} + \frac{1}{\sqrt{2}}T^{02}_{00} - \frac{3}{\sqrt{10}}T^{01}_{00} + \frac{1}{2\sqrt{6}}\right) -$$
$$- |\sqrt{2}\,T^{23}_{02} - \sqrt{2}\,T^{22}_{02} - T^{03}_{02} + T^{02}_{02}|^2 = 0,$$

$$\left(\frac{1}{\sqrt{5}}T^{23}_{20} + T^{22}_{20} + \frac{3}{\sqrt{5}}T^{21}_{20}\right)\left(\frac{3}{\sqrt{5}}T^{23}_{20} - T^{22}_{20} - \frac{1}{\sqrt{5}}T^{21}_{20}\right) -$$
$$- 2(T^{23}_{2-2} + T^{22}_{2-2})(T^{23}_{22} + T^{22}_{22}) = 0.$$

One can obtain two remaining relations by changing signs of all even-odd tensors. It should be emphasized that our relations do not depend on any dynamical assumptions.

Similar relations have been derived by Białas and Kotański [10], Ringland and Thews [11] and Thews [12] under specific dynamical assumptions, namely the exchange of a define set of quantum numbers in the t-channel and consequently, the common phase of all helicity amplitudes.

Now, we obtain our result on the dependence of the class (a) quark model predictions in the following way: The diagonal density matrix elements ϱ^{00}_{-3-3} and ϱ^{00}_{33} must be nonnegative and the relations (1) imply that their sum $\varrho^{00}_{-3-3}+\varrho^{00}_{33}$ vanishes. Thus, both ϱ^{00}_{-3-3} and ϱ^{00}_{33} must vanish. Consequently, we can write the relation

$$\varrho^{00}_{-3-3}-\varrho^{00}_{33} = 0$$

which is equivalent to the constraint (5).

Moreover, it follows from the relations (6) and (8) that also $\varrho^{00}_{-31} = 0$ and $\varrho^{00}_{-13} = 0$, thas is

$$\varrho^{00}_{-31}+\varrho^{00}_{-13} = 0, \qquad \varrho^{00}_{-31}-\varrho^{00}_{-13} = 0.$$

The last relations are just relations (2) and (4). To get equations (3) we use in the similar manner relations

$$\varrho^{++}_{33}\varrho^{00}_{-3-3}-|\varrho^{0+}_{-33}|^2 = 0, \qquad \varrho^{00}_{-3-3}\varrho^{--}_{33}-|\varrho^{-0}_{3-3}|^2 = 0$$

and the corresponding relations for the submatrix ϱ_{-}.

A similar analysis can be performed directly by expressing statistical tensors in terms of transversity amplitudes. One can see that relations (1) imply the vanishing of the double transversity flip amplitude, as noted by Gizbert-Studnicki, Golemo and Zalewski [13]. As a consequence some of the density matrix elements must vanish, and this imposes constraints (2), (3), (4) and (5) on the statistical tensors.

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